Reduction of the Supervisor Design Problem with Firing Vector Constraints

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Context

PNs developed in Computer Science to model concurrent processes (CPs).

Mutual exclusion: a common constraint type imposed on CPs.

Expressed in PNs by: $\sum_i \mu_i \leq c$.

Used also in the context of AGV coordination for manufacturing systems [Krogh and Holloway 1991] and batch chemical processes [Tittus and Lenartson, 1999].

More complex constraints $L\mu \leq b$ are necessary (L, b integer matrices)
- due to uncontrollability and unobervability
- to express more complex specifications

The constraints $L\mu \leq b$ have been used to describe liveness enforcing supervisors [Iordache and Antsaklis, 2000; Park and Reveliotis, 2002].

Context

In the context of chemical process control [Yamalidou and Kantor 1991], constraints describing how valves should be opened/closed result in:

$$L\mu + Hq \leq b.$$

The same arise also in railway networks [Giua and Seatzu 2001], describing safety constraints (trains should not collide).

The more general form

$$L\mu + Hq + Cv \leq b$$

used in a rather complex AGV coordination problem [Iordache and Antsaklis, 2003].

Fairness constraints of the form

$$Cv \leq b$$

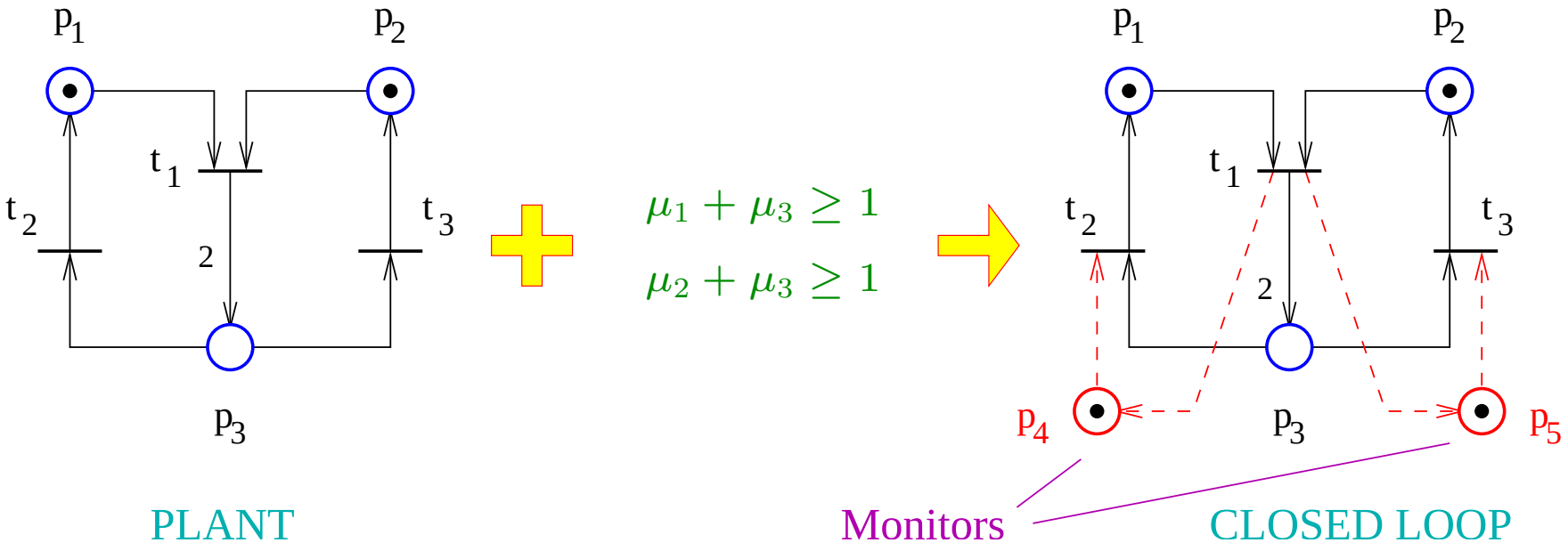
were used for a communication protocol [Genrich et al, 1980] and a manufacturing application [Li and Wonham, 1993].

The *supervision based on place invariants* developed for constraints $L\mu \leq b$.

(All reachable markings μ constrained to satisfy $L\mu \leq b$.)

Let D be the incidence matrix and μ_0 the initial marking.

The supervisor consists of *monitor places* connected according to $D_s = -LD$ and of initial marking $\mu_{s0} = b - L\mu_0$.

