$\mathcal{T}$-Liveness Enforcement in Petri Nets Based on Structural Net Properties

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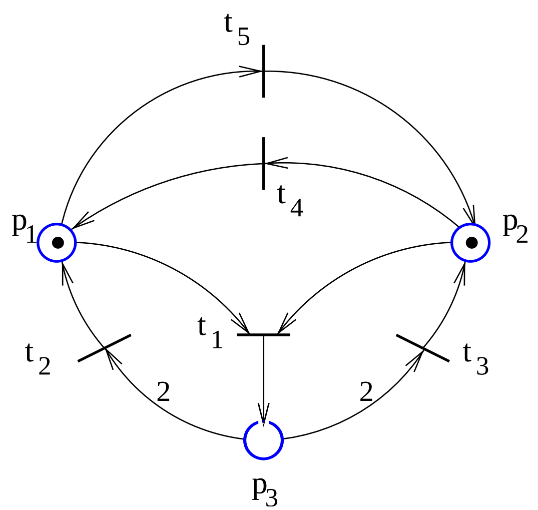
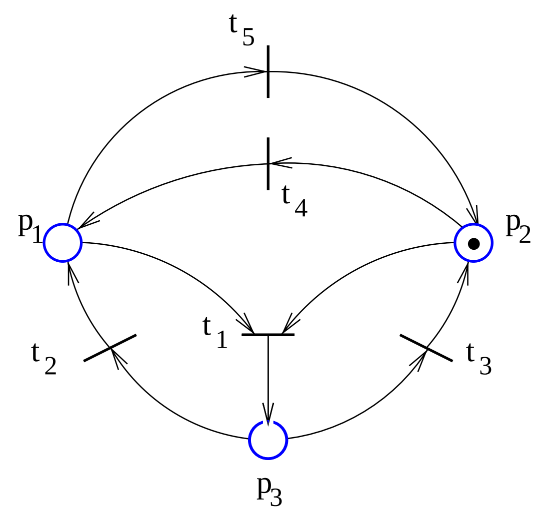
Introduction

Given a PN $\mathcal{N} = (P, T, F, W)$ and $\mathcal{T} \subseteq T$:

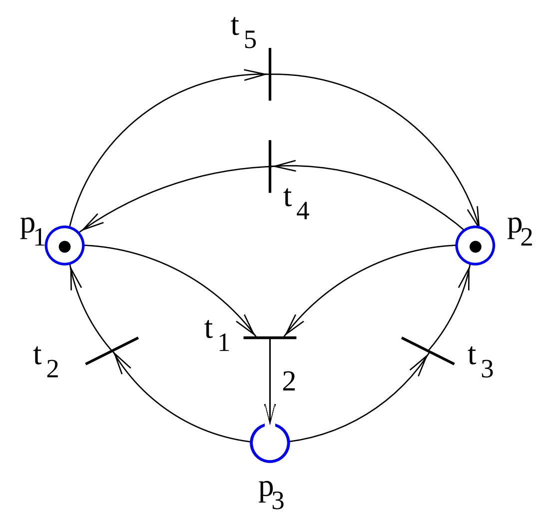
- $(\mathcal{N}, \mu_0)$ is **$\mathcal{T}$-live** if all transitions in $\mathcal{T}$ are live.
- $\mathcal{N}$ **can be made $\mathcal{T}$-live** (or **$\mathcal{T}$-liveness is enforcible** in $\mathcal{N}$) if
  $\exists \mu_0$ $\exists$ supervisor Ξ such that $(\mathcal{N}, \mu_0, \Xi)$ is $\mathcal{T}$-live.
- Liveness is $\mathcal{T}$-liveness for $\mathcal{T} = T$.



$\{t_4, t_5\}$-liveness enforcible



$\{t_4, t_5\}$-live PN



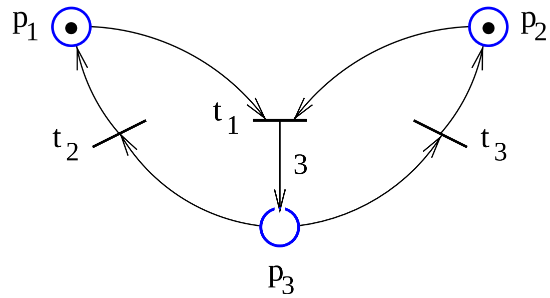
Live PN

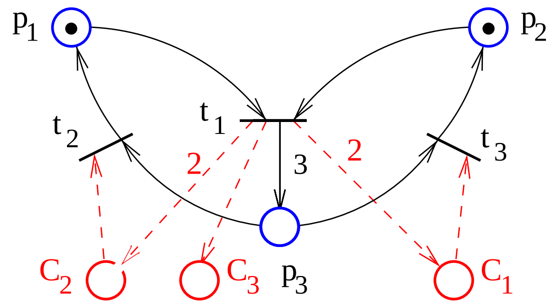
What are the initial markings for which a PN can be made $\mathcal{T}$-live?

