Decentralized Control of Petri Nets

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Outline

The goal of the paper is to

extend the supervision based on place invariants (SBPI) to a decentralized setting

Overview of the Supervision Based on Place Invariants

Supervision Based on Place Invariants: introduced by several researchers (Giua, Yamalidou, Moody, and others).

The specification of the SBPI is $L\mu \leq b$.

Case I: All transitions are controllable and observable.

Let D be the incidence matrix of the plant Petri net. The supervisor can be designed as a Petri net of incidence matrix

$$D_s = -LD$$

If μ_0 is the initial marking of the plant, the initial marking of the supervisor is

$$\mu_{s0} = b - L\mu_0$$

The places of the supervisor are called *control places*. The closed-loop is a Petri net of incidence matrix

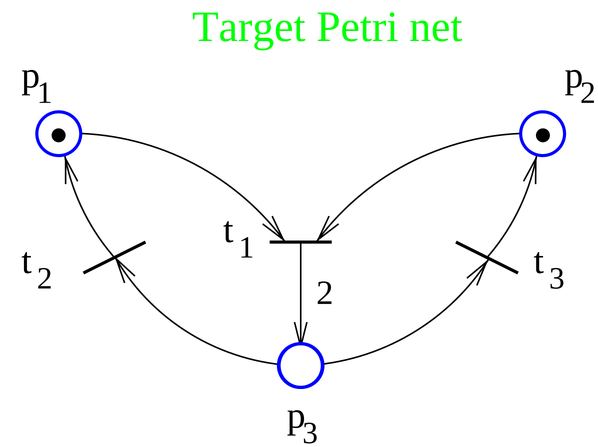
$$D_c = \begin{bmatrix} D \\ -LD \end{bmatrix}$$

The set of constraints

$$\mu(p_1) + \mu(p_3) \geq 1$$
$$\mu(p_2) + \mu(p_3) \geq 1$$

is described by $L\mu \leq b$ with:

$$L = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

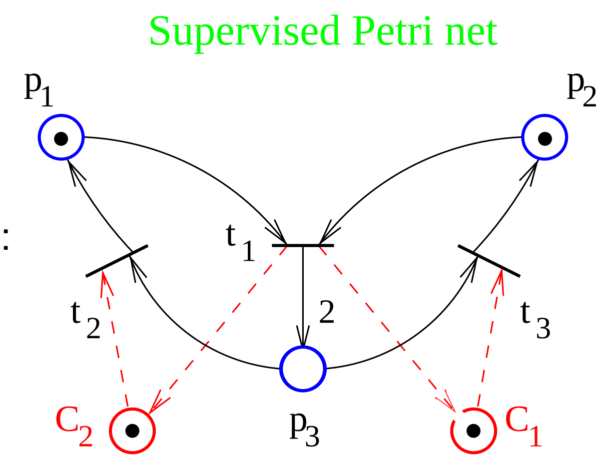


The incidence matrix is:

$$D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

The supervisor has two control places (as L has two rows):

$$D_s = -LD = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$



The initial marking of the supervisor is

$$\mu_{s0} = b - L\mu_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

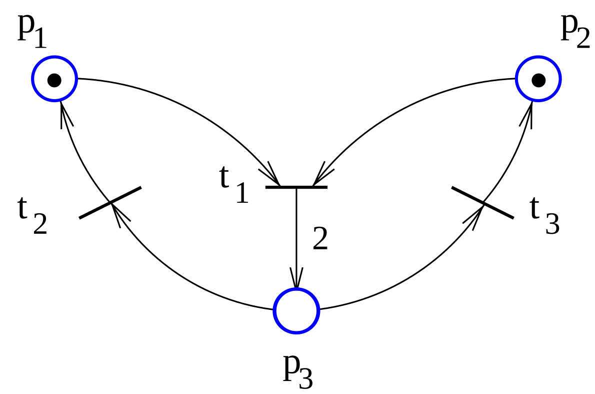
Note that for all reachable markings

$$\mu_s = b - L\mu$$

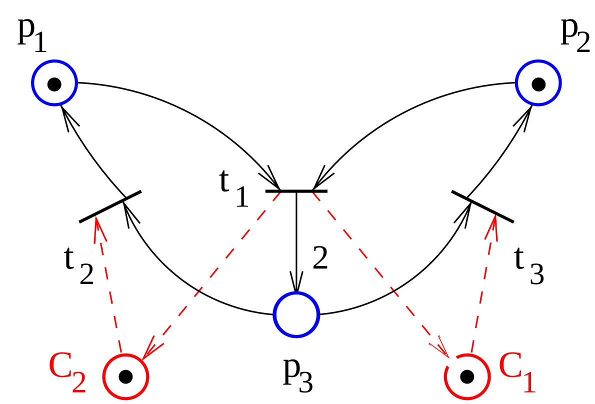
This approach is called *supervision based on place invariants*, as it creates for each row of L a place invariant. In particular:

$$\mu(p_1) + \mu(p_3) - \mu(C_1) = 1$$
$$\mu(p_2) + \mu(p_3) - \mu(C_2) = 1$$

Target Petri net



Supervised Petri net



Overview of the Supervision Based on Place Invariants

Case II: Not all transitions are controllable and observable.

A supervisor should not inhibit uncontrollable transitions or observe firings of unobservable transitions.

Then, the supervisory approach of Case I can still be used if (but not only if)

$$LD_{uo} = 0 \text{ and } LD_{uc} \leq 0 \tag{1}$$

where D_{uc} and D_{uo} are the restrictions of the incidence matrix D to the columns of the uncontrollable and unobservable transitions, respectively.

To enforce $L\mu \leq b$ we can proceed as follows:

1. If L satisfies (1), find the supervisor as in Case I. Otherwise:
2. Transform $L\mu \leq b$ to $L_a\mu \leq b_a$ such that $L_a\mu \leq b_a \Rightarrow L\mu \leq b$ and L_a satisfies (1). Then the supervised PN is obtained as in Case I by enforcing $L_a\mu \leq b_a$ instead of $L\mu \leq b$.

Assume t_1 unobservable and the same specification:

$$\mu(p_1) + \mu(p_3) \geq 1 \qquad \left(L = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right)$$
$$\mu(p_2) + \mu(p_3) \geq 1$$

As $D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}$, $D_{uo} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ and D_{uc} is empty.

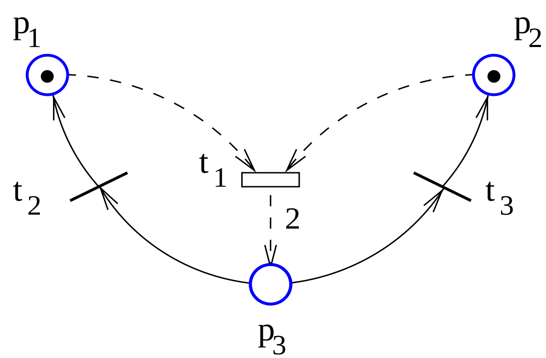
Note that $LD_{uo} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \neq 0$.

Therefore, the constraints are transformed to

$$2\mu(p_1) + \mu(p_3) \geq 1$$
$$2\mu(p_2) + \mu(p_3) \geq 1$$

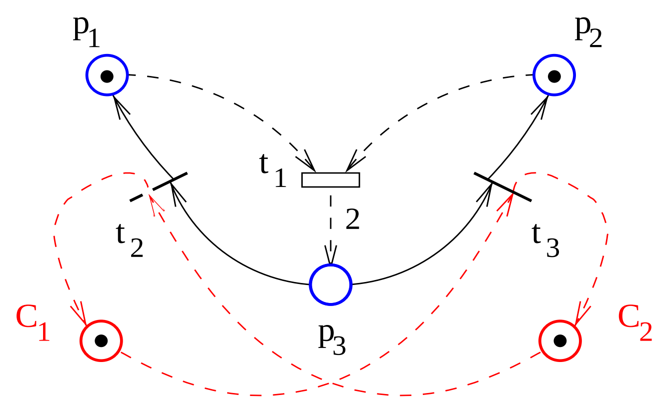
and enforced by the *control places* C_1 and C_2.

