A Survey of the Supervision of Petri Nets

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Introduction

PN models arise naturally in the context of DES containing subsystems operating in parallel.

Most supervisory control results are obtained for automata models.

Automata methods are a special case of PN methods:

1. Transform given PN to an equivalent automaton (reachability graph).
2. Apply automata methods on the reachability graph.

Drawback: the reachability graph may not be finite or its size may be exponentially related to the size of the PN.

Could it be that an exponential complexity PN method is faster than a polynomial complexity automata method?

This presentation focuses on PN methods that avoid direct reachability analysis.

PN structure

- unrestricted
- special
  - marked graphs ($|\bullet p| = |p \bullet| = 1$)
  - state machines ($|\bullet t| = |t \bullet| = 1$)
  - others . . .

Labeling

- free-labeled
- labeled
- double-labeled

Controllability/Observability type

- Individually controllable/observable transitions
- Controlled PNs
- State observation

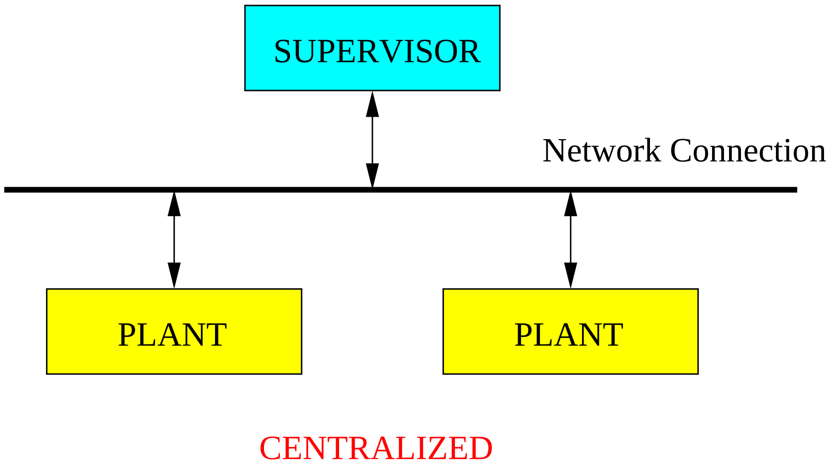
Concurrency:

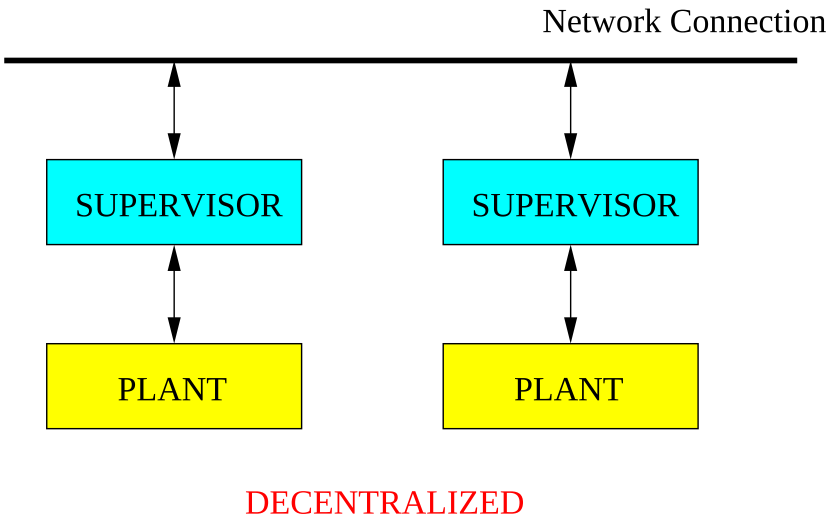
- no concurrency assumption *(*$q \in \{0,1\}^n$ *and* $\sum q_i \leq 1$*.)*
- concurrency assumption *(*$q \in \{0,1\}^n$*.)*
- transition bag assumption *(*$q \in \mathbb{N}^n$*.)*

- Marking Constraints
  - In particular, Linear Marking Constraints [$L\mu \leq b$, where $L \in \mathbb{Z}^{p \times m}$ and $b \in \mathbb{Z}^p$.]
- Extensions [Iordache and Antsaklis]
  - Generalized Linear Constraints [$L\mu + Hq + Cv \leq b$]
  - Language Constraints [the spec. is a labeled PN]
  - Disjunctive Constraints [$\bigvee_i L_i\mu \leq b_i$]

- Centralized
- Decentralized
  - with no communication
  - with communication

  [Iordache and Antsaklis, 2003]



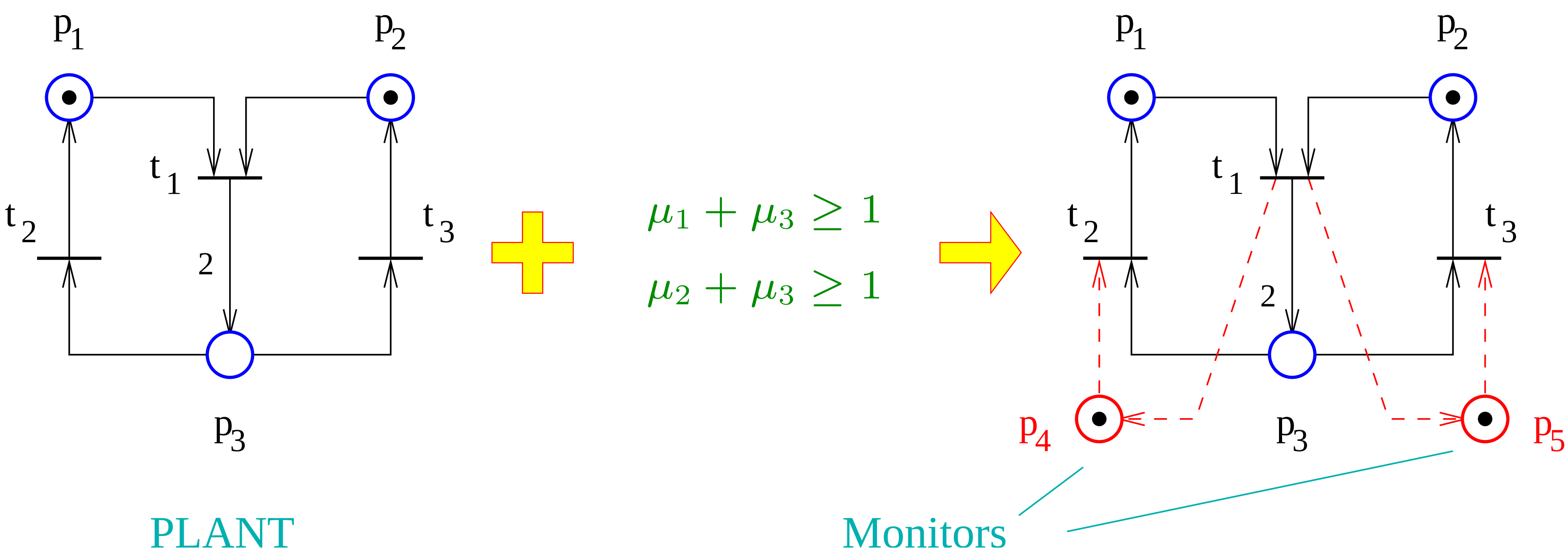


The *supervision based on place invariants* for constraints $L\mu \leq b$.