$$[\mathrm{C}_1 :] \qquad \mu_1 + \mu_3 \geq 1 \qquad (1)$$

$$[\mathrm{C}_2 :] \qquad \mu_2 + \mu_3 \geq 1 \qquad (2)$$

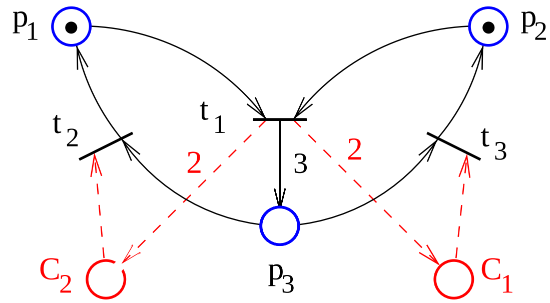
$$[\mathrm{C}_3 :] \qquad \mu_1 + \mu_2 + \mu_3 \geq 2 \qquad (3)$$





The control place (monitor) C_3 is useless.

- Let $L\mu \geq b$ describe (1) and (2).
- Let $L_0\mu \geq b_0$ describe (3).



The PN is live for all initial markings μ_0 satisfying

$$L\mu_0 \geq b \text{ and } L_0\mu_0 \geq b_0 \qquad (4)$$

when supervised according to the constraint $L\mu \geq b$.

Defining the $\mathcal{T}$-liveness enforcing procedure

Given an arbitrary PN and $\mathcal{T}$, the procedure finds matrices L, L_0, b, b_0, such that the PN is $\mathcal{T}$-live for all initial markings μ_0 satisfying

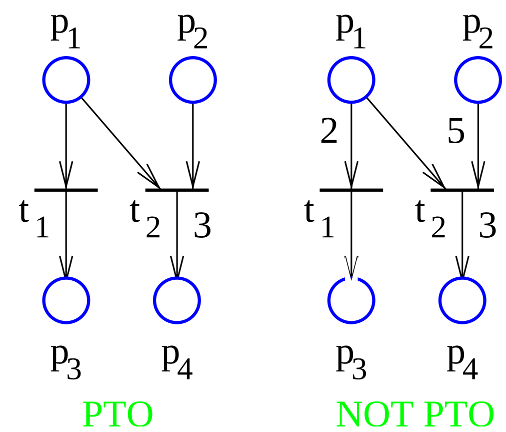
$$L\mu_0 \geq b \text{ and } L_0\mu_0 \geq b_0 \tag{5}$$

when supervised according to the constraint $L\mu \geq b$.

Let $\mathcal{N} = (P, T, F, W)$ be a PN.

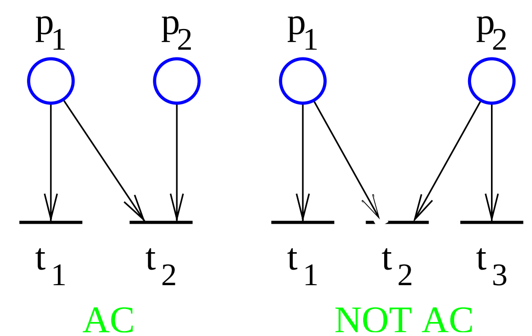
We call $\mathcal{N}$ PT**-ordinary** if

$$\forall (p, t) \in F : \ W(p, t) = 1$$



$\mathcal{N}$ has **asymmetric choice** if

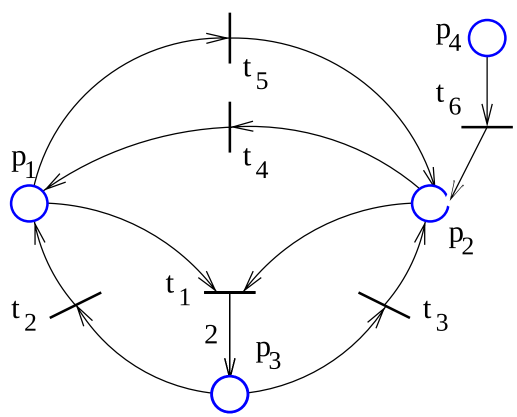
$$\forall p_1, \ p_2 \in P : p_1\bullet \cap p_2\bullet \neq \emptyset \Rightarrow p_1\bullet \subseteq p_2\bullet \vee p_2\bullet \subseteq p_1\bullet$$