Target Petri net



t_1 is unobservable

Supervised Petri net



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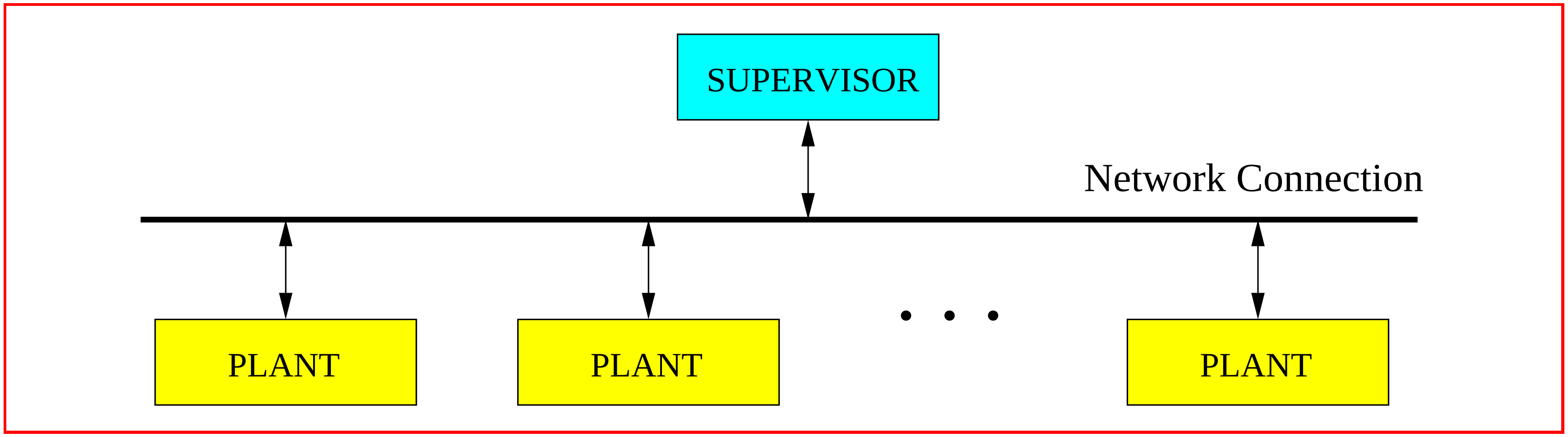
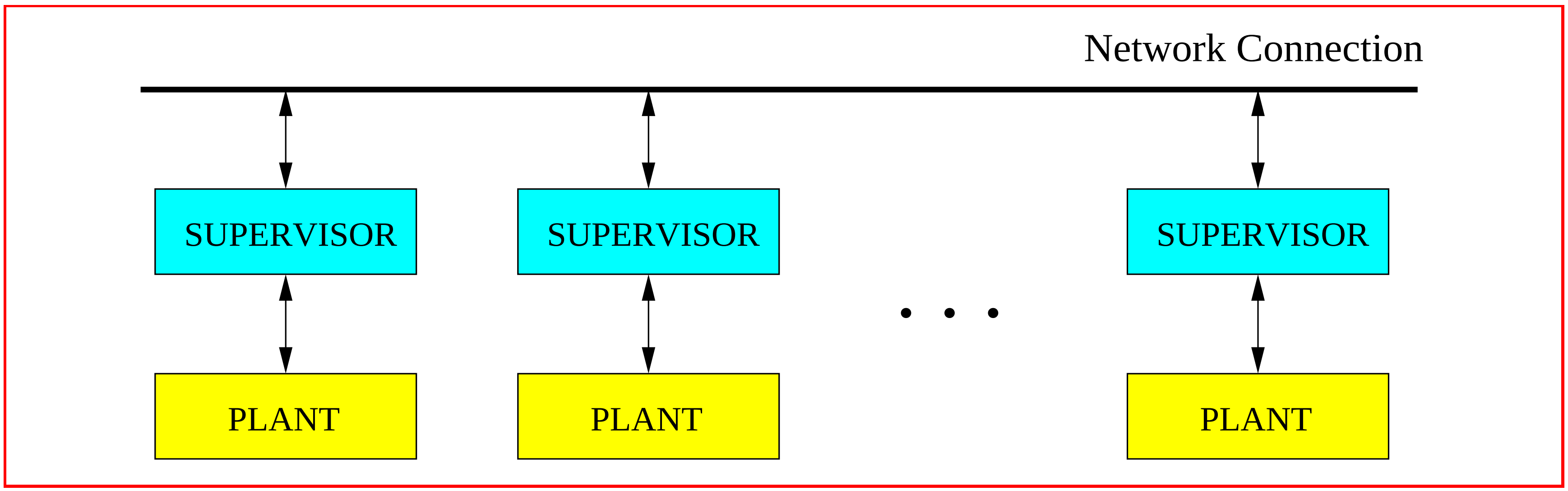
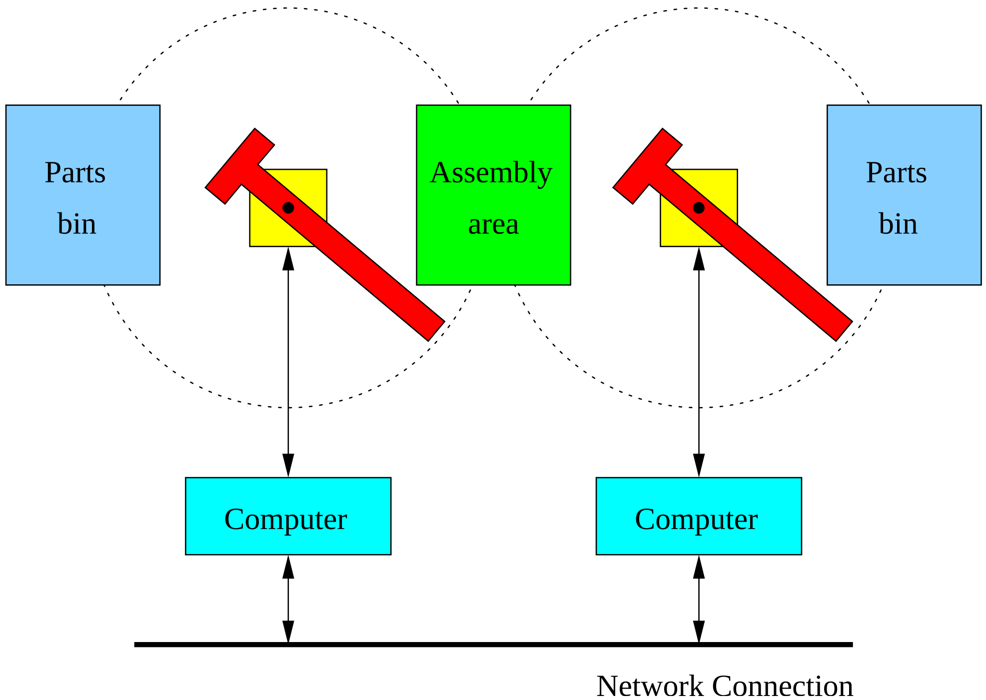
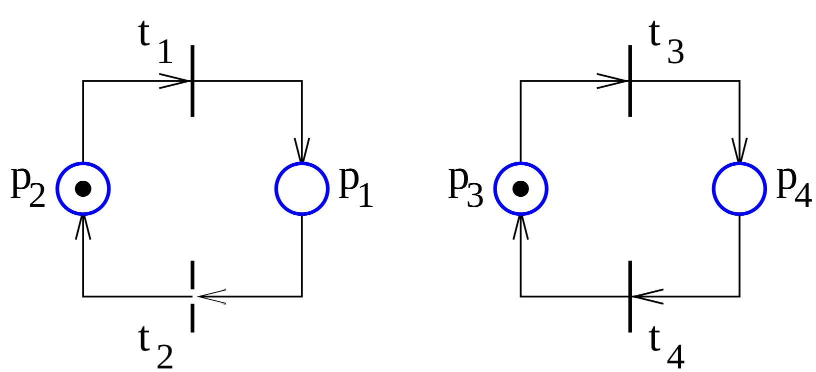
Centralized vs Decentralized