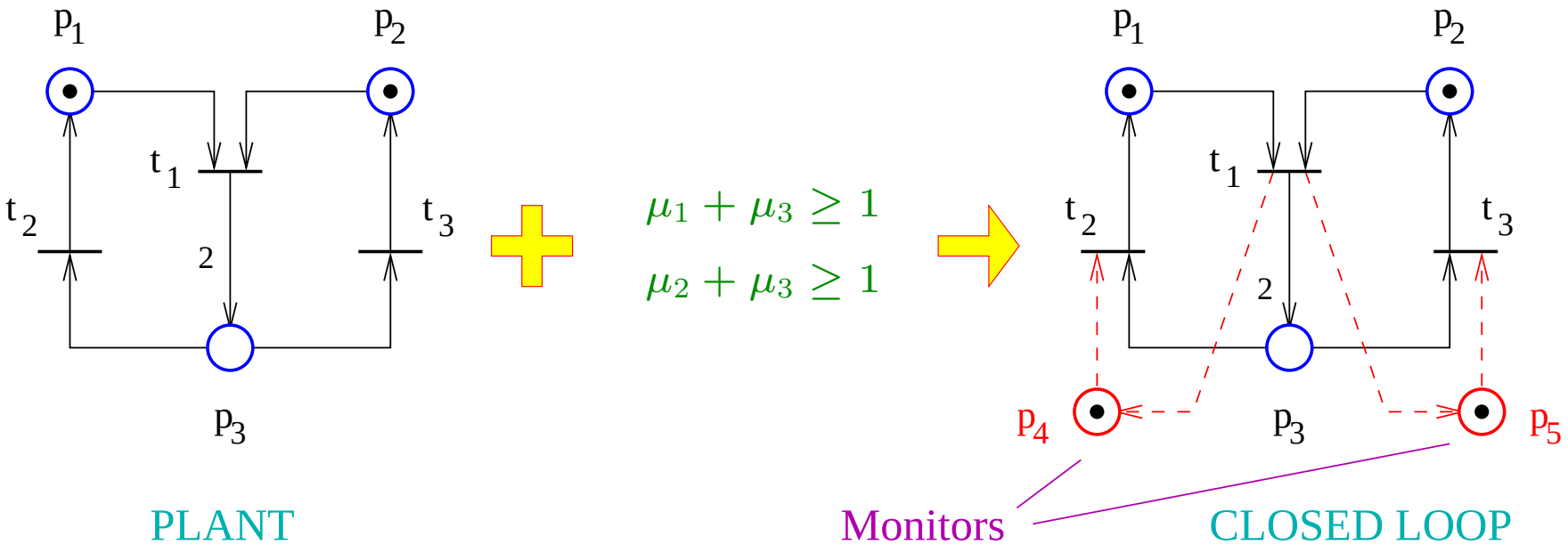


Conversely, given a set of monitors, what specification do they enforce?

$L\mu + Hq + Cv \leq b$, in general.

- The Cv term not needed if they participate in place invariants.

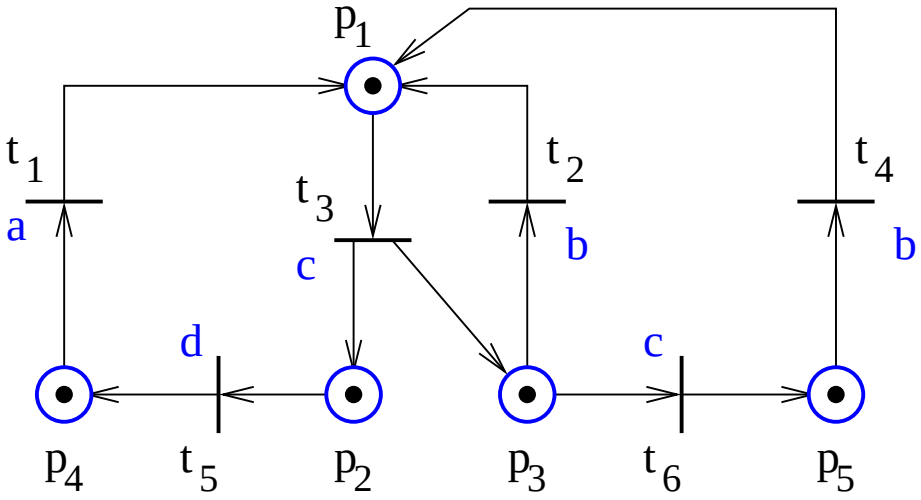
- The Hq term needed if they have selfloops.



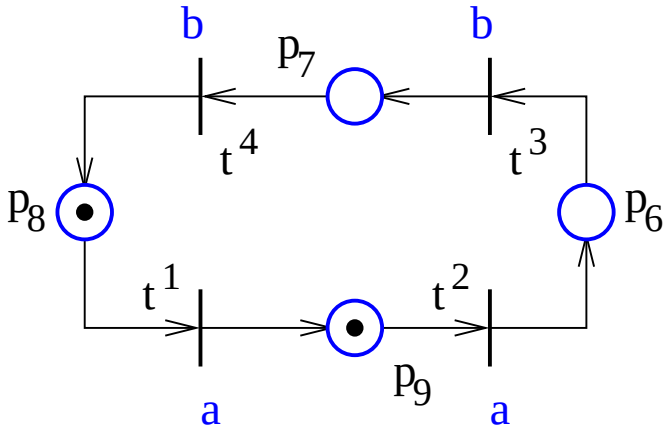
It is desirable to be able to deal with even more general specifications.

For what kind of specifications is the closed-loop still a Petri net?

- Prefix type language specifications.
- In particular, disjunctions $\bigvee_i L_i\mu \leq b_i$, under some boundedness assumptions.