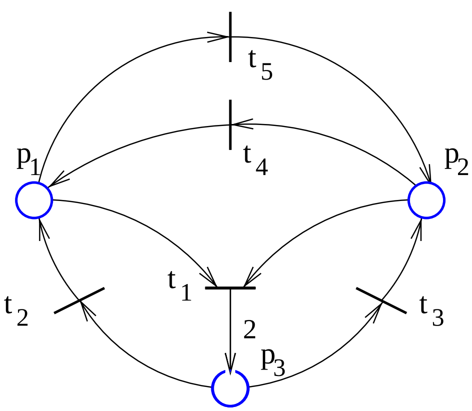
An *active subnet* is a PN subnet which can be made live. Formally:

Given $\mathcal{N} = (P, T, F, W)$ *of incidence matrix* D, $\mathcal{N}^A = (P^A, T^A, F^A, W^A)$ *is an* **active subnet** *of* $\mathcal{N}$ *if there is* $x \geq 0$, $x \neq 0$, *such that* $Dx \geq 0$ *and* $T^A = \|x\|$, $P^A = T^A\bullet$, $F^A = F \cap \{(T^A \times P^A) \times (P^A \times T^A)\}$ *and* W^A *is* W *restricted to* F^A.

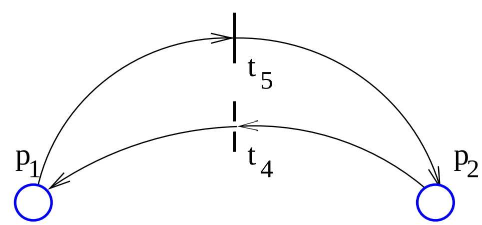
If $\mathcal{T} \subseteq T^A$ *and there is no active subnet* $\mathcal{N}_1^A = (P_1^A, T_1^A, F_1^A, W_1^A)$ *such that* $\mathcal{T} \subseteq T_1^A$ *and* $T_1^A \subset T^A$, *we say that* $\mathcal{N}^A$ *is a* $\mathcal{T}$**-minimal active subnet** *of* $\mathcal{N}$.



PN



An active subnet



$\{t_4, t_5\}$*-minimal active subnet*

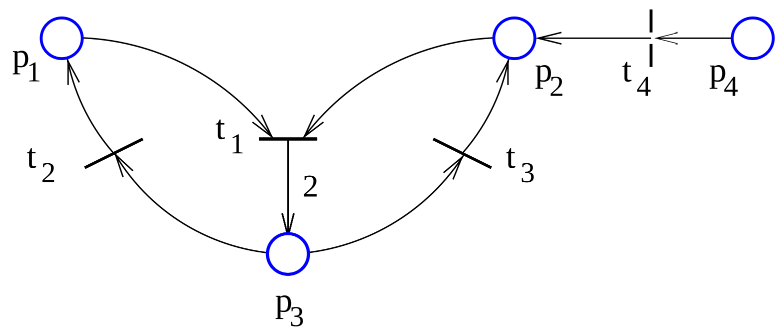
A **siphon** *is a set of places* $S \neq \emptyset$ *such that* $\bullet S \subseteq S \bullet$*.*

S *is an* **active siphon** *with respect to an active subnet, if it is a siphon which includes one or more places of that subnet.*

S *is a* **minimal active siphon**, *if there is no other siphon* $S' \subseteq S$ *active w.r.t. the same active subnet.*

The only nonempty active subnet has $T^A = \{t_1, t_2, t_3\}$.