Decentralized Supervision — Example 1

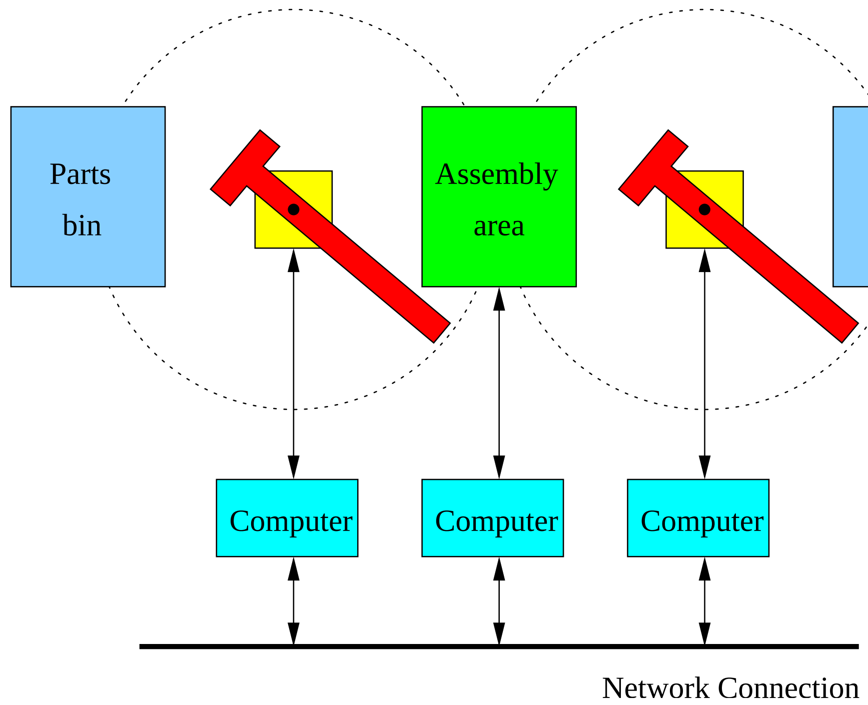


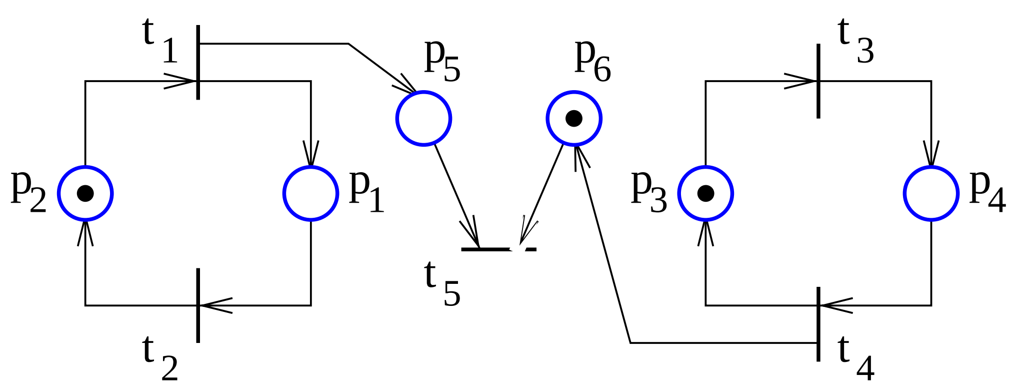


$$T_{c,1} = T_{o,1} = \{t_1, t_2\}$$

$$T_{c,2} = T_{o,2} = \{t_3, t_4\}$$

Specification: $\mu_1 + \mu_3 \leq 1$





$$T_{c,1} = T_{o,1} = \{t_1, t_2\}$$

$$T_{c,2} = T_{o,2} = \{t_3, t_4\}$$

$$T_{c,3} = \{t_5\}$$

$$T_{o,3} = \{t_1, t_2, t_3, t_4, t_5\}$$

Specification:

$$\begin{aligned} \mu_1 + \mu_3 &\leq 1 \\ \mu_5 &\leq 1 \\ \mu_6 &\leq 1 \end{aligned}$$

Decentralized Supervision

Given:

- the Petri net model of the system
- the sets of controllable and observable $T_{c,i}$ and $T_{o,i}$, $i = 1 \ldots p$.
- the specification $L\mu \leq b$.

Problem 1: *Find the supervisors $\mathcal{S}_1 \ldots \mathcal{S}_p$ such that*

1. *The joint operation of $\mathcal{S}_1 \ldots \mathcal{S}_p$ ensures the plant satisfies $L\mu \leq b$.*
2. *Each $\mathcal{S}_i$ controls only transitions in $T_{c,i}$ and observes only transitions in $T_{o,i}$.*

Decentralized Supervision with Communication

Problem 2: *Solve Problem 1 when communication is allowed.*

Communication can be used to enable $\mathcal{S}_i$ to

- control $t \in \bigcup_{j \neq i} T_{c,j}$, $t \notin T_{c,i}$.
- observe $t \in \bigcup_{j \neq i} T_{o,j}$, $t \notin T_{o,i}$.

Remark: *Centralized supervision assumes:*

$$T_c = \bigcup_{j=1\ldots p} T_{c,j} \quad and \quad T_o = \bigcup_{j=1\ldots p} T_{o,j}$$

that is, full (maximum) communication!

Optimality criteria:

- minimum communication.
- maximally permissive design.

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1. Overview of the SBPI
2. The Decentralized Setting
3. > **Decentralized Admissibility**
4. Enforcing D-Admissible Constraints
5. Enforcing D-Inadmissible Constraints
   (a) *Enforcement With Communication*
   (b) *Enforcement Without Communication*
   (c) *Enforcement With Restricted Communication*

Decentralized Admissibility

In centralized supervision:

- it is (computationally) easy to enforce constraints $L\mu \leq b$ on fully controllable and observable PNs.
- in partially controllable and observable PNs, we say that $L\mu \leq b$ is *c-admissible* if it can be enforced as if the PN were fully controllable and observable.
- constraints $L\mu \leq b$ that are not c-admissible are transformed to a c-admissible form $L_a\mu \leq b_a$ such that $L_a\mu \leq b_a \Rightarrow L\mu \leq b$.

In decentralized supervision:

- we extend c-admissibility to *d-admissibility*, such that
  - d-admissible constraints $L\mu \leq b$ are (computationally) easy to enforce.
  - checking whether a set of constraints is d-admissible is (computationally) tractable.
- the definition we propose allows us to
  - transform constraints $L\mu \leq b$ that are not d-admissible to d-admissible constraints $L_a\mu \leq b_a$ such that $L_a\mu \leq b_a \Rightarrow L\mu \leq b$.
  - enforce constraints that are not d-admissible by enabling communication

Let $L\mu \leq b$, $L \in \mathbb{Z}^{m\times|P|}$ and $b \in \mathbb{Z}^{m\times 1}$ be a set of constraints. A constraint of $L\mu \leq b$ is denoted by $l\mu \leq c$, $l \in \mathbb{Z}^{1\times|P|}$ and $c \in \mathbb{Z}$.

$l\mu \leq c$ is **d-admissible** with respect to $(\mathcal{N}, \mu_0, T_{c,1} \dots T_{c,n}, T_{o,1} \dots T_{o,n})$, if there is $\mathcal{C} \subseteq \{1, 2, \dots n\}$, $\mathcal{C} \neq \emptyset$, such that $l\mu \leq c$ is c-admissible with respect to $(\mathcal{N}, \mu_0, T_c, T_o)$, where $T_c = \bigcup_{i\in\mathcal{C}} T_{c,i}$ and $T_o = \bigcap_{i\in\mathcal{C}} T_{o,i}$.