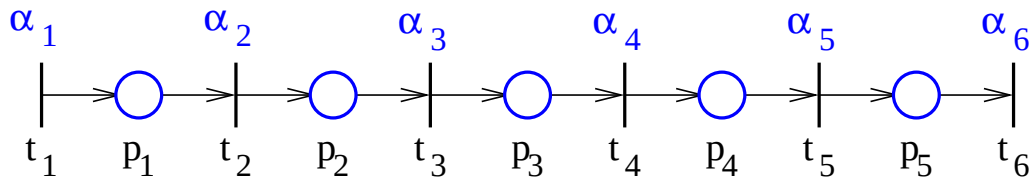
PLANT



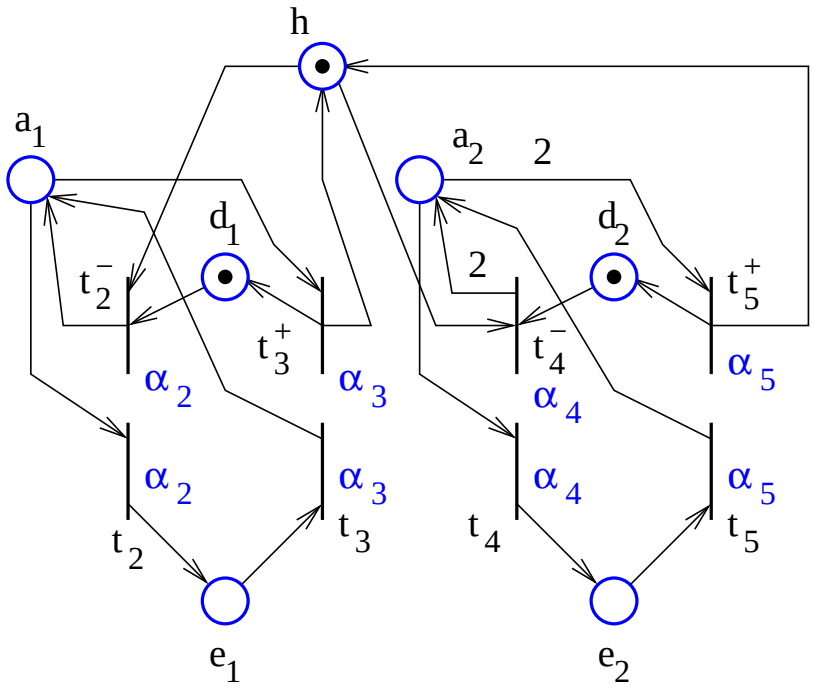
SPECIFICATION

As an example, this is the plant. The specification is

$$[\mu_2 \leq 0] \vee [\mu_4 \leq 0].$$



Assuming the bounds $\mu_2 \leq 2$ and $\mu_4 \leq 3$, this supervisor results:



Note that the supervisor is not free-labeled (though the plant is).

Difficulties arise due to partial uncontrollability and unobservability.

We have focused on structural and *suboptimal* methods of supervisor design.

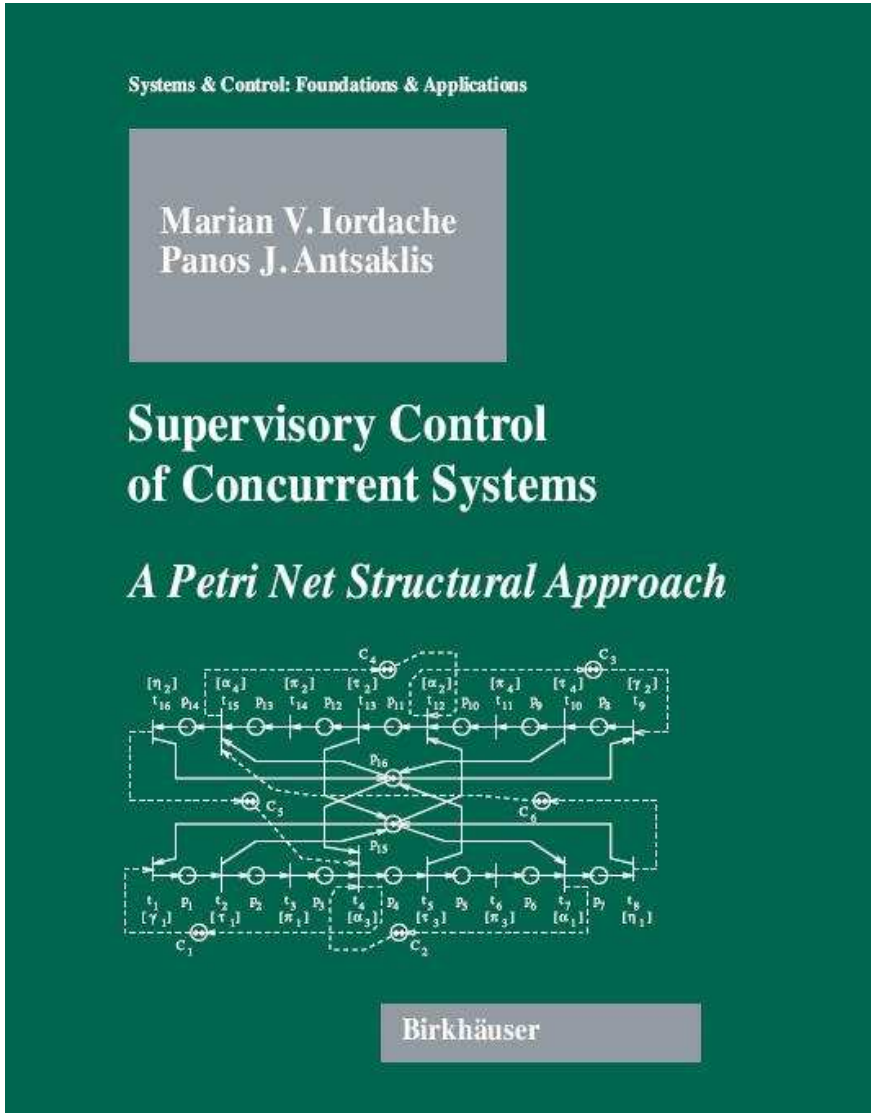
- Not necessarily least restrictive solutions; a solution may not be found even when one exists.
- The deadlock prevention problem needs to be dealt with separately.

+ The supervisor is parametrized by the initial marking μ_0:
  + a numerical value of μ_0 is not necessary to design a supervisor;
  + changes in the numerical value of μ_0 do not require recalculating the supervisor.
+ Potential computational benefits (reachability analysis avoided).

We have addressed:

- Centralized and decentralized control.
- Labeled PN plants with or without concurrency.
- Disjunctive, P-type language, generalized linear constraints. Also liveness specifications.

BOOK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Marian V. Iordache
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Supervisory Control
of Concurrent Systems
A Petri Net Structural Approach
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TOPIC

This presentation considers the supervision problem for specifications

$$\bigvee_{i=1}^{n_d} [L_i \mu + H_i q + C_i v \leq b_i] \tag{1}$$

By adding sink places, the Cv term can be incorporated in the $L\mu$ term.