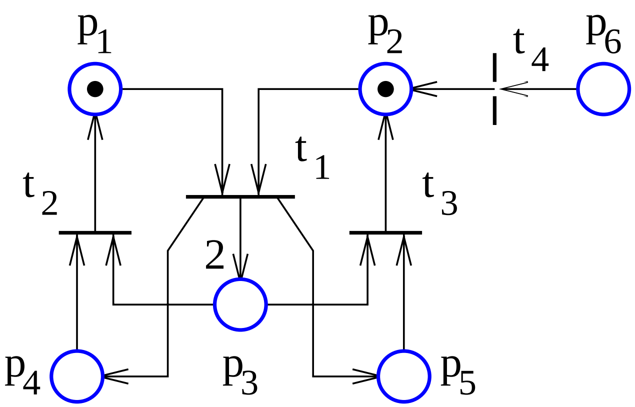
The active siphons are $\{p_1, p_3\}$, $\{p_2, p_3, p_4\}$ and $\{p_1, p_2, p_3, p_4\}$; the first two are also minimal.



A siphon S *is* **controlled** *w.r.t. a set of PN initial markings if for all reachable markings the total marking of* S *is nonzero.*

Theoretical Foundation

Theorem. *Given a PT-ordinary asymmetric-choice net $\mathcal{N}$, let $\mathcal{N}^A$ be a $\mathcal{T}$-minimal active subnet. If all minimal active siphons w.r.t. $\mathcal{N}^A$ are controlled, the PN is $\mathcal{T}$-live.*



The PN is $\mathcal{T}$-live for $\mathcal{T} = \{t_1, t_2, t_3\}$.

There is a single $\mathcal{T}$-minimal active subnet $\mathcal{N}^A$ (the one with $T^A = \mathcal{T}$.)

All minimal active siphons w.r.t. $\mathcal{N}^A$ are controlled: $\{p_1, p_3\}$, $\{p_1, p_4\}$, $\{p_2, p_3, p_6\}$, and $\{p_2, p_5, p_6\}$

$\mathcal{T}$-liveness supervisors are generated by iteratively correcting deadlock situations. This involves the following:

1. siphon control
2. transformations to PT-ordinary and asymmetric choice Petri nets
3. active subnet computation

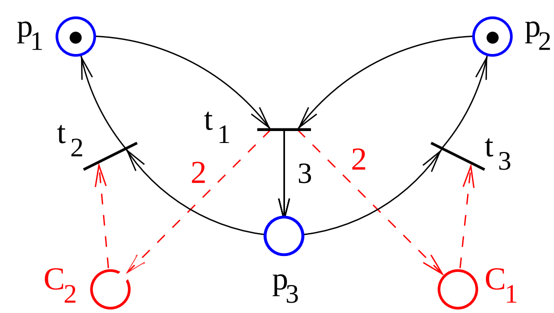
Siphon control: at every iteration, all uncontrolled minimal active siphons S are controlled by enforcing:

$$\sum_{p \in S} \mu(p) \geq 1 \tag{6}$$

Depending on the structural properties, (6) can be enforced by adding a **control place** (monitor) to the PN or by only requiring the initial marking to satisfy (6).

C_1 controls $\{p_1, p_3\}$ and C_2 controls $\{p_2, p_3\}$.

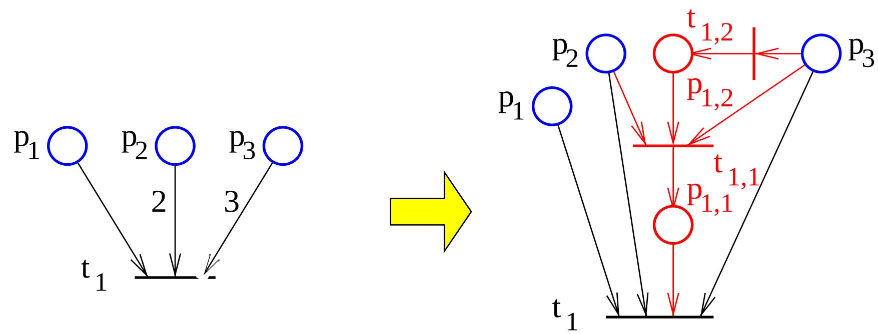
$\{C_1, p_2\}$ and $\{C_2, p_1\}$ controlled by requiring $\mu_0(C_1) + \mu_0(p_2) \geq 1$ and $\mu_0(C_2) + \mu_0(p_1) \geq 1$



Transformation to PT-ordinary PNs

In the example, any inequalities on the original PN are changed as follows:

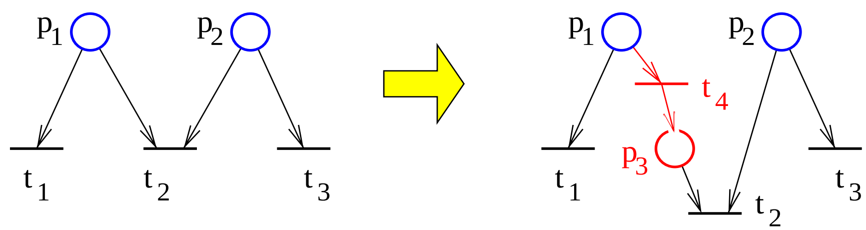
$$\mu(p_1) \longrightarrow \mu(p_1)$$
$$\mu(p_2) \longrightarrow \mu(p_2) + \mu(p_{1,1})$$
$$\mu(p_3) \longrightarrow \mu(p_3) + \mu(p_{1,2}) + 2\mu(p_{1,1})$$



Transformation to AC nets

In the example, any inequalities on the original PN are changed as follows:

$$\mu(p_1) \longrightarrow \mu(p_1) + \mu(p_3)$$
$$\mu(p_2) \longrightarrow \mu(p_2)$$



In general: $\mu(p_i) \longrightarrow \mu(p_i) + \sum_j k_j \mu(p_{i,j})$

The computation of a $\mathcal{T}$-minimal active subnet reduces to:

Find $x \geq 0$, $x_i > 0\ \forall t_i \in \mathcal{T}$, such that $Dx \geq 0$ and there is no other $y \geq 0$, $Dy \geq 0$, $y_i > 0\ \forall t_i \in \mathcal{T}$, such that $\|y\| \subset \|x\|$.

At every iteration the active subnet is *updated* by repeating the changes done to the PN in the active subnet.

Input: *The target PN $\mathcal{N}_0$ and the set $\mathcal{T}$*
Output: *Two sets of constraints (L, b) and (L_0, b_0)*

repeat

1. *Transform the current net to a PT-ordinary AC PN.*

2. *Compute the $\mathcal{T}$-minimal active subnet.*

3. **For** *every uncontrolled minimal active siphon S* **do**

If *S needs to be controlled with a $control\ place$* **then**
add control place to Petri net and inequality in (L, b).
Else
add inequality to (L_0, b_0).

until *no uncontrolled minimal siphon is found at 2.*

Restrict the constraints (L, b) and (L_0, b_0) to the places of $\mathcal{N}_0$.

$\mathcal{T}$-liveness is enforced for all initial markings μ_0 such that

$$L\mu_0 \geq b \text{ and } L_0\mu_0 \geq b_0$$

by supervising $\mathcal{N}_0$ according to $L\mu \geq b$.

Theoretical Results

Theorem. *The supervisors generated by the $\mathcal{T}$-liveness procedure enforce $\mathcal{T}$-liveness.*

Theorem. *Given a PN and $\mathcal{T}$, if the PN has a single $\mathcal{T}$-minimal active subnet and the procedure terminates, the generated supervisor is least restrictive.*

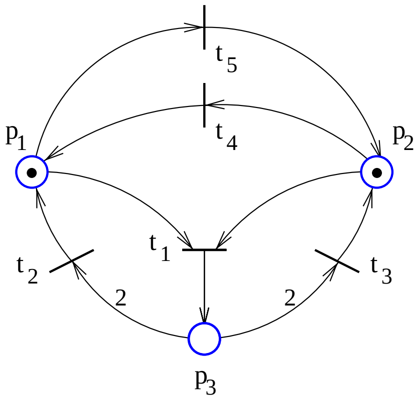
A supervisor generated by the procedure is said to be least restrictive when:

- The set of initial markings μ_0 for which liveness is enforcible is

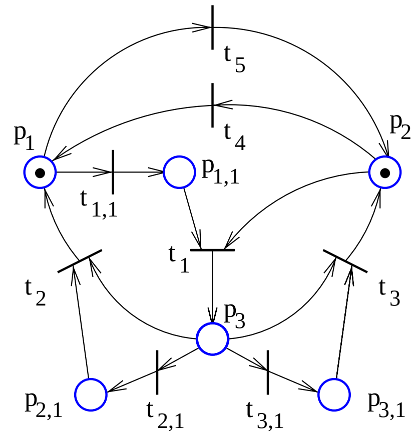
$$L\mu_0 \geq b \;\wedge\; L_0\mu_0 \geq b_0 \tag{7}$$

- For all initial markings μ_0 satisfying (7), there is no $\mathcal{T}$-liveness enforcing supervisor less restrictive.