(All reachable markings μ constrained to satisfy $L\mu \leq b$.)

Let D be the incidence matrix and μ_0 the initial marking.

The supervisor consists of *monitor places* connected according to $D_s = -LD$ and of initial marking $\mu_{s0} = b - L\mu_0$.



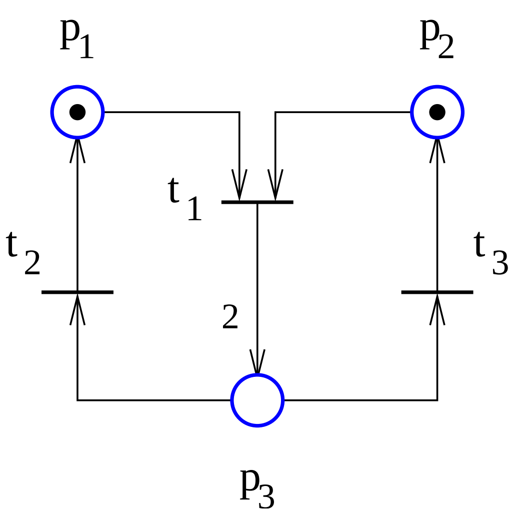
Marking Constraints — The SBPI

The SBPI has been studied by Giua et al (1992), Chen and Hu (1994), Li and Wonham (1994), Yamalidou et al (1996), Moody and Antsaklis (1998), Stremersch (2001).

Main difficulty: supervision with (individually) uncontrollable and unobservable transitions.

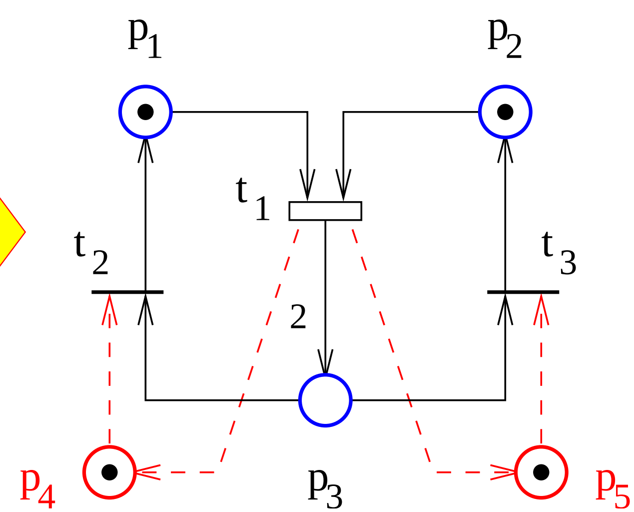
One approach has been to replace the original specification $L\mu \leq b$ with stronger constraints $L_a\mu \leq b_a$ subject to the structural admissibility conditions

$$L_a D(\cdot, T_{uo}) = 0 \text{ and } L_a D(\cdot, T_{uc}) \leq 0$$



$$\mu_1 + \mu_3 \geq 1$$

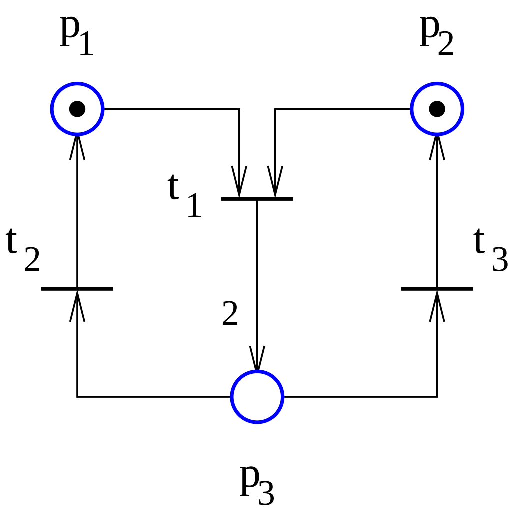
$$\mu_2 + \mu_3 \geq 1$$



Problem if t_1 unobservable!

Possible solution . . .

$$\begin{cases} \mu_1 + \mu_3 \geq 1 \\ \mu_2 + \mu_3 \geq 1 \end{cases}$$ (inadmissible) $$\Rightarrow \begin{cases} 2\mu_1 + \mu_3 \geq 1 \\ 2\mu_2 + \mu_3 \geq 1 \end{cases}$$ (admissible)

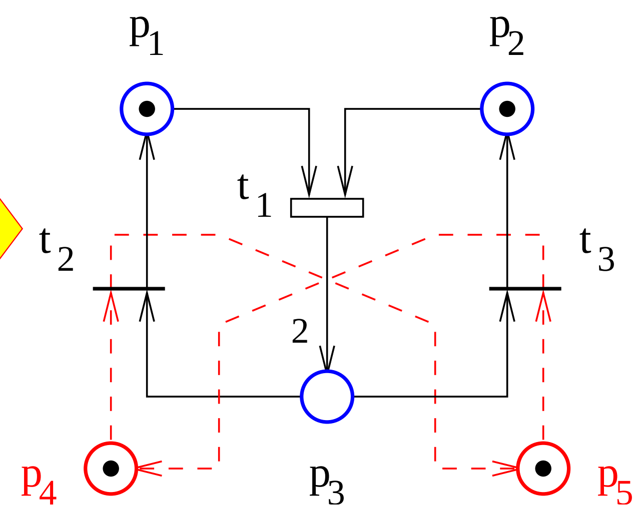


$+$

$$2\mu_1 + \mu_3 \geq 1$$

$$2\mu_2 + \mu_3 \geq 1$$

$\Rightarrow$



Parameterization based solution: Moody and Antsaklis, 1998.

ILP minimizing observation and control costs: Basile et al, 2000.

Solution in disjunctive form: Chen, 2000.

Optimal under the conditions of Chen and Hu 1994, and Chen 1998.

Solution in disjunctive form: Stremersch 2001.

Optimal w.r.t. the structural admissibility conditions.

Assumes all rows of L have the same sign.

Structural admissibility conditions for labeled PNs have the same general form as for free-labeled PNs (Iordache and Antsaklis, 2004).