Therefore, it is enough to focus on constraints

$$\bigvee_{i=1}^{n_d} [L_i \mu + H_i q \leq b_i] \tag{2}$$

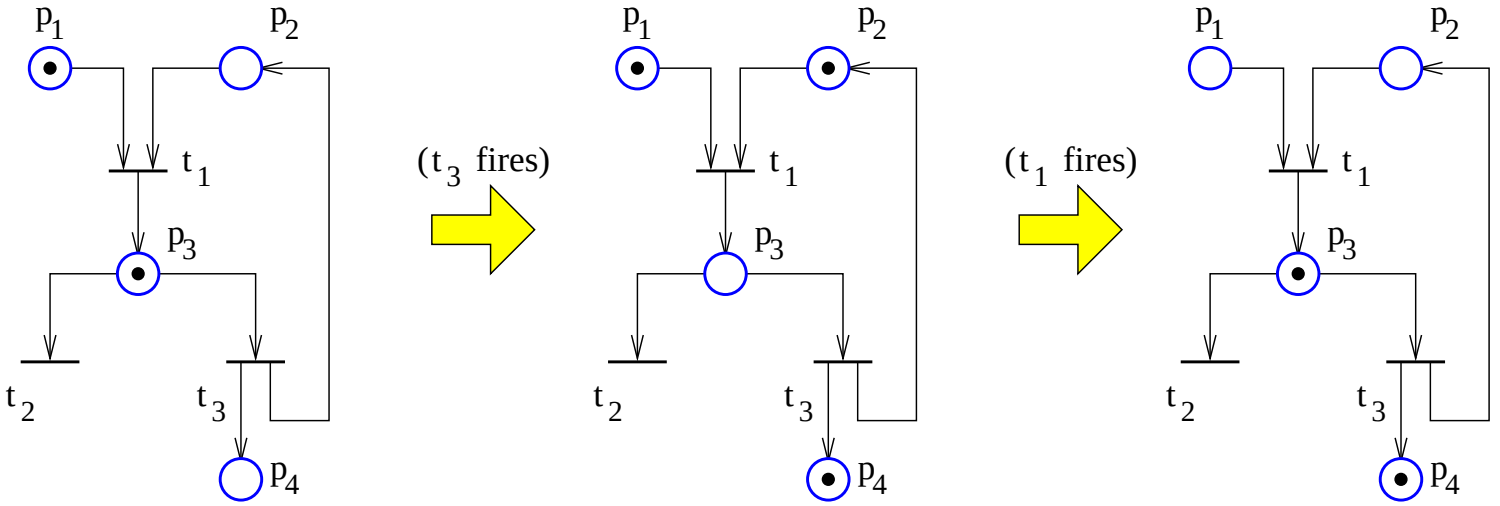
The paper shows that the problem of enforcing (2) can be reduced to the problem of enforcing (3) on a transformed Petri net, *without loss of permissiveness.*

$$\bigvee_{i=1}^{n_d} [L_{H,i} \mu_H \leq b_i] \tag{3}$$

Notation: μ – the marking, μ_0 – the initial marking, D – the incidence matrix, q – the firing vector, and v – the Parikh vector. Let $\mu_i = \mu(p_i)$, $q_j = q(t_j)$ and $v_j = v(t_j)$.

q_j: how many times t_j is to be fired; v_j: how many times t_j has been fired.

The state equation: $\mu = \mu_0 + Dv$.



$$\mu_0 = [\ 1 \quad 0 \quad 1 \quad 0\ ]^T \qquad \mu' = [\ 1 \quad 1 \quad 0 \quad 1\ ]^T \qquad \mu'' = [\ 0 \quad 0 \quad 1 \quad 1\ ]^T$$
$$v = [\ 0 \quad 0 \quad 0\ ]^T \qquad v = [\ 0 \quad 0 \quad 1\ ]^T \qquad v = [\ 1 \quad 0 \quad 1\ ]^T$$
$$q = [\ 0 \quad 0 \quad 1\ ]^T \qquad q = [\ 1 \quad 0 \quad 0\ ]^T$$

The *generalized linear constraints* can describe places arbitrarily connected to a PN.

They have the form:

$$L\mu + Hq + Cv \leq b \tag{4}$$

They require the initial state (μ_0, v_0) to satisfy

$$L\mu_0 + Cv_0 \leq b$$

Further, a transition t_i may fire from a current state (μ, v) iff

(a) $L\mu + Hq + Cv \leq b$ for $q(i) = 1$ and $q(j) = 0\ \forall j \neq i$.

(b) $L\mu' + Cv' \leq b$, where $v' = v + q$ and $\mu \xrightarrow{t_i} \mu'$.

The generalized linear constraints describe the P-type languages of free-labeled PNs.

The enforcement of the *generalized linear constraints* has been studied by Iordache and Antsaklis [ACC 2002, TAC 48(11)].

