A Structural Approach to the Enforcement of Language and Disjunctive Constraints

Marian V. Iordache
School of Engineering and Eng. Tech.
LeTourneau University
Longview, TX 75607-7001

Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

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Outline

There are already numerous results on the enforcement of linear constraints $L\mu \leq b$.

Can we deal with labeled PNs?

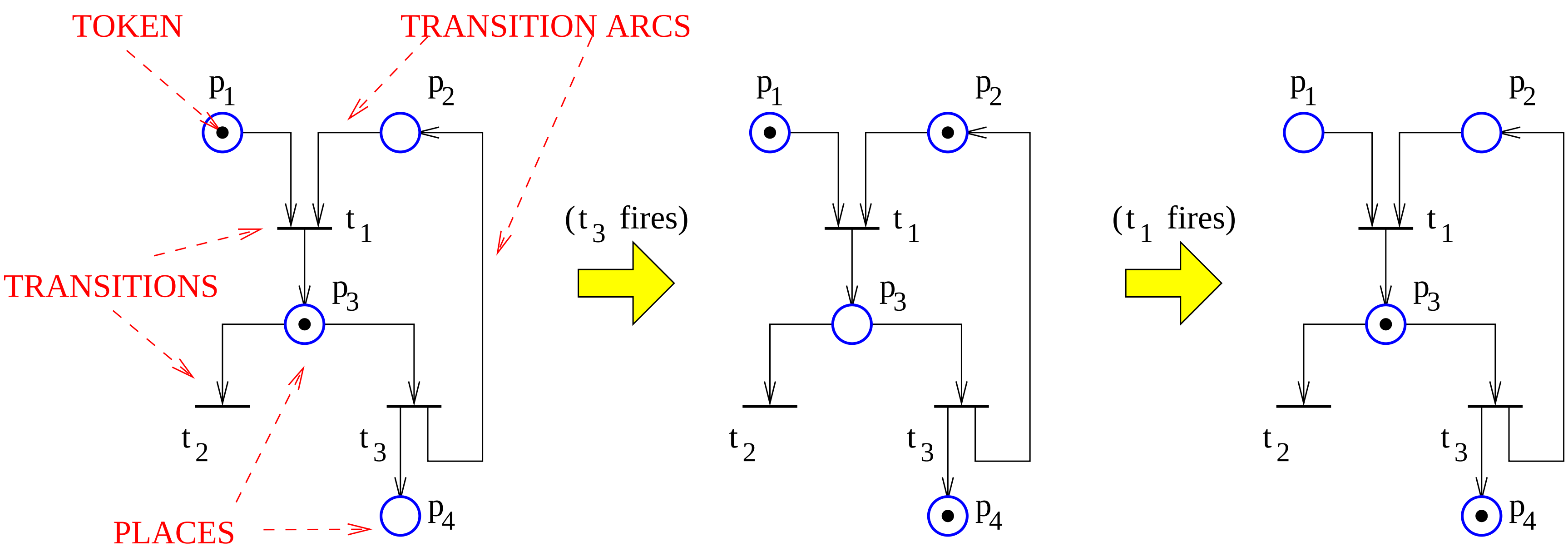
Can we deal with more general specifications?

We address the following topics:

- Background: Generalized Linear Constraints ($L\mu + Hq + Cv \leq b$)
- Language Constraints
- Disjunctive Constraints ($\bigvee_i L_i\mu \leq b_i$)

Notation: μ – the marking, μ_0 – the initial marking, D – the incidence matrix, q – the firing vector, and v – the Parikh vector. Let μ_i denote $\mu(p_i)$ and v_j denote $v(t_j)$.

The state equation: $\mu = \mu_0 + Dv$.



$$\mu_0 = [\ 1\quad 0\quad 1\quad 0\ ]^T \qquad \mu' = [\ 1\quad 1\quad 0\quad 1\ ]^T \qquad \mu'' = [\ 0\quad 0\quad 1\quad 1\ ]^T$$
$$v = [\ 0\quad 0\quad 0\ ]^T \qquad v = [\ 0\quad 0\quad 1\ ]^T \qquad v = [\ 1\quad 0\quad 1\ ]^T$$
$$q = [\ 0\quad 0\quad 1\ ]^T \qquad q = [\ 1\quad 0\quad 0\ ]^T$$

The *generalized linear constraints* can describe places arbitrarily connected to a PN.