This means that the above methods can be applied to labeled PNs as well.

Marking Constraints

A different idea: compute the *maximal controlled invariant set*

$$\mathcal{A}_F = \{\mu : \mathcal{R}(\mathcal{N}_u, \mu) \cap \mathcal{M}_F = \emptyset\} \text{ (Krogh and Holloway, 1991)}$$

$\mathcal{N}_u$: the uncontrollable subnet; $\mathcal{M}_F$: the set of forbidden markings.
Computing $\mathcal{A}_F$ solves our problem, assuming full observability.

Chen and Hu, 1994, obtain the set $\mathcal{A}_F$ in disjunctive form.

L: assumed to have nonnegative elements.
the influential subnets (certain subnets of $\mathcal{N}_u$) should be marked graphs.

Chen, 1998, obtains the set $\mathcal{A}_F$ in conjunctive form.

L: assumed to have nonnegative elements.
the influential subnets (certain subnets of $\mathcal{N}_u$) should be state machines.

Li and Wonham, 1994, obtain $\mathcal{A}_F$ in conjunctive and disjunctive form, assuming certain subnets to have a particular form.

Marking Constraints

Reachability graph approach in Ghaffari et al, 2003b.

- *A monitor based supervisor is designed; each monitor found by linear programming.*
- *Blocking states are avoided.*

Results for PNs that are marked graphs:

Ghaffari et al, 2003a, propose solving online a linear program after each firing of a controllable transition.

Darondeau and Xie, 2003, approach the supervision of live marked graphs. The paper shows

- *how to find the least restrictive supervisor*
- *that the least restrictive supervisor corresponds, in general, to a disjunctive form*
- *that even in the conjunctive form, the number of constraints may be exponential in terms of the size of* $D(\cdot, T_{uc})$.

The setting allows for both uncontrollable and unobservable transitions.

Marking Constraints

CtlPN results:

- *Typically: under the concurrency assumption and assuming full observability.*
- *The specification is to avoid a set of forbidden markings $\mathcal{M}_F$.*

Holloway and Krogh, 1990 and 1991: safe, cyclic marked graph structure assumed. A supervisor evaluates online predicates based on paths found in offline analysis.

Zhang and Holloway, 1995, consider also unobservable transitions.

Holloway and Krogh, 1992, sufficient conditions for the closed-loop to be live.

Boel et al, 1995, approach the problem for CtlPNs with a state machine structure and forbidden set described by a disjunction $\bigvee_i L_i\mu \geq b_i$, all L_i having nonnegative elements.

Holloway et al, 1996, extension to arbitrary PN structures. Assumes a particular form of $\mathcal{M}_F$.

Other Constraints

We have looked at marking constraints in the following two cases

I. requiring $L\mu \leq b$.

II. requiring a set $\mathcal{M}_F$ to be avoided.

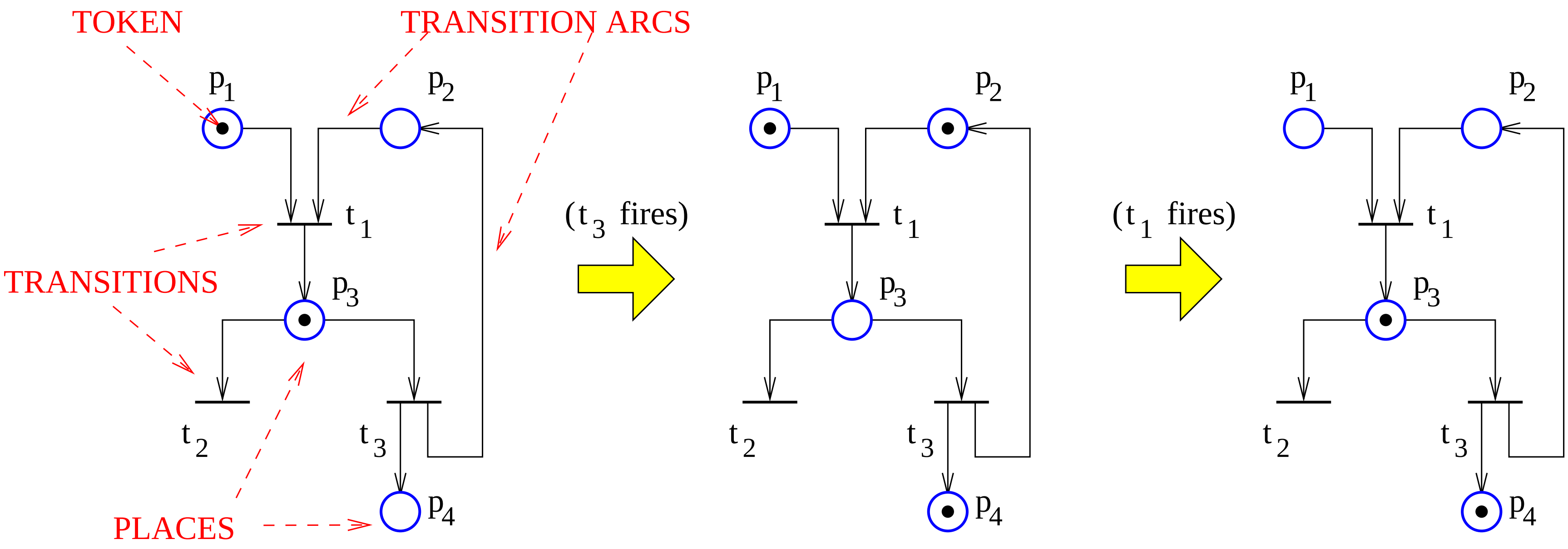
Some of our recent results approach the enforcement of a wider class of constraints

- Generalized Linear Constraints $(L\mu + Hq + Cv \leq b)$
- Language Constraints *(the specification is a labeled PN)*
- Disjunctive Constraints $(\bigvee L_i\mu \leq b_i)$

Generalized Linear Constraints

Notation: μ – the marking, μ_0 – the initial marking, D – the incidence matrix, q – the firing vector, and v – the Parikh vector. Let μ_i denote $\mu(p_i)$ and v_j denote $v(t_j)$.

The state equation: $\mu = \mu_0 + Dv$.



$\mu_0 = [\ 1\ \ 0\ \ 1\ \ 0\ ]^T$ $\quad$ $\mu' = [\ 1\ \ 1\ \ 0\ \ 1\ ]^T$ $\quad$ $\mu'' = [\ 0\ \ 0\ \ 1\ \ 1\ ]^T$

$v = [\ 0\ \ 0\ \ 0\ ]^T$ $\quad$ $v = [\ 0\ \ 0\ \ 1\ ]^T$ $\quad$ $v = [\ 1\ \ 0\ \ 1\ ]^T$

$q = [\ 0\ \ 0\ \ 1\ ]^T$ $\quad$ $q = [\ 1\ \ 0\ \ 0\ ]^T$

Generalized Linear Constraints

The *generalized linear constraints* can describe places arbitrarily connected to a PN.