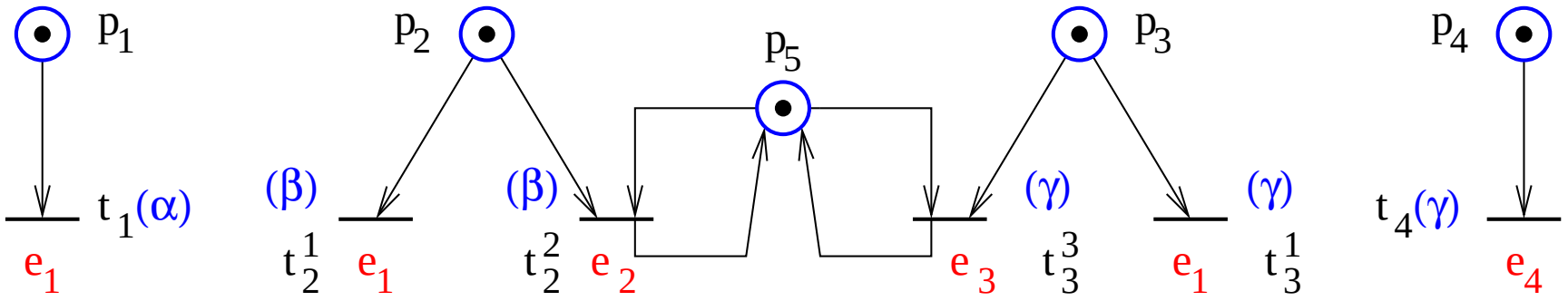
Setting

The plant is assumed to be a *double-labeled PN*. $\rho : T \rightarrow \mathcal{K}$ ($o : T \rightarrow \mathcal{O}$) associates control (observation) events to transitions.

$\mathcal{K}_c \subseteq \mathcal{K}$ ($\mathcal{O}_o \subseteq \mathcal{O}$) denote the sets of controllable (observable) events.

A double-labeled PN can model both the uncontrollability of CtlPNs (Krogh and Holloway) and the unobservability of labeled PNs.

The results are obtained under the concurrency setting (transition-bag assumption, $q \in \mathbb{N}$).



Above, $\mathcal{K} = \{e_1, e_2, e_3, e_4\}$ and $\mathcal{O} = \{\alpha, \beta, \gamma\}$.

GLCs and Concurrency

Interpreting $\bigvee_i L_i\mu + H_i q + C_i v \leq b_i$ is a nontrivial issue.

Let's begin with $L\mu + Hq + Cv \leq b$, *under concurrency.*

Let $H_d = \max(0, LD + C, H)$. Consider the monitors enforcing $L\mu + Hq + Cv \leq b$ that are constructed under the no concurrency assumption (Iordache and Antsaklis, 2003).

The monitors enable q when $H_d q \leq \mu_c = b - L\mu - Cv$.

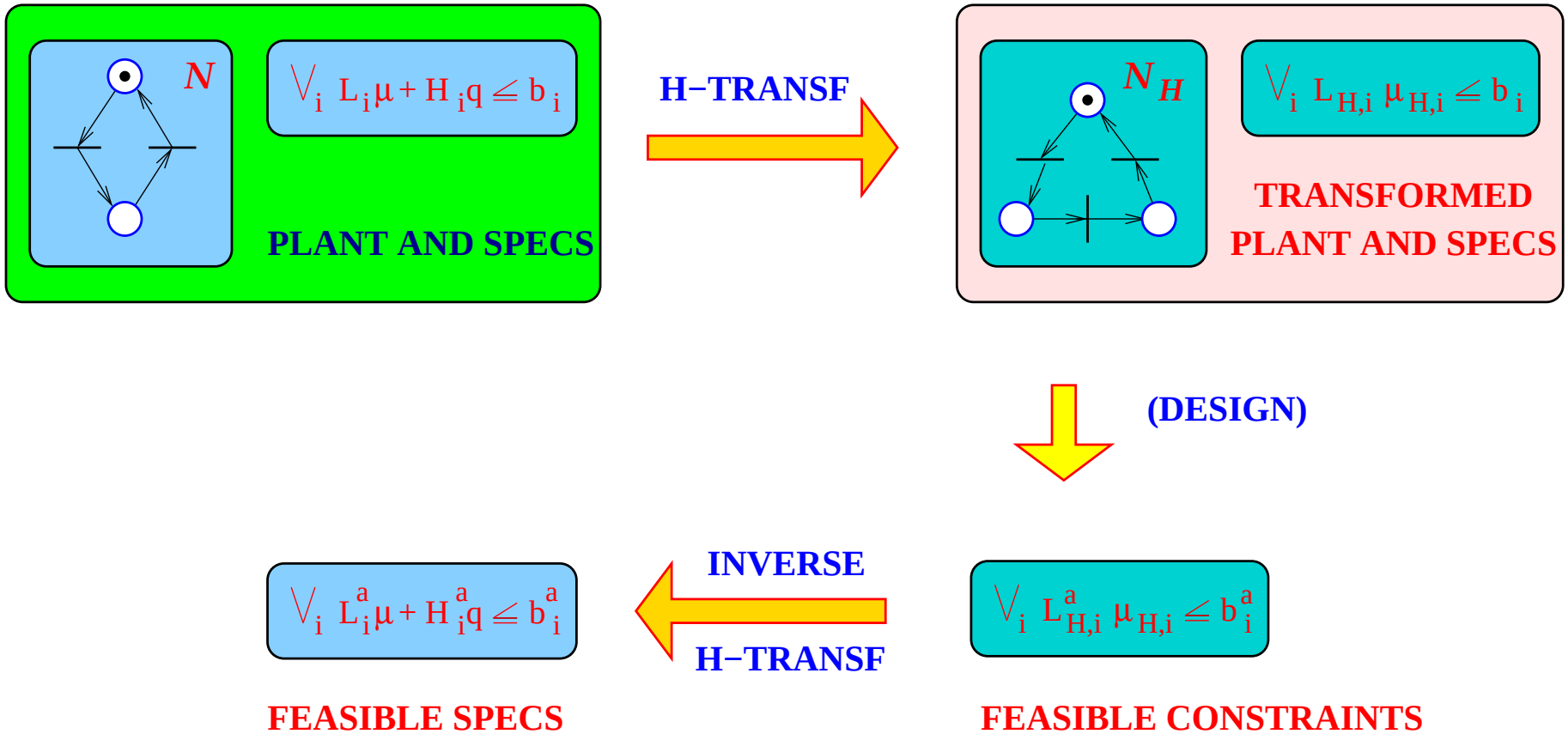
This will be the concurrency interpretation of $L\mu + Hq + Cv \leq b$: at any time, μ, q and v are permissible if they satisfy $H_d q \leq b - L\mu - Cv$.