$L\mu \leq b$ is **d-admissible** if each of its constraints $l\mu \leq c$ is d-admissible.

- c-admissibility is a special case of d-admissibility, in the sense that if $l\mu \leq c$ is c-admissible w.r.t. $(\mathcal{N}, T_{c,i}, T_{o,i})$, $l\mu \leq c$ is d-admissible (set $\mathcal{C} = \{i\}$).
- $l\mu \leq c$ d-admissible implies
  - If firing a plant-enabled transition t violates $l\mu \leq c$ then $\exists i \in \mathcal{C}$: $t \in T_{c,i}$.
  - All supervisors $\mathcal{S}_i$ with $i \in \mathcal{C}$ are able to know the value of $c - l\mu$.
- an algorithm checking whether a set of constraints is d-admissible is in the paper.

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Enforcement of D-admissible Constraints

Let D and μ_0 be the incidence matrix and the initial marking of a PN $\mathcal{N}$.

Recall the centralized enforcement of a c-admissible constraint $l\mu \leq c$ on $(\mathcal{N}, \mu_0)$:

- A control place C is generated such that for all t:
  1. If $lD(\cdot, t) > 0$, then $C \in \bullet t$ and the weight is $W(C, t) = lD(\cdot, t)$.
  2. If $lD(\cdot, t) < 0$, then $C \in t\bullet$ and the weight is $W(t, C) = -lD(\cdot, t)$.
- The initial marking of C is $c - l\mu_0$.

In the decentralized enforcement of a d-admissible constraint $l\mu \leq c$, for all $i \in \mathcal{C}$:

- Define $x_i \in \mathbb{N}$, as the state variable of $\mathcal{S}_i$.
- Initialize x_i to $c - l\mu_0$.
- $\mathcal{S}_i$ disables a transition t if $t \in T_{c,i}$ and $x_i < lD(\cdot, t)$.
- If $t \in T_{o,i}$ fires and $lD(\cdot, t) \neq 0$, then $x_i = x_i - lD(\cdot, t)$.

It can be proved that the decentralized supervisor $\bigwedge_{i \in \mathcal{C}} \mathcal{S}_i$ enforces $l\mu \leq c$ and that it is equally permissive to the centralized supervisor $\mathcal{S}$ enforcing $l\mu \leq c$ in the fully controllable and observable version of $\mathcal{N}$.

Desired constraint: $\mu_1 + \mu_3 \leq 1$. Initial marking $\mu_0 = [0, 1, 1, 0]^T$.

Decentralized setting: $T_{c,1} = \{t_1, t_2\}$, $T_{c,2} = \{t_3, t_4\}$, $T_{o,1} = T_{o,2} = \{t_1, t_2, t_3, t_4\}$.

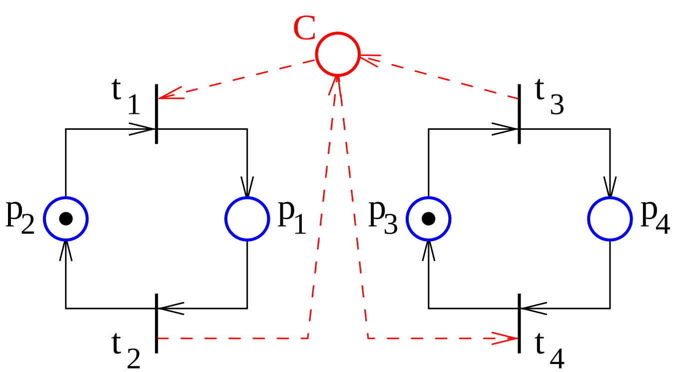
The supervisor $\mathcal{S}_1$:

- initializes x_1 to 0.
- disables t_1 if $x_1 = 0$
- increments x_1 if t_2 or t_3 fires.
- decrements x_1 if t_1 or t_4 fires.

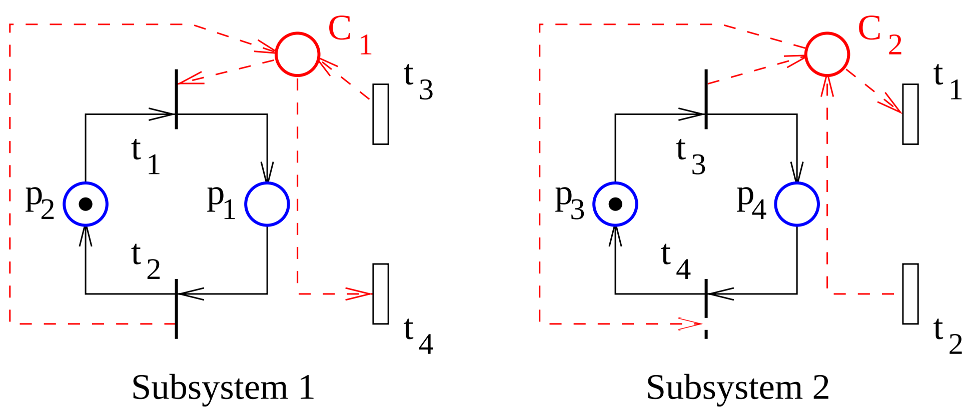
The supervisor $\mathcal{S}_2$:

- initializes x_2 to 0.
- disables t_4 if $x_2 = 0$
- increments x_2 if t_2 or t_3 fires.
- decrements x_2 if t_1 or t_4 fires.

A graphical representation is possible, however it may be both helpful and misleading.



Centralized control



Decentralized control

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5. **Enforcing D-Inadmissible Constraints**

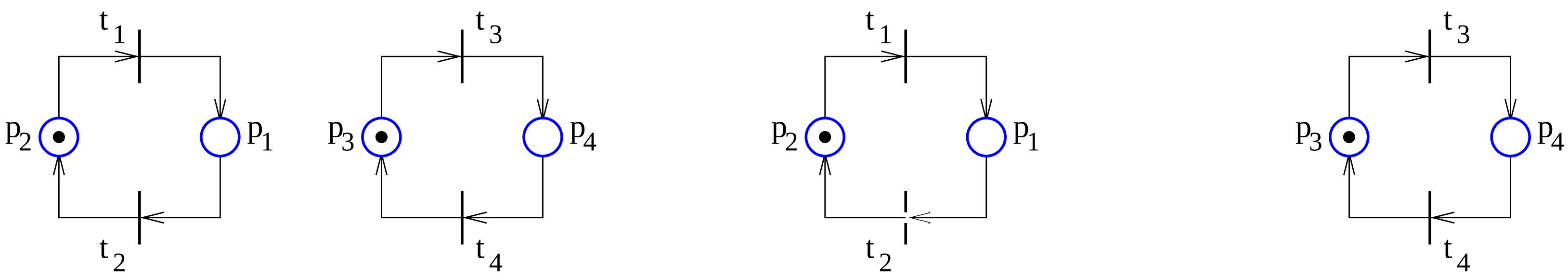
$>$ *(a)* Enforcement With Communication

(b) Enforcement Without Communication

(c) Enforcement With Restricted Communication

Enforcement of D-Inadmissible Constraints via Communication

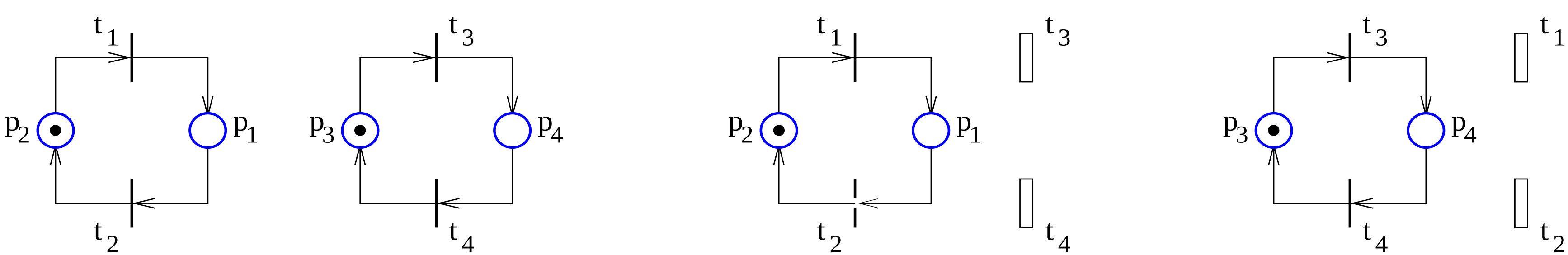
$\mu_1 + \mu_3 \leq 1$ is d-inadmissible for $T_{c,1} = T_{o,1} = \{t_1, t_2\}$ and $T_{c,2} = T_{o,2} = \{t_3, t_4\}$.