They have the form:

$$L\mu + Hq + Cv \leq b \tag{1}$$

They require the initial state (μ_0, v_0) to satisfy

$$L\mu_0 + Cv_0 \leq b$$

Further, a transition t_i may fire from a current state (μ, v) iff

(a) $L\mu + Hq + Cv \leq b$ for $q(i) = 1$ and $q(j) = 0 \ \forall j \neq i$.

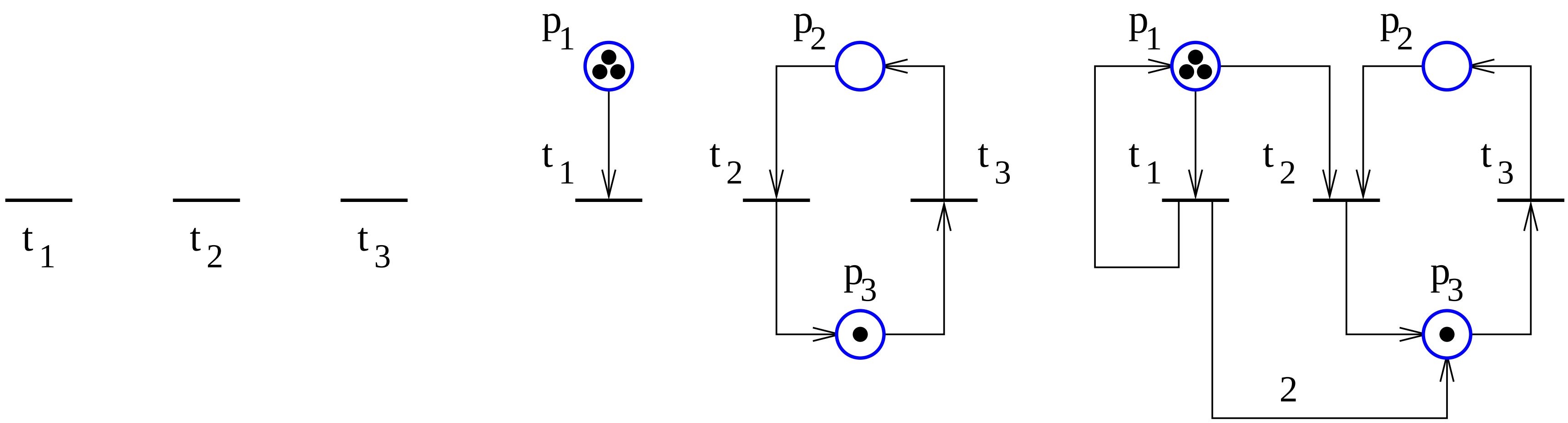
(b) $L\mu' + Cv' \leq b$, where $v' = v + q$ and $\mu \xrightarrow{t_i} \mu'$.

The generalized linear constraints describe the P-type languages of free-labeled PNs.

The enforcement of the *generalized linear constraints* has been studied by Iordache and Antsaklis [ACC 2002, TAC 48(11)].

Generalized Linear Constraints

The generalized linear constraints describe the P-type languages of free-labeled PNs.



(a) (b) (c)

unconstrained operation

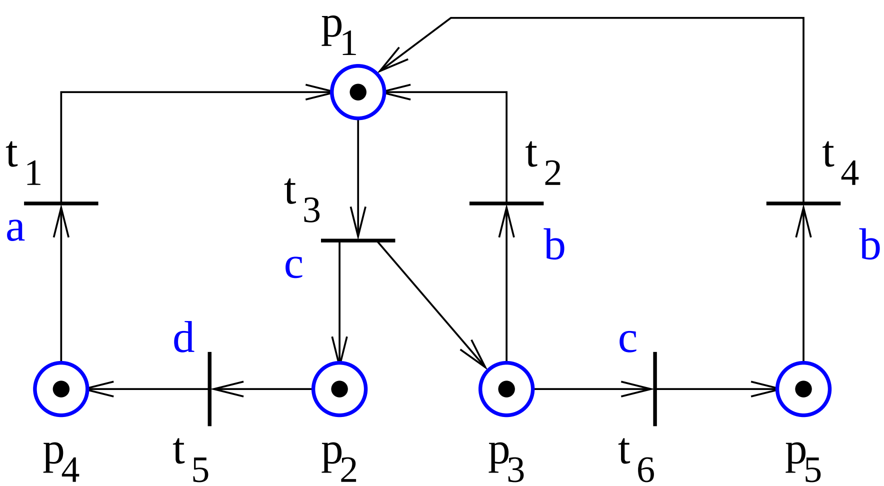
(p_1)	v_1	$\leq$	3	$q_1 + v_2$	$\leq$	3
(p_2)	$v_2 - v_3$	$\leq$	0	$v_2 - v_3$	$\leq$	0
(p_3)	$-v_2 + v_3$	$\leq$	1	$-2v_1 - v_2 + v_3$	$\leq$	1

Language Constraints

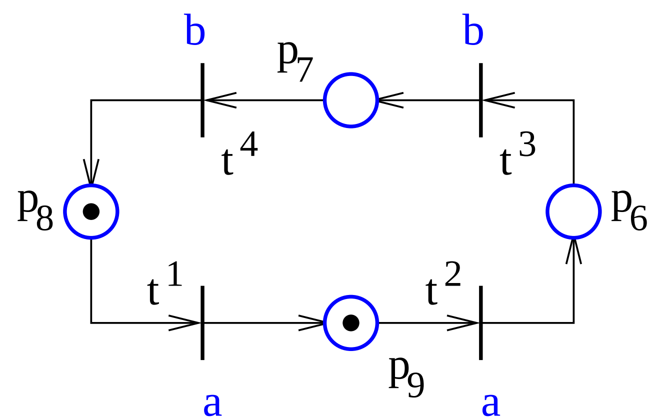
Generalized linear constraints correspond to P-type languages of *free-labeled* PNs.

We can deal also with the enforcement of P-type languages of general labeled PNs.

In the following example, the event a is *uncontrollable*.