Intuitively, this corresponds to q enabled iff at all intermediary stages of the firing of q, the specification is satisfied (Lemma 2.1).

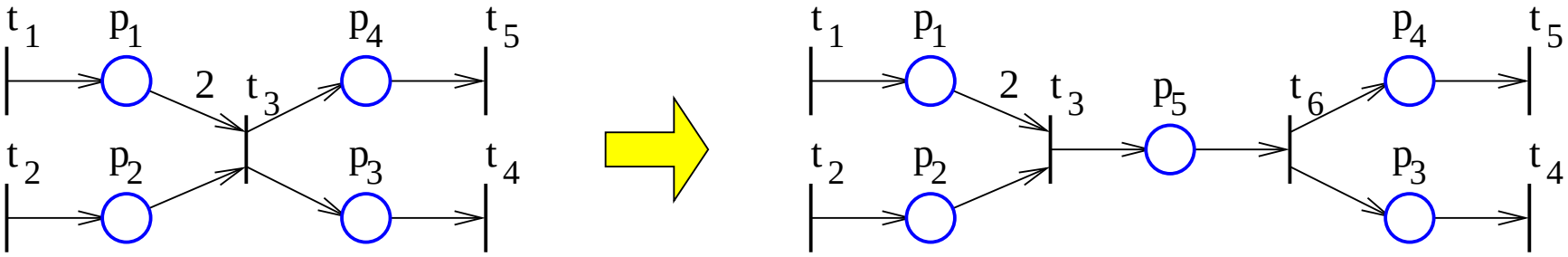
This will be the concurrency interpretation of $\bigvee_i L_i\mu + H_i q + C_i v \leq b_i$: at any time, μ, q and v are permissible if they satisfy $H_{d,i} q \leq b_i - L_i\mu - C_i v$ for some i.

Approach

Given $\bigvee_{i=1}^{n} L_i\mu + H_i q \leq b_i$, the approach is to find $\bigvee_{i=1}^{m} L_i^a\mu + H_i^a q \leq b_i^a$ that is feasible such that $\bigvee_{i=1}^{m} L_i^a\mu + H_i^a q \leq b_i^a \Rightarrow \bigvee_{i=1}^{n} L_i\mu + H_i q \leq b_i$.



The transformation splits transitions to substitute a q_i term by a marking term.



This transformation maps

$$\mu_1 + \mu_2 + 2\mu_3 + q_3 \leq 5 \tag{5}$$

into

$$\mu_1 + \mu_2 + 2\mu_3 + 4\mu_5 \leq 5 \tag{6}$$

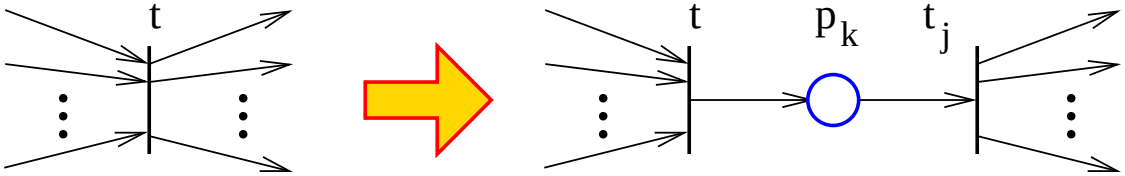
The term $4\mu_5$ is obtained as follows. Consider firing t_3 in the transformed net: $\mu \xrightarrow{t_3} \mu'$. The coefficient a of t_3 is to satisfy that

$$a + \mu'_1 + \mu'_2 + 2\mu'_3 = 1 + \mu_1 + \mu_2 + 2\mu_3$$

Input: $\mathcal{N} = (P, T, D^-, D^+)$, $L\mu + Hq \leq b$, and optionally μ_0 and a set $T_{s,H} \subseteq T$.

Output: $\mathcal{N}_H = (P_H, T_H, D_H^-, D_H^+)$, $L_H\mu_H \leq b$, and μ_{H0}.

1. Let $T_s = \{t \in T : \rho(t) = \rho(t') \text{ for some } t' \in T \text{ s.t. } t' \in T_{s,H} \text{ or } H_d(\cdot, t') \neq 0\}$, where $H_d = \max(LD, H, 0)$.
2. Let $\mathcal{N}_H = \mathcal{N}$, $L_H = L$, and $\mu_{H0} = \mu_0$.
3. For all $t \in T_s$:



   (a) Split t in t, p_k, and t_j.
   (b) Let $L_H(\cdot, p_k) = H_d(\cdot, t_i) + LD^-(\cdot, t_i)$ and $\mu_{H0}(p_k) = 0$.
4. For all $t \in T_s$:
   (a) $o(t \bullet \bullet) = o(t)$.
   (b) Extend the set of control events s.t. $\rho(t \bullet \bullet) \notin \{\rho(t) : t \in T\}$.
   (c) $\rho(t \bullet \bullet)$ is controllable iff $\rho(t)$ is controllable.
   (d) For all $t' \in T_s$, $\rho(t \bullet \bullet) = \rho(t' \bullet \bullet)$ iff $\rho(t) = \rho(t')$.