Global system Subsystem 1 Subsystem 2

The constraint becomes d-admissible if the transitions t_1 and t_2 are communicated to subsystem 2 and the transitions t_3 and t_4 to subsystem 1.

Then $T_{o,1} = T_{o,2} = \{t_1, t_2, t_3, t_4\}$, $T_{c,1} = \{t_1, t_2\}$ and $T_{c,2} = \{t_3, t_4\}$.



Global system Subsystem 1 Subsystem 2

Enforcement of D-Inadmissible Constraints via Communication

D-inadmissible constraints can be made admissible by communication:

1. Let $T_{c,L} = \bigcup_{i=1\ldots n} T_{c,i}$ and $T_{o,L} = \bigcup_{i=1\ldots n} T_{o,i}$.
2. Is the specification c-admissible with respect to $(\mathcal{N}, T_{c,L}, T_{o,L})$? If not, transform it to be c-admissible.
3. Let $\mathcal{S}$ be the centralized SBPI supervisor enforcing the specification. Let T_c be the set of transitions controlled by $\mathcal{S}$ and T_o the set of transitions detected by $\mathcal{S}$.
4. Find a set $\mathcal{C}$ such that $\bigcup_{i \in \mathcal{C}} T_{c,i} \supseteq T_c$.
5. The communication can be designed as follows: for all $t \in T_o \setminus (\bigcap_{i \in \mathcal{C}} T_{o,i})$, a subsystem j such that $t \in T_{o,j}$ transmits the firings of t to all supervisors $\mathcal{S}_k$ with $t \notin T_{o,k}$ and $k \in \mathcal{C}$.
6. Design the decentralized supervisor according to the algorithm for d-admissible constraints.

Enforcement of D-Inadmissible Constraints via Communication

In the algorithm

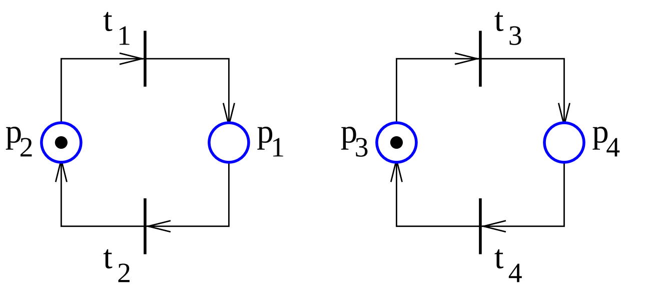
- No communication restrictions considered. These are considered later.
- The supervisor is equally permissive to the centralized supervisor.

In the communication policy proposed in the algorithm:

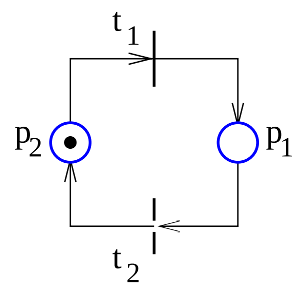
- The control decisions are taken locally (no control decisions are communicated).
- Assuming broadcast, there is less communication traffic than in the centralized solution, which remotely observes and controls the transitions in T_o and T_c, respectively.
- Better communication policies may be possible. (The optimal policy can be obtained by solving an integer program.)

Enforcement of D-Inadmissible Constraints via Communication

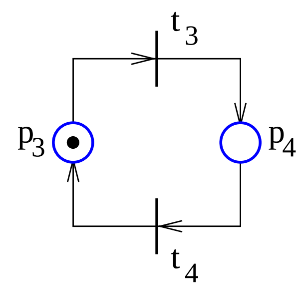
$\mu_1 + \mu_3 \leq 1$ is d-inadmissible for $T_{c,1} = T_{o,1} = \{t_1, t_2\}$ and $T_{c,2} = T_{o,2} = \{t_3, t_4\}$.



Global system



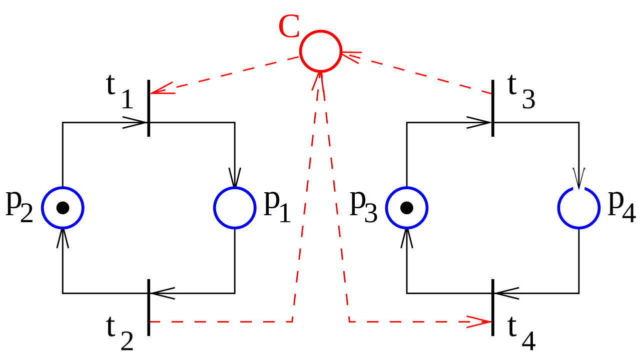
Subsystem 1



Subsystem 2

$T_{c,L} = T_{o,L} = \{t_1, t_2, t_3, t_4\}$; $\mu_1 + \mu_3 \leq 1$ is c-admissible w.r.t. $(\mathcal{N}, T_{c,L}, T_{o,L})$.

T_c and T_o found from the centralized SBPI:

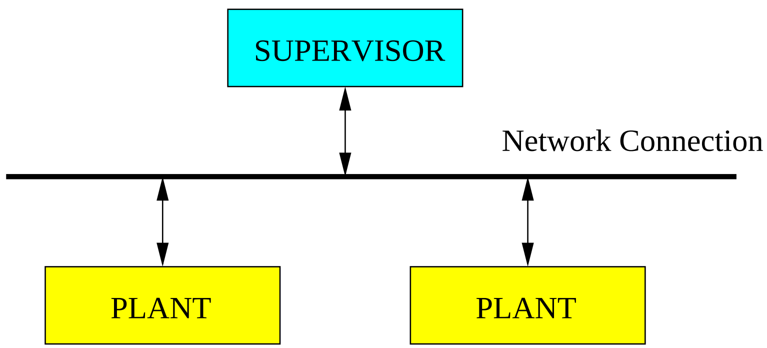


$T_c = \{t_1, t_4\}$

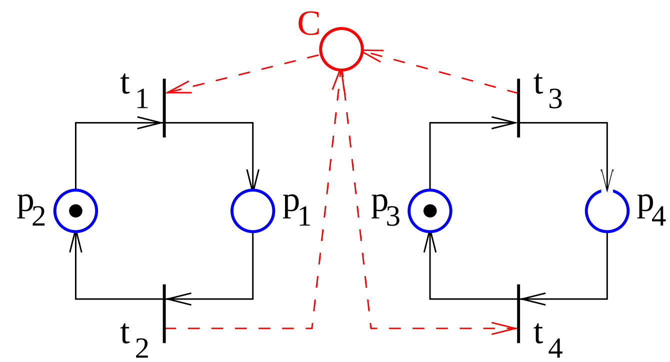
$T_o = \{t_1, t_2, t_3, t_4\}$

$\mathcal{C} = \{1, 2\}$

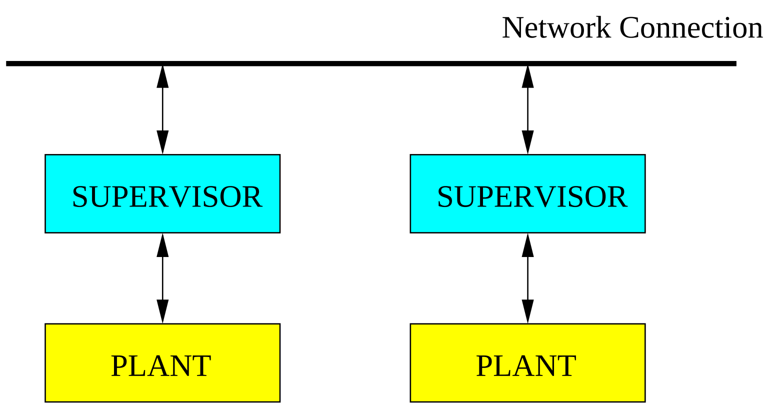
Enforcement of D-Inadmissible Constraints via Communication