They have the form:

$$L\mu + Hq + Cv \leq b \tag{1}$$

They require the initial state (μ_0, v_0) to satisfy

$$L\mu_0 + Cv_0 \leq b$$

Further, a transition t_i may fire from a current state (μ, v) iff

(a) $L\mu + Hq + Cv \leq b$ for $q(i) = 1$ and $q(j) = 0\ \forall j \neq i$.

(b) $L\mu' + Cv' \leq b$, where $v' = v + q$ and $\mu \xrightarrow{t_i} \mu'$.

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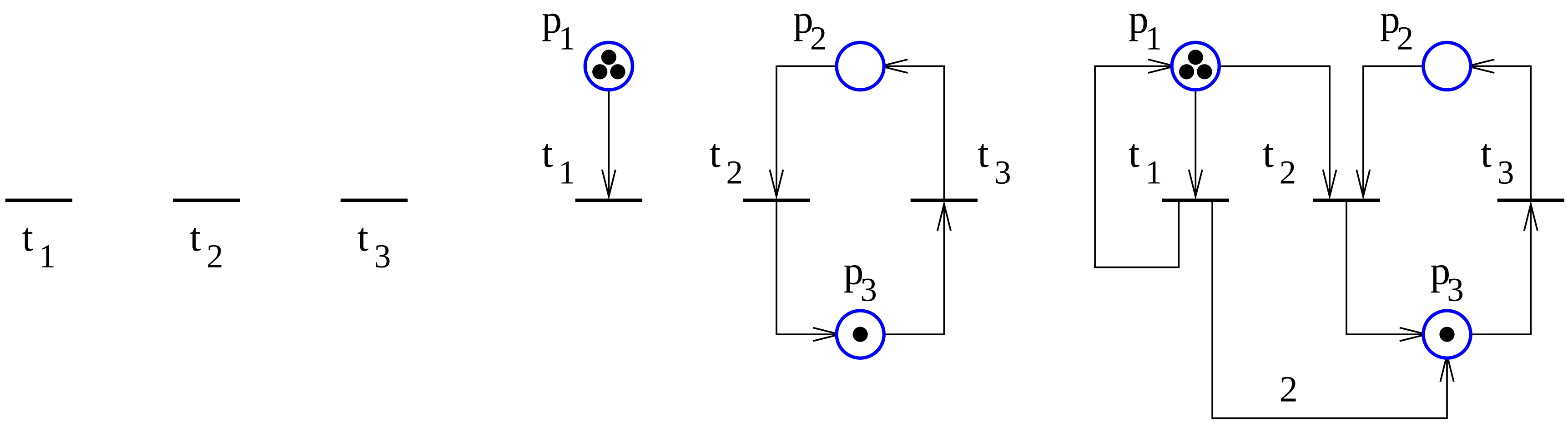


(a) (b) (c)

unconstrained operation

$$
\begin{array}{lrcl}
(p_1) & v_1 & \leq & 3 \\
(p_2) & v_2 - v_3 & \leq & 0 \\
(p_3) & -v_2 + v_3 & \leq & 1
\end{array}
\qquad
\begin{array}{rcl}
q_1 + v_2 & \leq & 3 \\
v_2 - v_3 & \leq & 0 \\
-2v_1 - v_2 + v_3 & \leq & 1
\end{array}
$$

The enforcement of the *generalized linear constraints* has been studied by Iordache and Antsaklis [ACC 2002, TAC 48(11)].

This approach has been followed:

Enforce admissible $L\mu + Hq + Cv \leq b$ *by simple matrix operations.*

If $L\mu + Hq + Cv \leq b$ *are not admissible, find* $L_a\mu + H_aq + C_av \leq b$ *such that*

1. $L_a\mu + H_aq + C_av \leq b \Rightarrow L\mu + Hq + Cv \leq b$
2. $L_a\mu + H_aq + C_av \leq b$ *is admissible.*