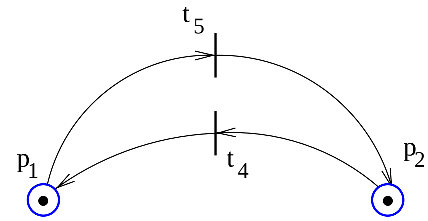
$\mathcal{T}$-Liveness Enforcement Example



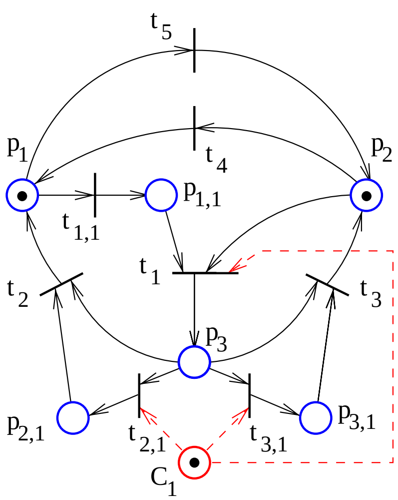
Target Petri net



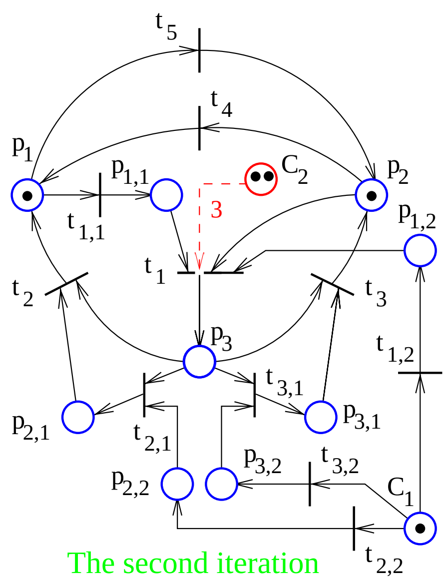
Transformation to PT−ordinary and AC PN



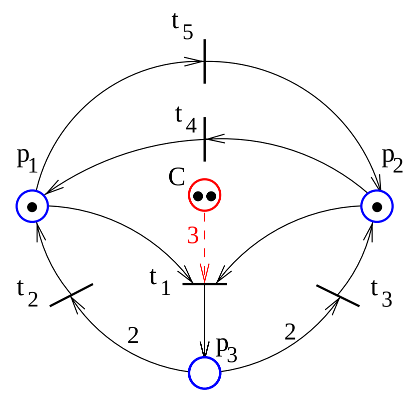
Active subnet



The first iteration



The second iteration



The final PN supervised
for $\{t_1, t_2\}$−liveness

$$L = [2, 2, 1],\ b = 2,\ L_0 = [\,] \text{ and } b_0 = [\,]$$

Performance

+ The procedure makes no assumption on the PN structure; it is applicable to PNs which may be unbounded and generalized. Furthermore, it can be extended to PNs with uncontrollable and unobservable transitions.

+ The procedure does not assume a given initial marking, but rather provides the constraints that the initial markings must satisfy for the supervisor to be effective.

+ If the procedure terminates and the PN has a single $\mathcal{T}$-minimal active subnet, the procedure provides the least restrictive $\mathcal{T}$-liveness enforcing supervisor.

+ When the procedure is used for liveness enforcement, the whole net is the single $\mathcal{T}$-minimal active subnet. Therefore, the supervisors generated by the procedure in this case are least restrictive.

− Procedure termination is not guaranteed.

− The procedure will not terminate for any PN with a single $\mathcal{T}$-minimal active subnet and with the property that the set of markings for which $\mathcal{T}$-liveness can be enforced is not the set of integer points of a polyhedron.

Performance

− The procedure may perform in each iteration computationally expensive operations (checking whether a siphon is uncontrolled may involve solving integer programs; finding the minimal siphons of a PN may also be computationally complex).

+ All computations are performed off-line. Very little computation is required to run a supervisor on-line.

+ The procedure allows fully automated computer implementation (and we have implemented it).