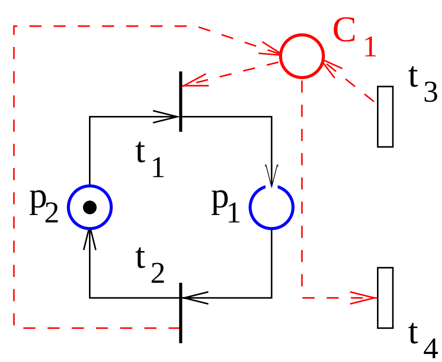
CENTRALIZED



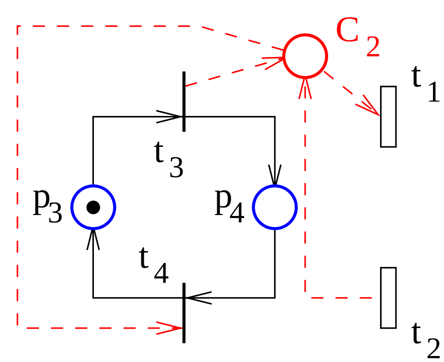
Broadcast: t_1, t_2, t_3, *and* t_4.
Remotely control: t_1 *and* t_4.



DECENTRALIZED



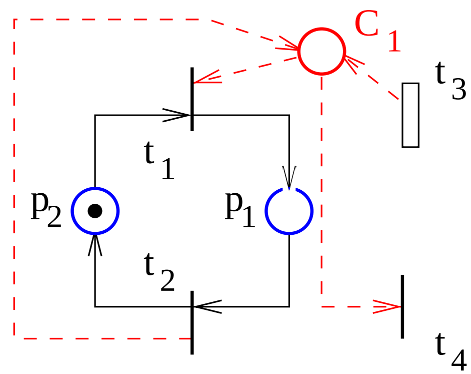
Broadcast: t_1 *and* t_2.
Remotely control: —



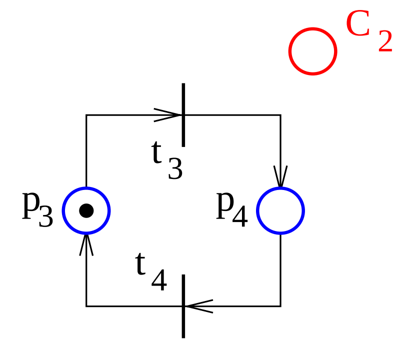
Broadcast: t_3 *and* t_4.
Remotely control: —

Enforcement of D-Inadmissible Constraints via Communication

Still another solution ...



Broadcast: —
Remotely control: t_4.



Broadcast: t_3 *and* t_4.
Remotely control: —

In general, several equally permissive and decentralized solutions are possible.

The optimal solution depends on the relative cost of broadcast/remote control.

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Enforcement of D-Inadmissible Constraints via Transformations

Specification: $L\mu \leq b$ (d-inadmissible)

Goal: Find $L_1\mu \leq b_1 \; \ldots \; L_m\mu \leq b_m$ that are d-admissible such that

$$(L_1\mu \leq b_1 \wedge L_2\mu \leq b_2 \wedge \ldots L_m\mu \leq b_m) \Rightarrow L\mu \leq b \tag{2}$$

Remarks:

- Each $L_i\mu \leq b_i$ has a different set $\mathcal{C}_i$.
- The sets $\mathcal{C}_i$ are given.
- Any solution can be found if all sets $\mathcal{C}_i$ are given. If so, $m = 2^n - 1$.
- However, we could discard the sets $\mathcal{C}_i$ with $T_o^{(i)} = \bigcap_{i \in \mathcal{C}_i} T_{o,i} = \emptyset$.
- In practice, we expect most sets $\mathcal{C}_i$ to have $T_o^{(i)} = \emptyset$.

We propose to simplify (1) to:

$$[(L_1 + L_2 + \ldots L_m)\mu \leq (b_1 + b_2 + \ldots b_m)] \Rightarrow L\mu \leq b \tag{3}$$

Enforcement of D-Inadmissible Constraints via Transformations

The following parametrization is used:

$$L_1 + L_2 + \dots L_m = R_1 + R_2 L \quad (4)$$

$$b_1 + b_2 + \dots b_m = R_2(b + \mathbf{1}) - \mathbf{1} \quad (5)$$

for $R_1 \in \mathbb{N}^{m \times |P|}$, $R_2 \in \mathbb{N}^{m \times m}$ such that $R_2 > 0$ and R_2 is diagonal.

Admissibility constraints

$$L_i D(\cdot, T_{uc}^{(i)}) \leq 0 \quad (6)$$

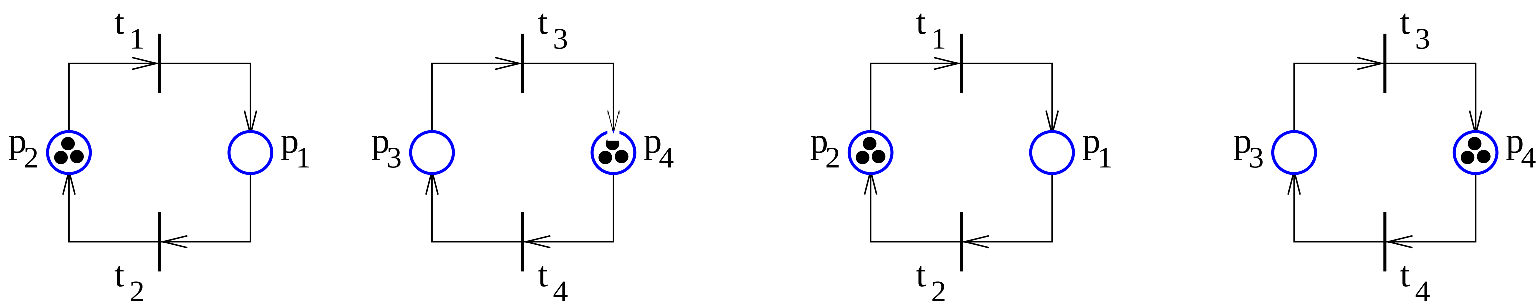
$$L_i D(\cdot, T_{uo}^{(i)}) = 0 \quad (7)$$

where $T_{uc}^{(i)} = \bigcap_{i \in \mathcal{C}_i} T_{uc,i}$ and $T_{uo}^{(i)} = \bigcup_{i \in \mathcal{C}_i} T_{uo,i}$.

Then the problem is to *find a feasibile solution of (4–7).* The unknowns are R_1, R_2, L_i, and b_i, and integer programming can be used to find them.

Drawbacks: The computational complexity of ILP and the fact that a permissivity requirement seems rather hard to be encoded as linear constraints.