The H-transformation for a set of constraints $\bigvee_i L_i\mu + H_i q \leq b_i$:

1. Let $H_{d,i} = \max(L_i D, H_i, 0)$ and $T_{s,H} \rightarrow T_{s,H} \cup \bigcup_i \{t \in T : \ H_{d,i}(\cdot, t) \neq 0\}$.
2. For all i, apply the H-transformation to the constraints $L_i\mu + H_i q \leq b_i$ with the argument $T_{s,H}$ above. Let $L_{H,i}\mu_H \leq b_i$, $\mathcal{N}_H$, and μ_{H0} be the result.
3. Final result: $\bigvee_i L_{H,i}\mu_H \leq b_i$, $\mathcal{N}_H$, and μ_{H0}.

The H^{-1}-Transformation of $L_H\mu_H \leq b$

Input: $\mathcal{N} = (P, T, D^-, D^+)$, $\mathcal{N}_H = (P_H, T_H, D_H^-, D_H^+)$, and $L_H\mu_H \leq b$.

Output: The H^{-1}-transformed constraints $L\mu + Hq \leq b$.

1. Set $L(\cdot, p) = L_H(\cdot, p)\ \forall p \in P$ and H to the null matrix.
2. For all $p_k \in P_H \setminus P$
   (a) Let t_i be the transition such that $\{t_i\} = {\bullet}p_k$.
   (b) Set $H(\cdot, t_i) = L_H(\cdot, p_k) - L_H D_H^-(\cdot, t_i)$.

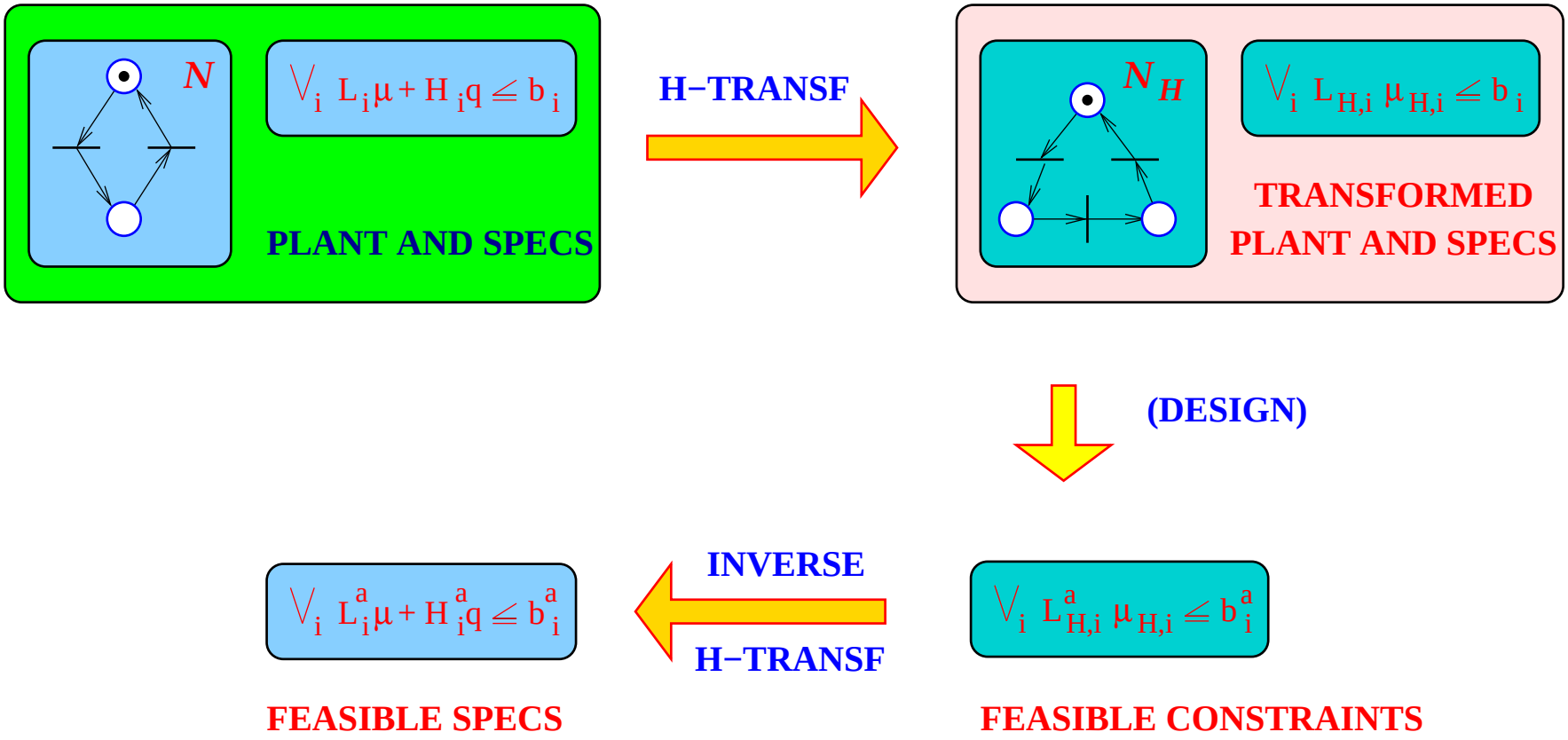
The H^{-1}-Transformation of $\bigvee_i L_{H,i}\mu_H \leq b_i$

1. For all i, apply the H^{-1}-transformation to the constraints $L_{H,i}\mu_H \leq b_i$. Let $L_i\mu + H_iq \leq b_i$ be the transformed constraints.
2. Final result: $\bigvee_i L_i\mu + H_iq \leq b_i$.

Notation

$\mathcal{S} \equiv \bigvee_i L_i\mu + H_i q \leq b_i$, $\mathcal{S}_H \equiv \bigvee_i L_{H,i}\mu_H \leq b_i$, $\mathcal{S}_H^a \equiv \bigvee_i L_{H,i}^a\mu_H \leq b_i^a$,

and $\mathcal{S}^a \equiv \bigvee_i L_i^a\mu + H_i^a q \leq b_i^a$.