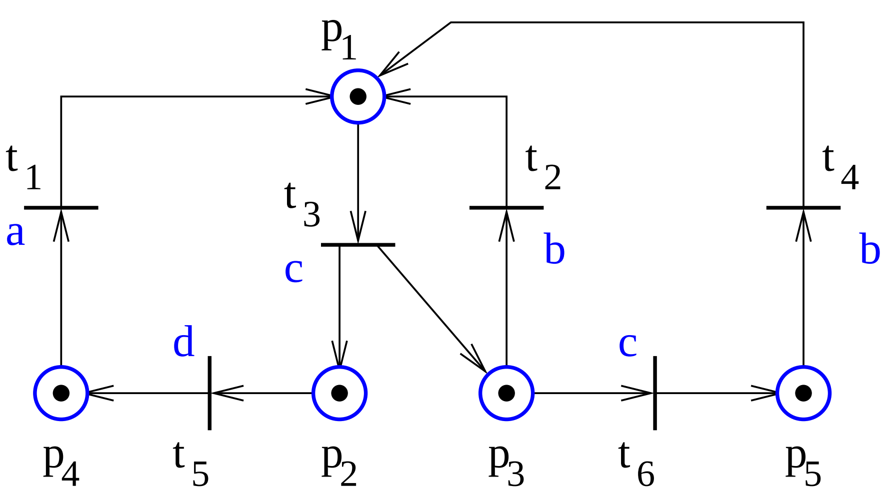
The approach used to find $L_a\mu + H_aq + C_av \leq b$ *reduces the problem to the enforcement of constraints* $L_t\mu_t \leq b$ *in a transformed PN, for which numerous methods are available.*

Language Constraints

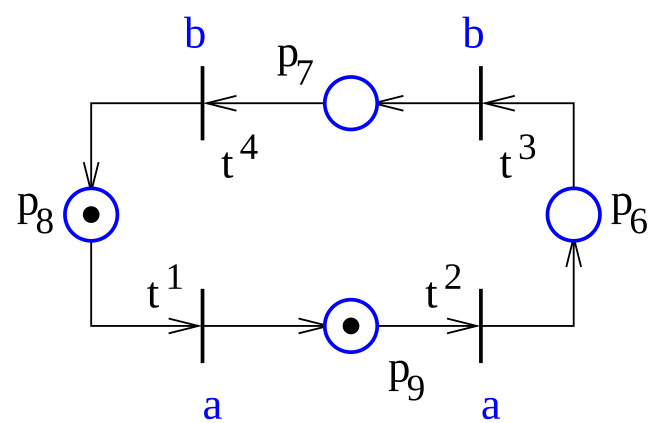
Generalized linear constraints correspond to P-type languages of *free-labeled* PNs.

We can deal also with the enforcement of P-type languages of general labeled PNs.

In the following example, the event a is *uncontrollable*.

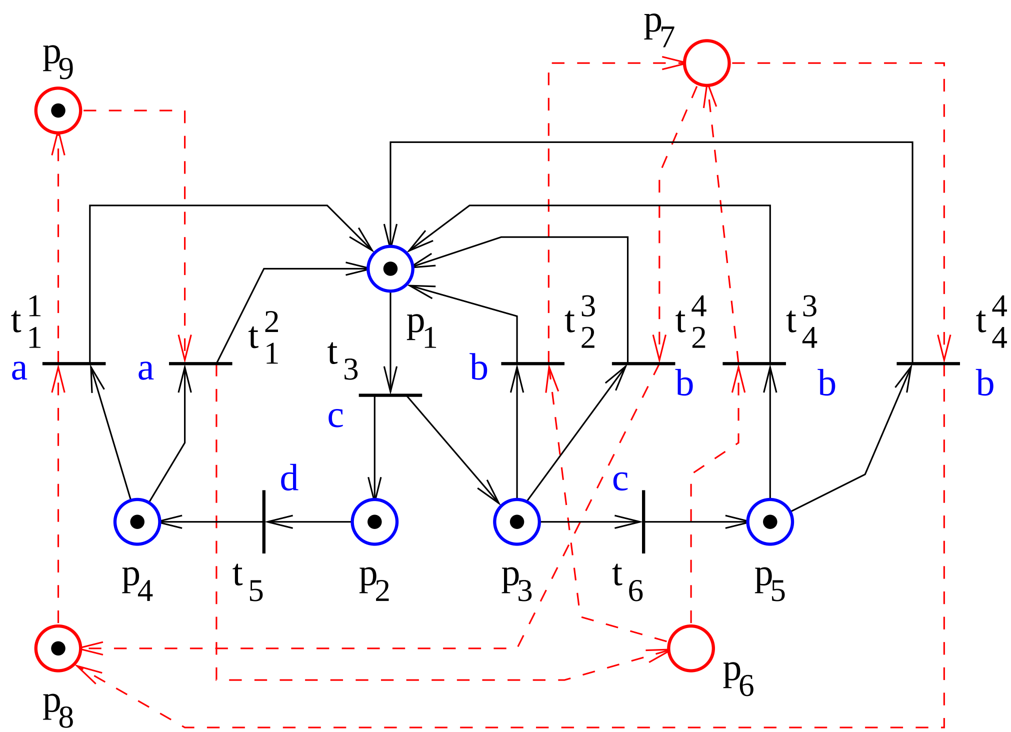


PLANT



SPECIFICATION

The first step is to compose the plant and specification models.