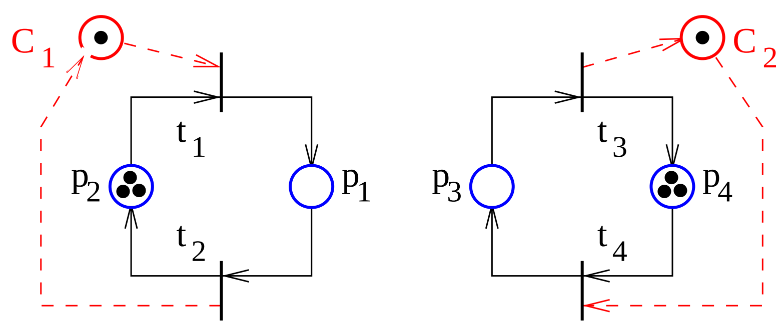
Specification: $\mu_1 + \mu_3 \leq 2$; $T_{c,1} = T_{o,1} = \{t_1, t_2\}$ and $T_{c,2} = T_{o,2} = \{t_3, t_4\}$.



Global system Subsystem 1 Subsystem 2

Take $m = 2$, $\mathcal{C}_1 = \{1\}$ and $\mathcal{C}_2 = \{2\}$.

Decentralized solution: $\mu_1 \leq 1$ (as $L_1\mu \leq b_1$) and $\mu_3 \leq 1$ (as $L_2\mu \leq b_2$).



Outline

The goal of the paper is to

extend the supervision based on place invariants (SBPI) to a decentralized setting

1. Overview of the SBPI
2. The Decentralized Setting
3. Decentralized Admissibility
4. Enforcing D-Admissible Constraints
5. **Enforcing D-Inadmissible Constraints**
   (a) *Enforcement With Communication*
   (b) *Enforcement Without Communication*
   > (c) Enforcement With Restricted Communication

Restricted Communication

The previous ILP approach can be used with communication extensions.

Note that: - Communication allows improving permissivity.
- Some constraints are not enforcible without communication.

Extensions:

- The binary variables α_{ij} and ε_{ij} are introduced.
  - $\alpha_{ij} = 1$ iff the firing of t_j is announced to the supervisors of $\mathcal{C}_i$.
  - $\varepsilon_{ij} = 1$ iff a supervisor from $\mathcal{C}_i$ remotely controls t_j.
- In particular, in the broadcast case
  - $\alpha_{ij} = \alpha_j \ \forall i = 1 \ldots m$ ($\alpha_j = 1$ iff each firing of t_j is broadcast, i.e., all supervisors are announced when t_j fires).
  - $\varepsilon_{ij} = \varepsilon_j \ \forall i = 1 \ldots m$ ($\varepsilon_j = 1$ iff all supervisors are allowed to remotely control t_j).
- Communication constraints can be incorporated as expressions of α_{ij} and ε_{ij}.

Restricted Communication

- Define B_U^i and B_L^i as upper and lower bounds of L_iD.
- Let $A = [\alpha_{ij}]$ and $E = [\varepsilon_{ij}]$.

The admissibility constraints $L_iD(\cdot, T_{uc}^{(i)}) \leq 0$ and $L_iD(\cdot, T_{uo}^{(i)}) = 0$ are replaced by:

$$L_iD(\cdot, T_{uo}^{(i)}) \leq [B_U^i diag(A(i,\cdot))]|_{T_{uo}^{(i)}} \quad (8)$$

$$L_iD(\cdot, T_{uo}^{(i)}) \geq [B_L^i diag(A(i,\cdot))]|_{T_{uo}^{(i)}} \quad (9)$$

$$L_iD(\cdot, T_{uc}^{(i)}) \leq [B_U^i diag(E(i,\cdot))]|_{T_{uc}^{(i)}} \quad (10)$$

Given the weight matrices C and F, the objective of the ILP can be set to

$$\min_{A,E,L_i,b_i,R_1,R_2} Trace(CA + FE) \quad (11)$$

to minimize communication.

C/F may reflect statistics on how often the transitions t_j are fired/require control.

Manufacturing Example (Adapted from [Lin, 1990])

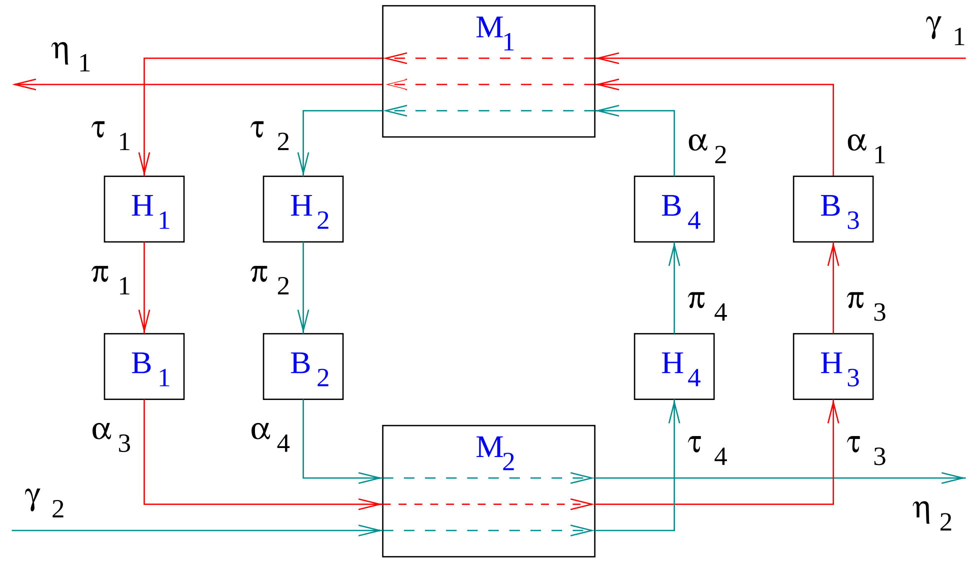
Machines: M_1 and M_2.
Buffers: $B_1 \ldots B_4$.
Robots: $H_1 \ldots H_4$.

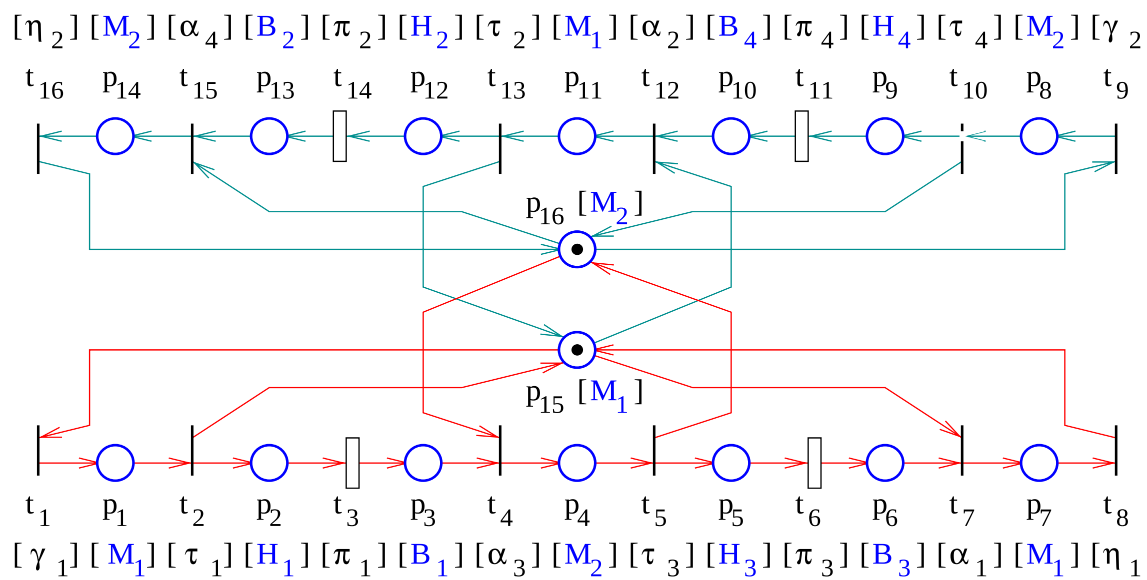
Two possible manufacturing sequences:

- $\gamma_1\tau_1\pi_1\alpha_3\tau_3\pi_3\alpha_1\eta_1$
- $\gamma_2\tau_4\pi_4\alpha_2\tau_2\pi_2\alpha_4\eta_2$

B_1 and B_2 share common buffer space.