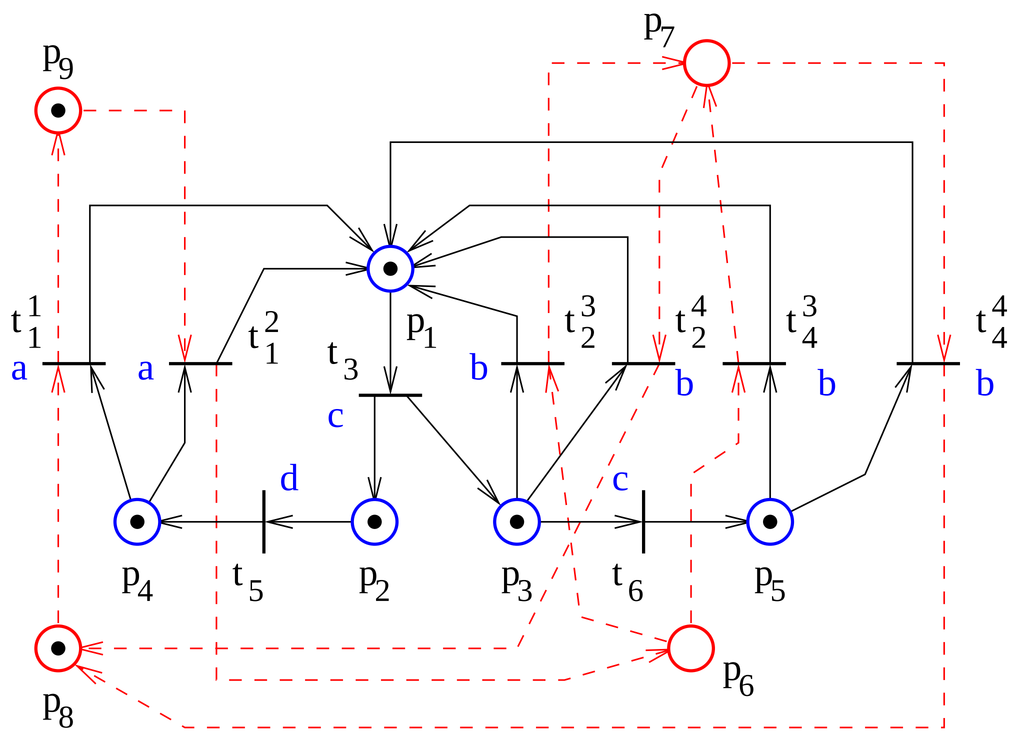
PLANT



SPECIFICATION

Language Constraints — Method–Step 1

The first step is to compose the plant and specification models.



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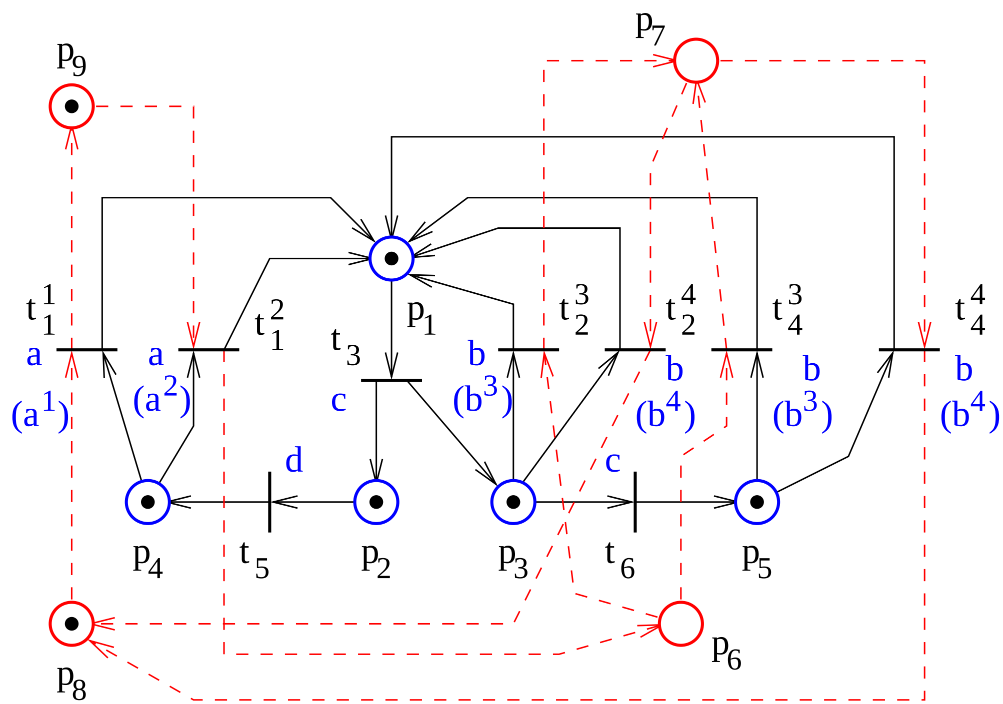


Because the supervisor can distinguish between its own transitions, we can relabel the net to take in account this fact.

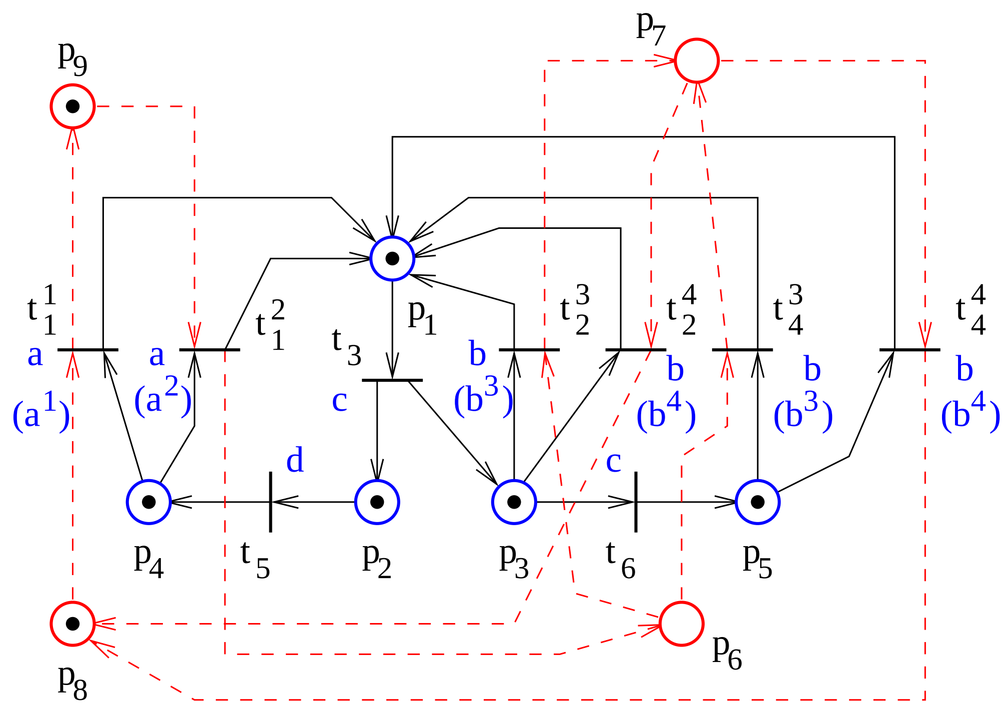
Next, we can identify the constraints associated with the supervisor places:

(p_6) $v_2^3 + v_4^3 - v_1^2 \leq 0$

(p_7) $v_2^4 + v_4^4 - v_2^3 - v_4^3 \leq 0$

(p_8) $v_1^1 - v_2^4 - v_4^4 \leq 1$

(p_9) $v_1^2 - v_1^1 \leq 1$



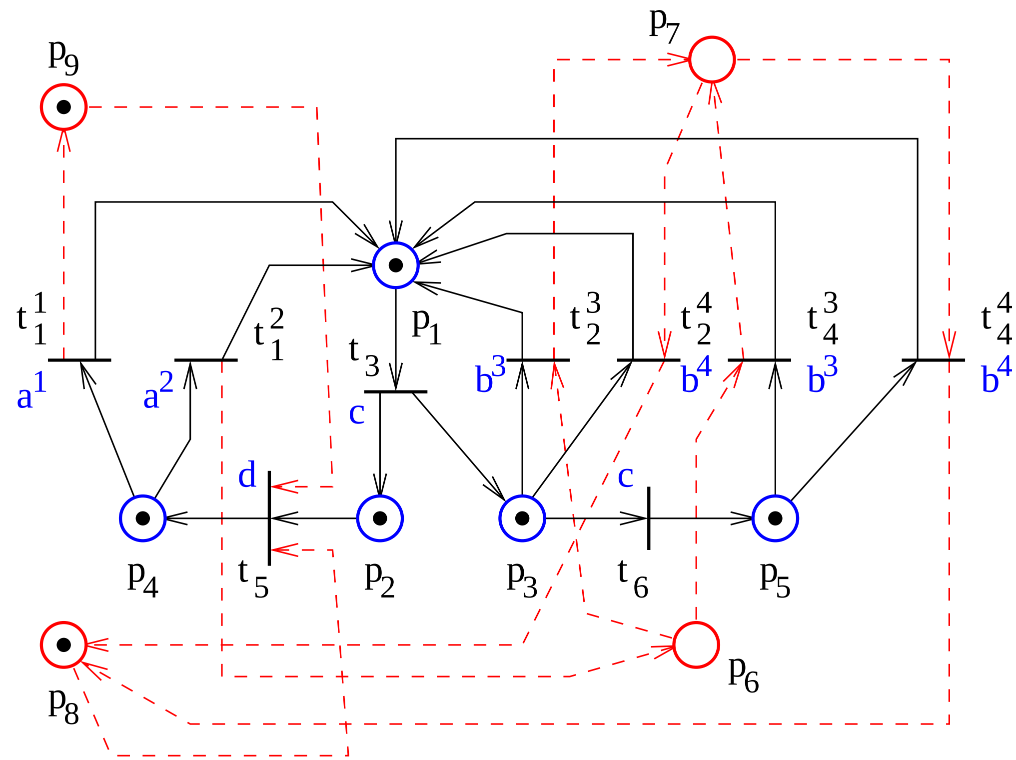
Then, the constraints can be transformed to an admissible form

$(p_6) \quad v_2^3 + v_4^3 - v_1^2 \leq 0$

$(p_7) \quad v_2^4 + v_4^4 - v_2^3 - v_4^3 \leq 0$

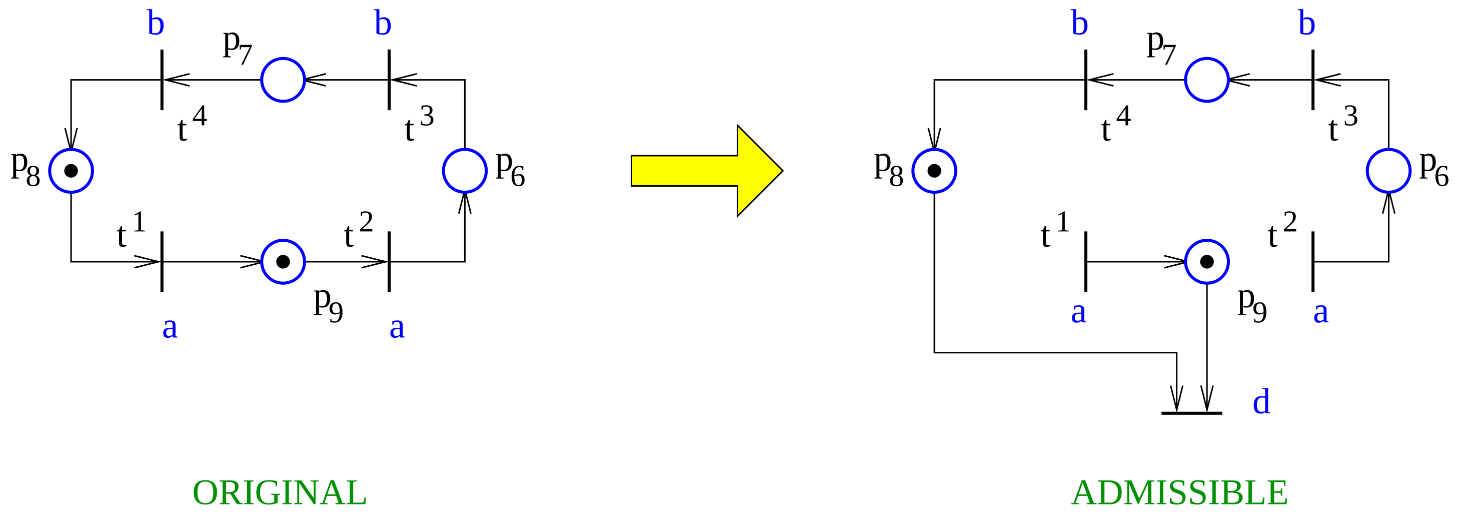
$(p_8) \quad v_1^1 - v_2^4 - v_4^4 + \mu_4 \leq 1$

$(p_9) \quad v_1^2 - v_1^1 + \mu_4 \leq 1$



Language Constraints

This process has changed the specification such that it is admissible:



The closed-loop generates a sublanguage of the original specification.

Disjunctive Constraints

Disjunctions have the form

$$\bigvee_i L_i \mu \leq b_i$$

where $L_i \in \mathbb{Z}^{m_i \times n}$ and $b_i \in \mathbb{Z}_i^m$, or equivalently

$$\bigwedge_j \bigvee_{i \in A_j} l_i \mu \leq c_i$$

where $l_i \in \mathbb{Z}^{1 \times n}$, $c_i \in \mathbb{Z}$ and A_j is a set of integers.