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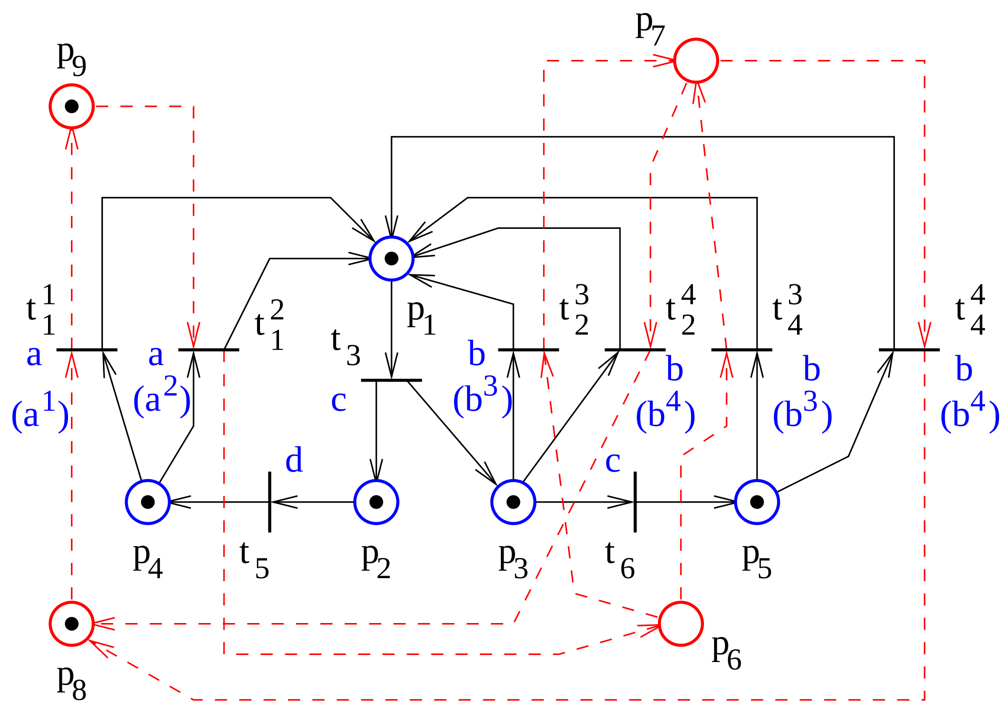


Because the supervisor can distinguish between its own transitions, we can relabel the net to take in account this fact.

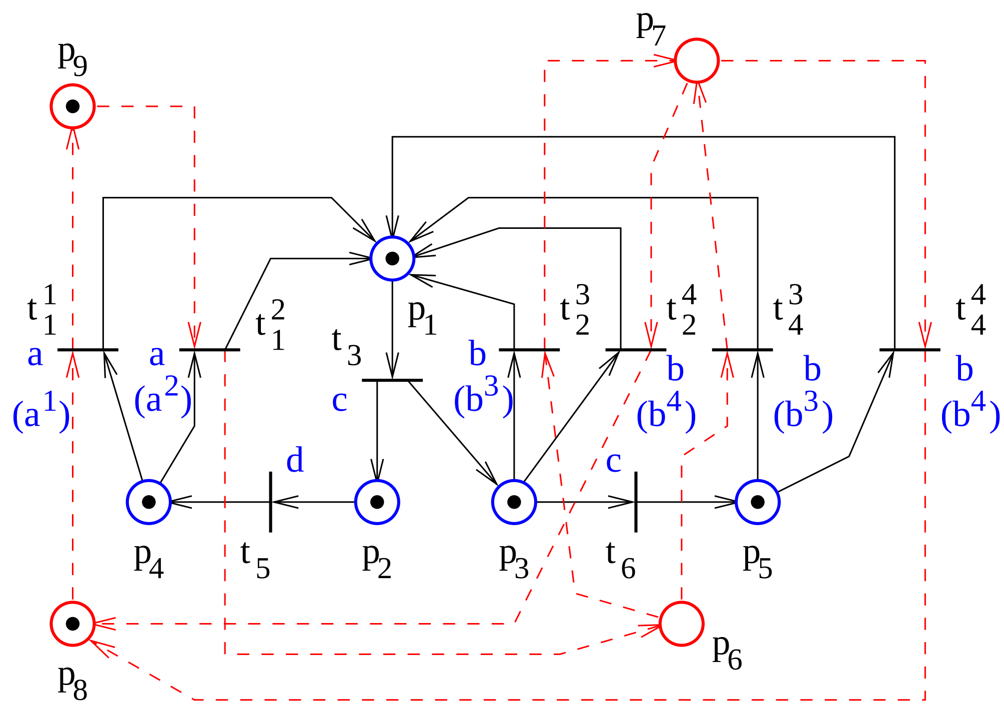
Next, we can identify the constraints associated with the supervisor places:

(p_6) $v_2^3 + v_4^3 - v_1^2 \leq 0$

(p_7) $v_2^4 + v_4^4 - v_2^3 - v_4^3 \leq 0$

(p_8) $v_1^1 - v_2^4 - v_4^4 \leq 1$

(p_9) $v_1^2 - v_1^1 \leq 1$



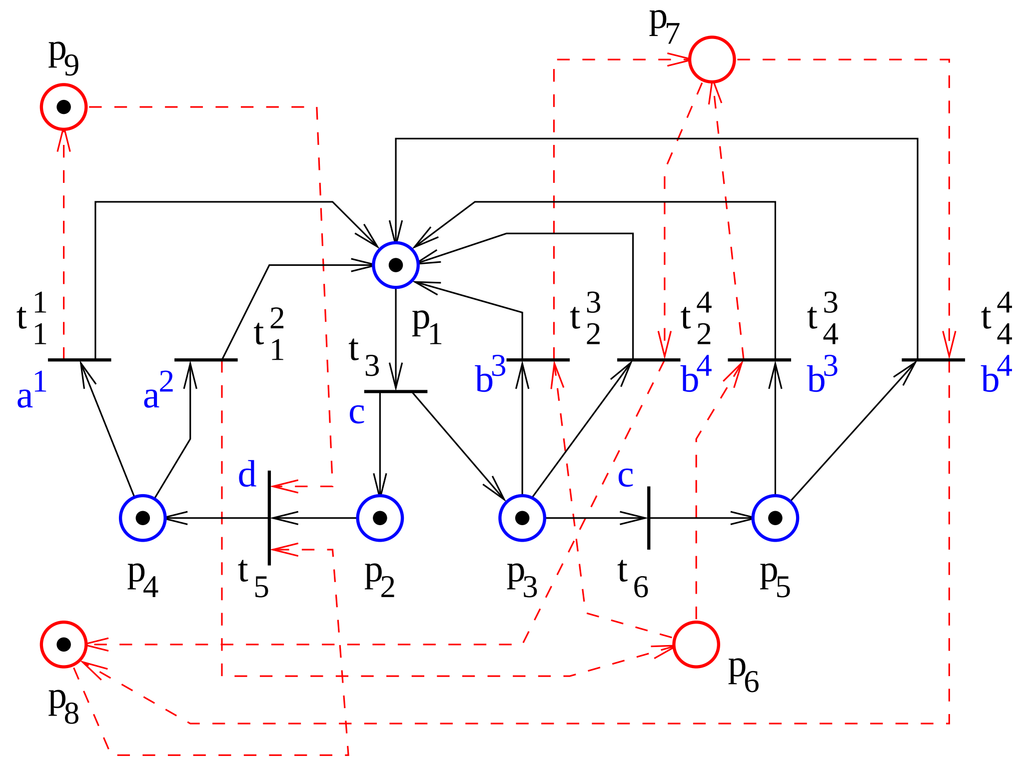
Then, the constraints can be transformed to an admissible form