B_3 and B_4 share also common space.

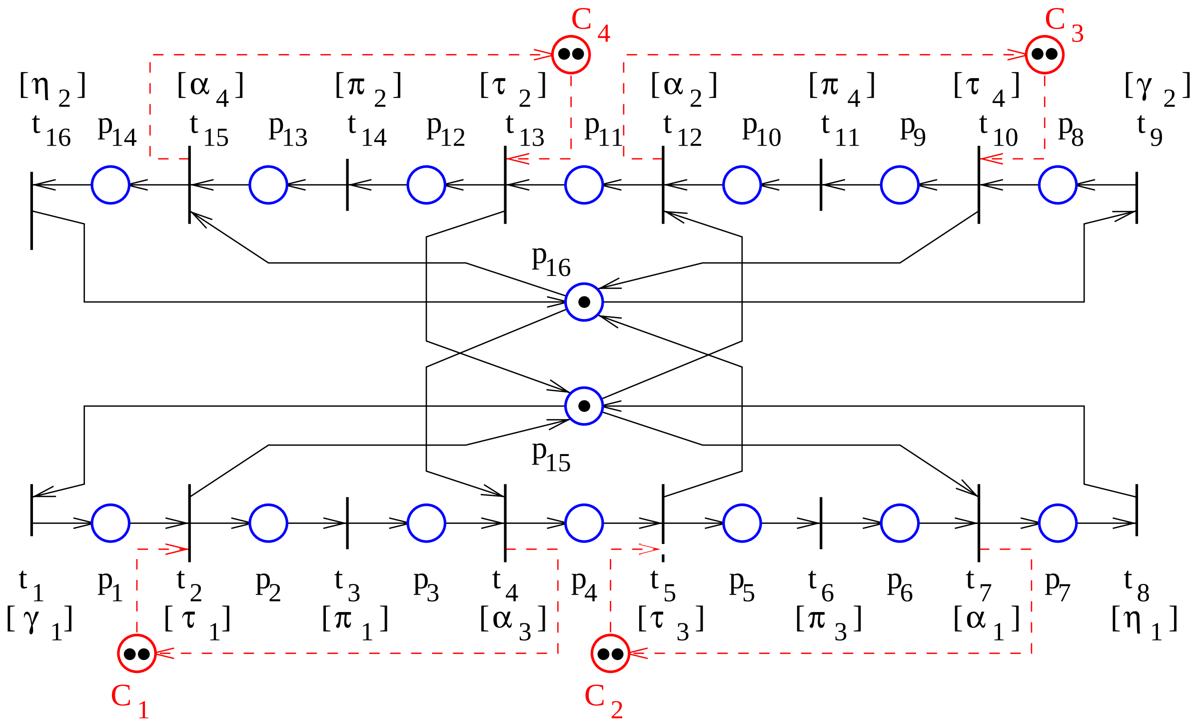




Decentralized Supervision

$$
\begin{aligned}
T_{c,1} &= \{t_2\} & T_{o,1} &= \{t_2, t_3, t_4\} \\
T_{c,2} &= \{t_5\} & T_{o,2} &= \{t_5, t_6, t_7, t_8\} \\
T_{c,3} &= \{t_{10}\} & T_{o,3} &= \{t_{10}, t_{11}, t_{12}\} \\
T_{c,4} &= \{t_{13}, t_{16}\} & T_{o,4} &= \{t_{13}, t_{14}, t_{15}, t_{16}\}
\end{aligned}
$$

Avoid buffer overflow: $\mu_3 + \mu_{13} \leq 4$ and $\mu_6 + \mu_{10} \leq 4$.



Take $m = 4$ and $\mathcal{C}_i = \{i\}$, $i = 1 \ldots 4$.

Solution without communication:

$\mu_2 + \mu_3 \leq 2$ (sub-1)

$\mu_5 + \mu_6 \leq 2$ (sub-2)

$\mu_9 + \mu_{10} \leq 2$ (sub-3)

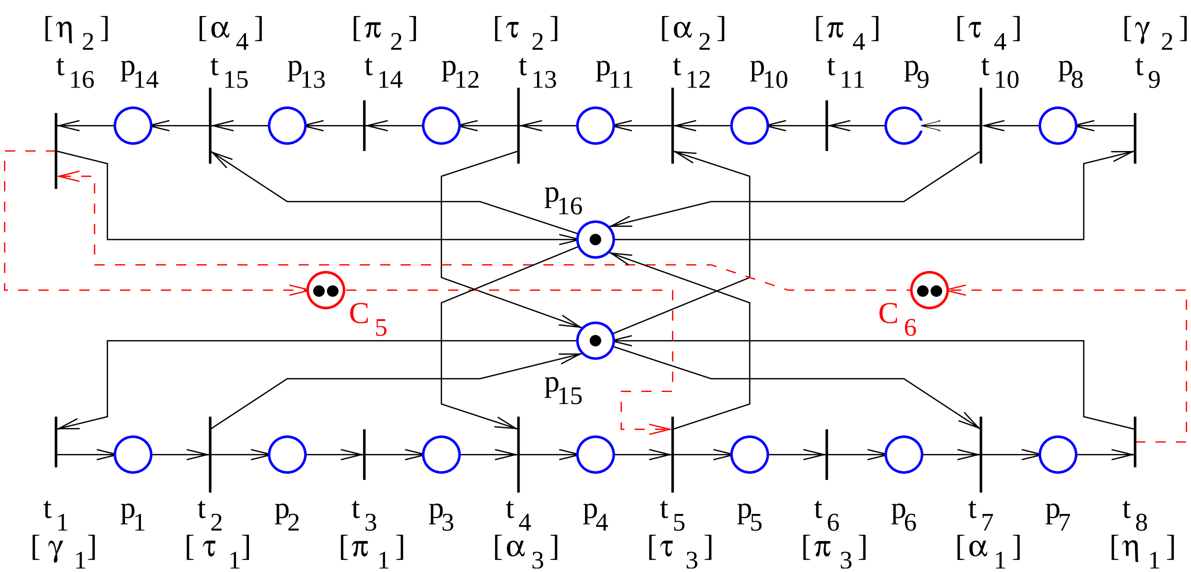
$\mu_{12} + \mu_{13} \leq 2$ (sub-4)

Decentralized Supervision

$$
\begin{array}{lllllll}
T_{c,1} & = & \{t_2\} & \quad & T_{o,1} & = & \{t_2, t_3, t_4\} \\
T_{c,2} & = & \{t_5\} & & T_{o,2} & = & \{t_5, t_6, t_7, t_8\} \\
T_{c,3} & = & \{t_{10}\} & & T_{o,3} & = & \{t_{10}, t_{11}, t_{12}\} \\
T_{c,4} & = & \{t_{13}, t_{16}\} & & T_{o,4} & = & \{t_{13}, t_{14}, t_{15}, t_{16}\}
\end{array}
$$

Fairness: $v_8 - v_{16} \leq 2$ and $v_{16} - v_8 \leq 2$.

(v_i: the number of firings of t_i.)



No acceptable solution without communication!

Result:

subsystem 2: broadcast t_8 and enforce

$$\mu_5 + \mu_6 + \mu_7 + v_8 - v_{16} \leq 2$$

subsystem 4: broadcast t_{16} and enforce

$$v_{16} - v_8 \leq 2$$

Conclusion

This paper extends the SBPI to the decentralized setting.

The supervisors can be designed by constraint transformation for:

- no communication
- restricted communication
- minimal communication

This work shows that the decentralized supervision of PNs can be tractable.

On the negative side:

- Our ILP approach is suboptimal.
- Difficult to include permissivity requirements in the ILP.