We can apply here literature results that reduce propositional logic to inequalities, by means of auxiliary variables.

Disjunctive Constraints

Let δ_i be auxiliary variables:

$$[\delta_i = 1] \leftrightarrow [l_i\mu \le c_i] \tag{2}$$

Thus,

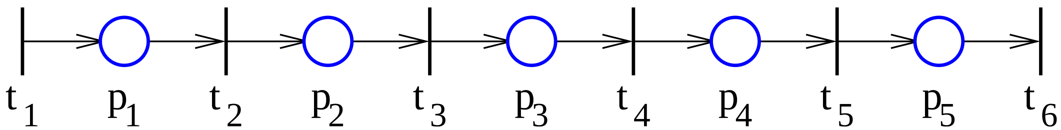
$$\sum_{i \in A_j} \delta_i \ge 1 \leftrightarrow \bigvee_{i \in A_j} l_i\mu \le c_i \tag{3}$$

Assuming $l_i\mu$ is bounded, $m_i \le l_i\mu \le M_i$, $[\delta_i = 1] \leftrightarrow [l_i\mu \le c_i]$ becomes

$$l_i\mu + (M_i - c_i)\delta_i \quad \le \quad M_i \tag{4}$$

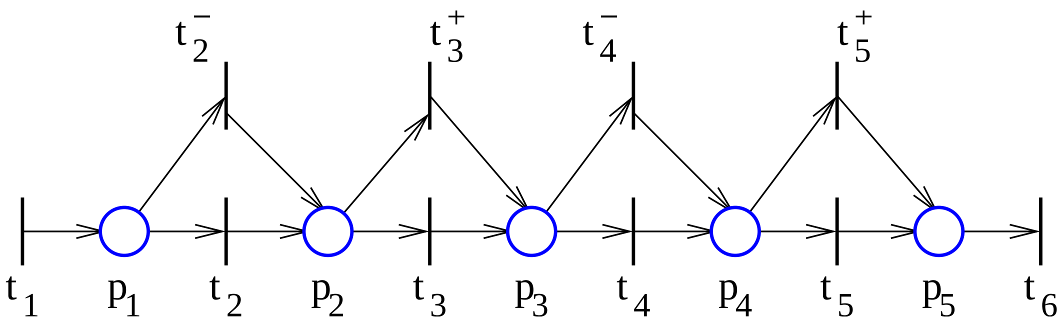
$$l_i\mu + (c_i + 1 - m_i)\delta_i \quad \ge \quad c_i + 1 \tag{5}$$

Given are a PN and a specification



$$[\mu_2 \leq 0] \vee [\mu_4 \leq 0] \quad (6)$$

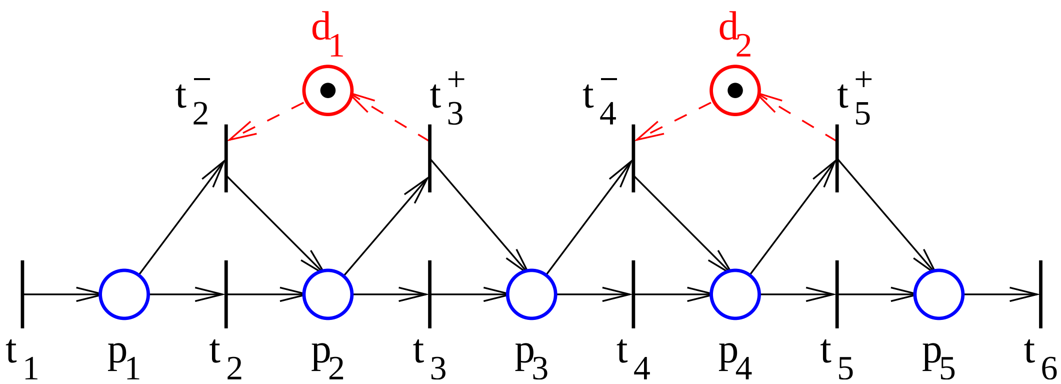
The first step is to identify and create copies of the transitions that increase/decrease $l_i\mu$



For $\mu_2 \leq 0$: $T_1^- = \{t_2^-\}$ and $T_1^+ = \{t_3^+\}$.

For $\mu_4 \leq 0$: $T_2^- = \{t_4^-\}$ and $T_2^+ = \{t_5^+\}$.

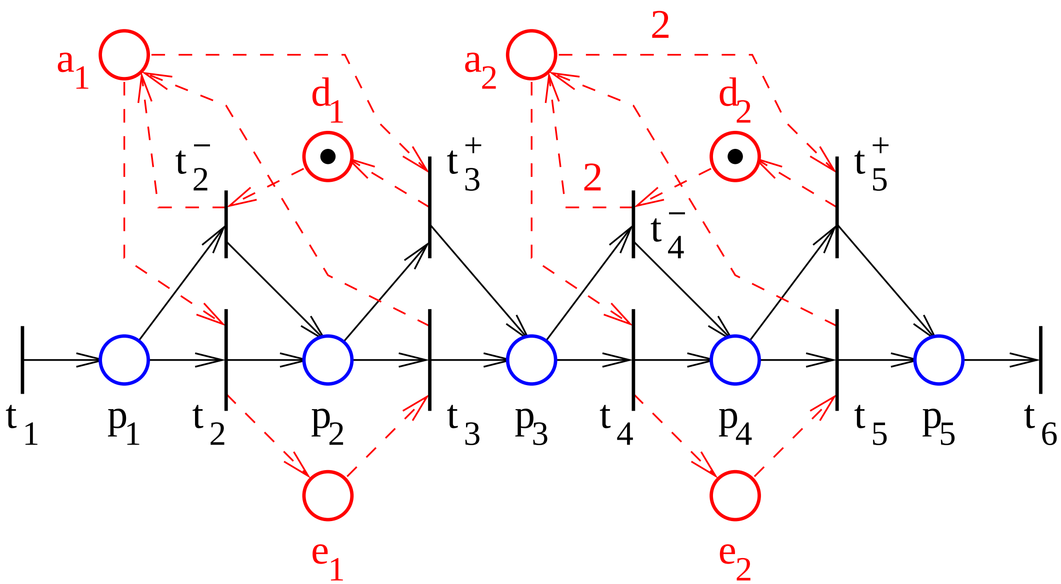
The second step is to add places d_i as input (output) places to T_i^- (T_i^+):



We intend to achieve $\delta_i = \mu(d_i)$.

The next step is to enforce the constraints involving the auxiliary variables.