$(p_6)\quad v_2^3 + v_4^3 - v_1^2 \leq 0$

$(p_7)\quad v_2^4 + v_4^4 - v_2^3 - v_4^3 \leq 0$

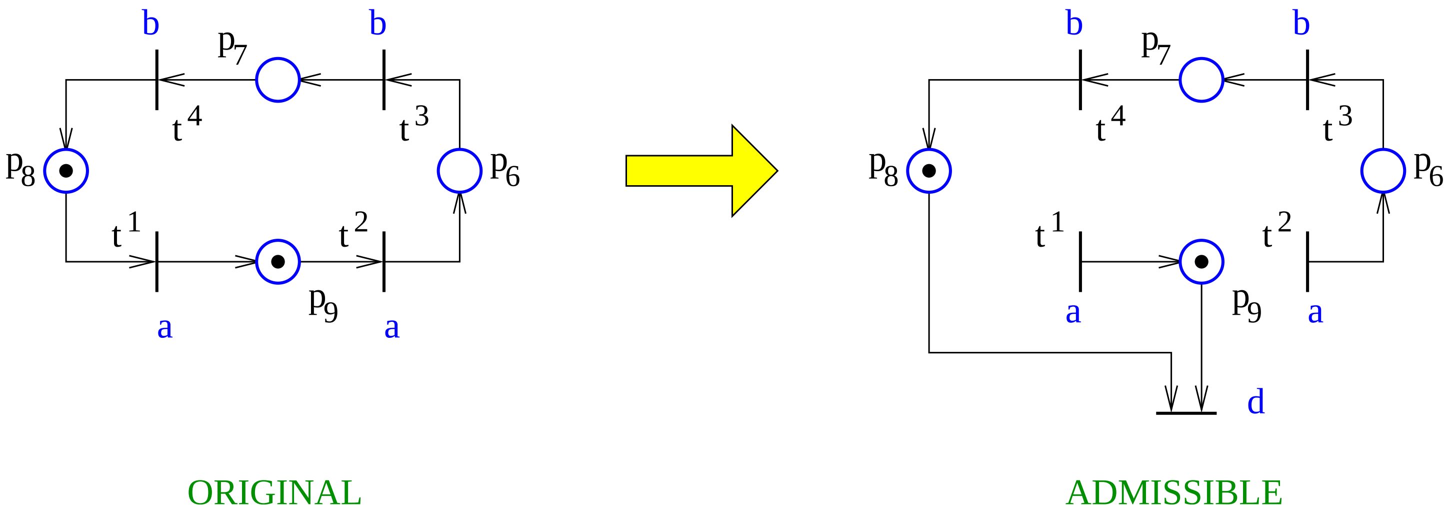
$(p_8)\quad v_1^1 - v_2^4 - v_4^4 + \mu_4 \leq 1$

$(p_9)\quad v_1^2 - v_1^1 + \mu_4 \leq 1$



Language Constraints

This process has changed the specification such that it is admissible:



The closed-loop generates a sublanguage of the original specification.

Language Constraints

Admissibility: A language specification is *admissible* if the composition plant-supervisor is such that the supervisor never attempts to inhibit plant-enabled *uncontrollable* transitions, or detect closed-loop-enabled *unobservable* transitions, or distinguish between plant transitions with the same label.

The generalized constraint approach is extended to labeled PNs to:

- Reduce the enforcement of $L\mu + Hq + Cv \leq b$ in a labeled PN to the enforcement of $L_e\mu_e \leq b_e$ in a different labeled PN.
- Find an admissible solution $L_{e,a}\mu_e \leq b_{e,a}$, and use it to derive an admissible specification $L_a\mu + H_aq + C_av \leq b_a$ of the original problem.

Structural methods could be used with the following sufficient conditions for admissibility

$$\forall t_1, t_2 \in T,\ \rho(t_1) = \rho(t_2) \Rightarrow L_eD(\cdot, t_1) = L_eD(\cdot, t_2) \tag{2}$$

$$\forall t \in T,\ \rho(t) \in \Sigma_{uc} \cup \{\lambda\} \Rightarrow L_eD(\cdot, t) \leq 0 \tag{3}$$

$$\forall t \in T,\ \rho(t) \in \Sigma_{uo} \cup \{\lambda\} \Rightarrow L_eD(\cdot, t) = 0 \tag{4}$$

Disjunctive Constraints

Disjunctions have the form

$$\bigvee_i L_i \mu \leq b_i$$

where $L_i \in \mathbb{Z}^{m_i \times n}$ and $b_i \in \mathbb{Z}_i^m$, or equivalently

$$\bigwedge_j \bigvee_{i \in A_j} l_i \mu \leq c_i$$

where $l_i \in \mathbb{Z}^{1 \times n}$, $c_i \in \mathbb{Z}$ and A_j is a set of integers.