Notation

Fact: $\mathcal{S} \xrightarrow{H} \mathcal{S}_H$ and $\mathcal{S}_H \xrightarrow{H^{-1}} \mathcal{S}' \Rightarrow \mathcal{S} = \mathcal{S}'$.

But is it true that $\mathcal{S}_H \xrightarrow{H^{-1}} \mathcal{S}$ and $\mathcal{S} \xrightarrow{H} \mathcal{S}'_H \Rightarrow \mathcal{S}_H = \mathcal{S}'_H$?

Theorem 3.3 (a) The H-transformation of any $\mathcal{S}$ satisfies

$$\forall p \in P_H \setminus P: \quad \begin{cases} L_H(\cdot, p) \geq L_H D_H^+(\cdot, p\bullet) \\ L_H(\cdot, p) \geq L_H D_H^-(\cdot, \bullet p) \end{cases} \tag{7}$$

$$\forall t \in T \setminus \bullet(P_H \setminus P): \quad L_H D_H(\cdot, t) \leq 0. \tag{8}$$

(b) Given $\mathcal{N}_H$ and $\mathcal{S}_H$, assume $\mathcal{S}_H \xrightarrow{H^{-1}} \mathcal{S}$ and $\mathcal{S} \xrightarrow{H} (\mathcal{S}'_H, \mathcal{N}'_H)$. If L_H satisfies (7–8) and the H-transformation has $T_{s,H} = \bullet(P_H \setminus P)$, then $\mathcal{N}_H = \mathcal{N}'_H$ and $\mathcal{S}_H = \mathcal{S}'_H$.

Feasibility

Let Ξ be the optimal supervisor for $\mathcal{S}$ (i.e. without restriction), Ξ_H for $\mathcal{S}_H$, Ξ_H^a for $\mathcal{S}_H^a$, and Ξ^a for $\mathcal{S}^a$.

Ξ is feasible if it respects the controllability and observability constraints of $(\mathcal{N}, \mu_0)$.

$\mathcal{S}$ is feasible when Ξ is feasible.

H-feasibility: *weaker* than feasibility. Defined for $\mathcal{S}_H$ such that the following result holds true.

Theorem 3.1 Assume $\mathcal{S} \xrightarrow{H} \mathcal{S}_H$. Then $\mathcal{S}$ is feasible iff $\mathcal{S}_H$ is h-feasible.

Permissiveness

The **joint H-transformation** of $\mathcal{S}$ and $\mathcal{S}'$ selects $T_{s,H}$ s.t. $\mathcal{S} \xrightarrow{H} (\mathcal{S}_H, \mathcal{N}_H)$ and $\mathcal{S}' \xrightarrow{H} (\mathcal{S}'_H, \mathcal{N}'_H) \Rightarrow \mathcal{N}_H = \mathcal{N}'_H$.

Notation:

$\Xi \preceq \Xi'$: Ξ is at least as restrictive as Ξ'.

$\Xi \prec \Xi'$: Ξ is more restrictive than Ξ'.

Theorem 3.2 $\Xi \preceq \Xi'$ ($\Xi \prec \Xi'$) iff $\Xi_H \preceq \Xi'_H$ ($\Xi_H \prec \Xi'_H$).

Procedure

1. $(\mathcal{S}, \mathcal{N}) \xrightarrow{H} (\mathcal{S}_H, \mathcal{N}_H)$.
2. Find h-feasible $\mathcal{S}_H^a$ that satisfy (7–8) and $\Xi_H^a \preceq \Xi_H$.
3. $\mathcal{S}_H^a \xrightarrow{H^{-1}} \mathcal{S}^a$. $\underline{\mathcal{S}^a \text{ is the solution.}}$

The **total H-transformation** splits all transitions: $T_{s,H} = T$.

$\mathcal{X}$: the set of supervisors optimally enforcing feasible constraints $\mathcal{S}$.

$\mathcal{X}_H$: the set of supervisors optimally enforcing h-feasible constraints $\mathcal{S}_H$ that satisfy (7–8).

Theorem 3.4

a) $\mathcal{S}^a$ is feasible and $\Xi^a \preceq \Xi$.

Assume that the *total* H-transformation is applied at step one.

b) Ξ^a is least restrictive among the supervisors of $\mathcal{X}$ enforcing $\mathcal{S}$ iff Ξ_H^a is least restrictive among the supervisors of $\mathcal{X}_H$ enforcing $\mathcal{S}_H$.

c) There is no supervisor $\Xi^* \succ \Xi^a$ of $\mathcal{X}$ that enforces $\mathcal{S}$ if there is no supervisor $\Xi_H^* \succ \Xi_H^a$ of $\mathcal{X}_H$ that enforces $\mathcal{S}_H$.

Remarks

The problem of enforcing $\bigvee_i L_i\mu + H_i q \leq b_i$ can be solved in terms of the simpler $\bigvee_i L_{H,i}\mu_H \leq b_i$ in a transformed PN, without loss of permissiveness.

The results were derived under the transition-bag concurrency setting. A loss of permissiveness is possible when this approach is used for other concurrency settings.

The results obtained under a very general uncontrollability and unobservability setting.

The problem of enforcing $\bigvee_i L_{H,i}\mu_H \leq b_i$ is still complex.

- Under certain assumptions, including no concurrency, a solution is available (Stremerssch and Boel, 2002).
- More work has been done on the particular form $L_H\mu_H \leq b$.
- A structural and suboptimal solution to the enforcement of $\bigvee_i L_{H,i}\mu_H \leq b_i$ appears in the book of Iordache and Antasklis, 2006. It applies to double-labeled PNs and the most common concurrency settings.