Assume $m_1 = m_2 = 0$ and $M_1 = 2$ and $M_2 = 3$.



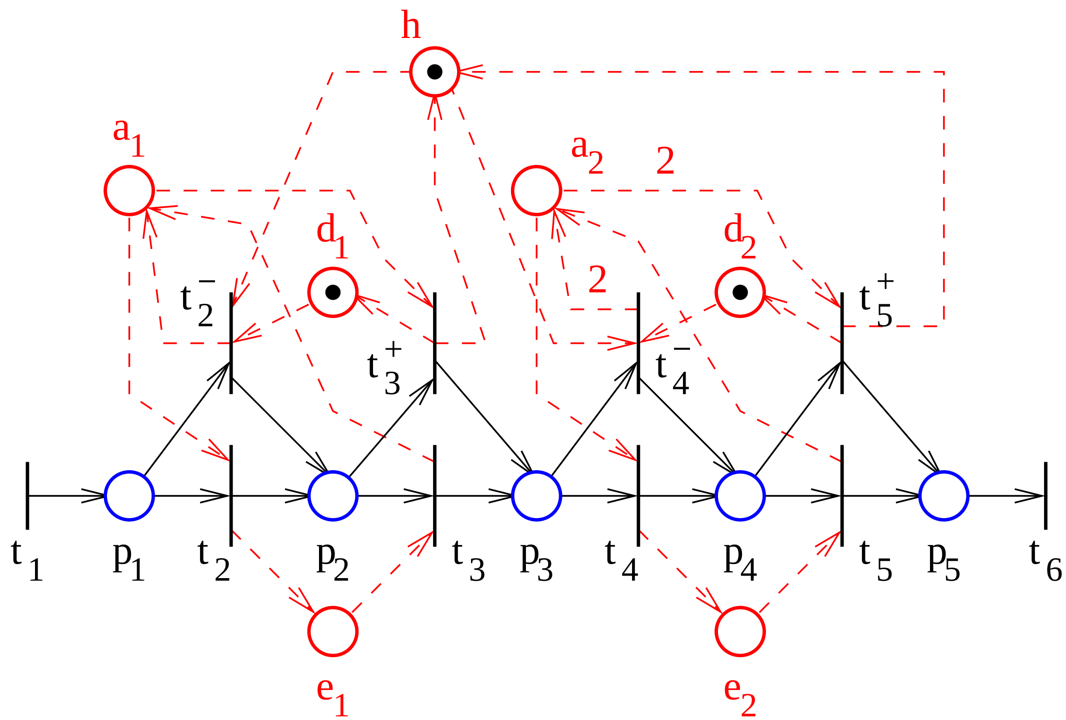
$$\begin{array}{rcll} \mu_2 + 2\delta_1 & \leq & 2 & [a_1] \\ \mu_2 + \delta_1 & \geq & 1 & [e_1] \\ & & & \\ \mu_4 + 3\delta_2 & \leq & 3 & [a_2] \\ \mu_4 + \delta_2 & \geq & 1 & [e_2] \end{array}$$

Now, $[\mu(d_i) = 1] \leftrightarrow [l_i \mu \leq c_i]$ is enforced.

We only need to enforce $\mu(d_1) + \mu(d_2) \geq 1$ to finish our problem.

Since $[\mu(d_i) = 1] \leftrightarrow [l_i \mu \leq c_i]$ is already enforced, we only need to enforce

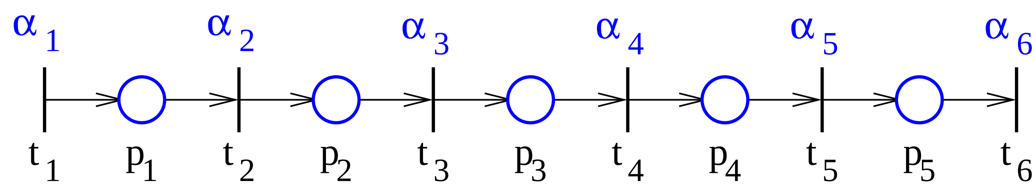
$$\mu(d_1) + \mu(d_2) \geq 1$$



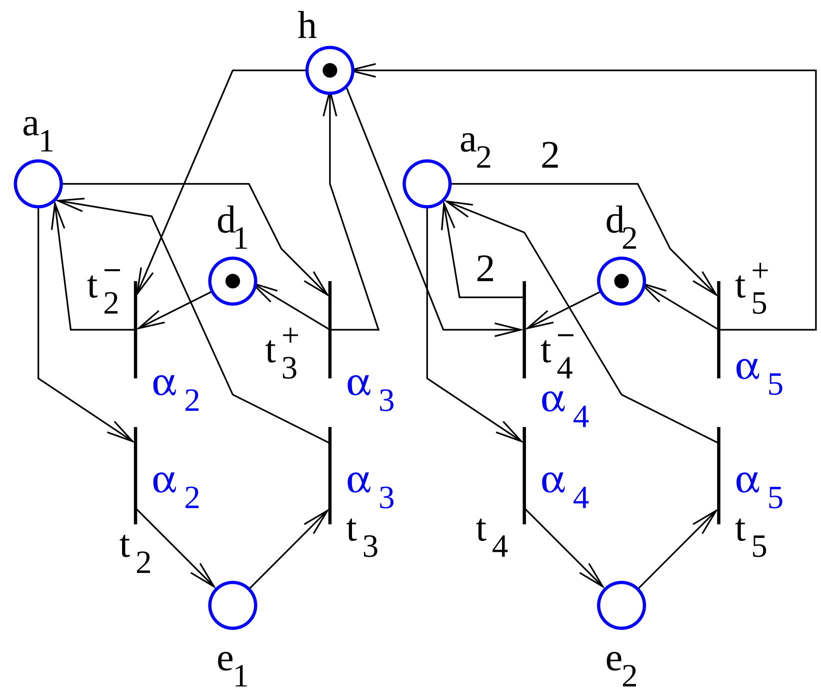
The place h is obtained.

Disjunctive Constraints

Our previuos developments describe a closed-loop system corresponding to the composition of the plant



with the following supervisor:



Note that the supervisor is not free-labeled (though the plant is).

Final Remarks

PN supervision promises computational benefits.

Various methods are available for various classes of PNs and specifications.

A class of structural methods that are suboptimal are available for

- conjunctive linear constraints
- disjunctive linear constraints (under certain boundedness assumptions)
- P-language specifications

Typically, PN supervision methods do not incorporate deadlock-prevention in the specification. Additional methods need to be applied to ensure the closed-loop is live.

Supervision methods that result in PN supervisors have the benefit that they allow the application of liveness enforcement procedures developed for PNs.