We can apply here literature results that reduce propositional logic to inequalities, by means of auxiliary variables.

Disjunctive Constraints

Let δ_i be auxiliary variables:

$$[\delta_i = 1] \leftrightarrow [l_i\mu \leq c_i] \tag{5}$$

Thus,

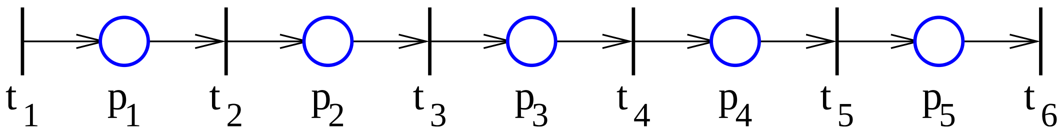
$$\sum_{i \in A_j} \delta_i \geq 1 \leftrightarrow \bigvee_{i \in A_j} l_i\mu \leq c_i \tag{6}$$

Assuming $l_i\mu$ is bounded, $m_i \leq l_i\mu \leq M_i$, $[\delta_i = 1] \leftrightarrow [l_i\mu \leq c_i]$ becomes

$$l_i\mu + (M_i - c_i)\delta_i \leq M_i \tag{7}$$

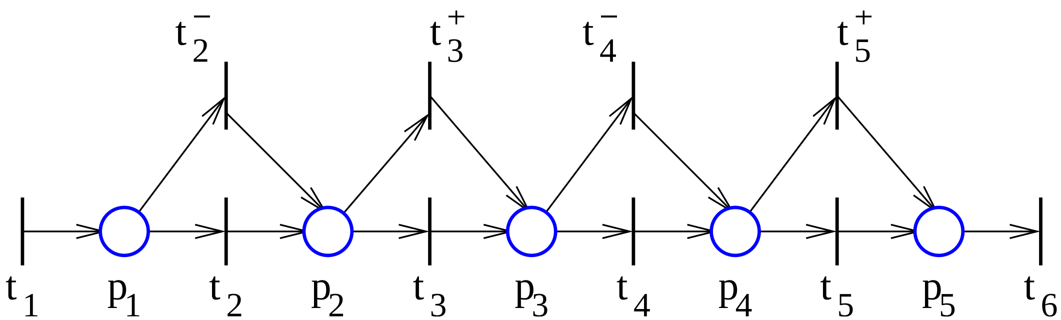
$$l_i\mu + (c_i + 1 - m_i)\delta_i \geq c_i + 1 \tag{8}$$

Given are a PN and a specification



$$[\mu_2 \leq 0] \vee [\mu_4 \leq 0] \quad (9)$$

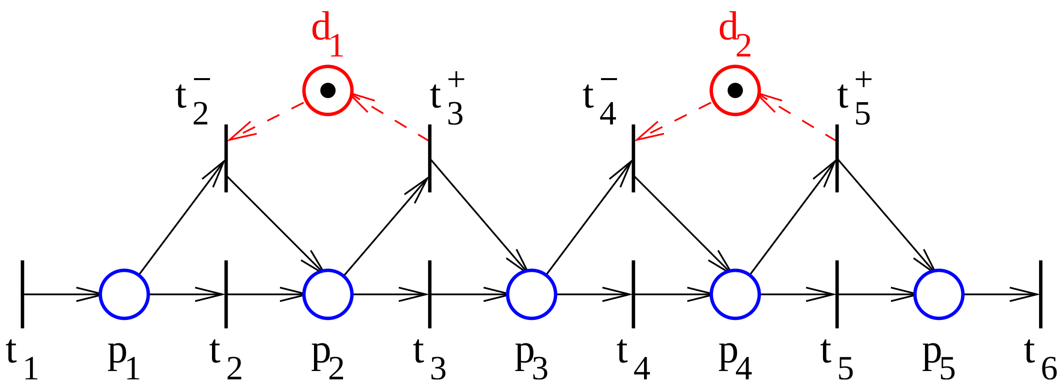
The first step is to identify and create copies of the transitions that increase/decrease $l_i\mu$



For $\mu_2 \leq 0$: $T_1^- = \{t_2^-\}$ and $T_1^+ = \{t_3^+\}$.

For $\mu_4 \leq 0$: $T_2^- = \{t_4^-\}$ and $T_2^+ = \{t_5^+\}$.

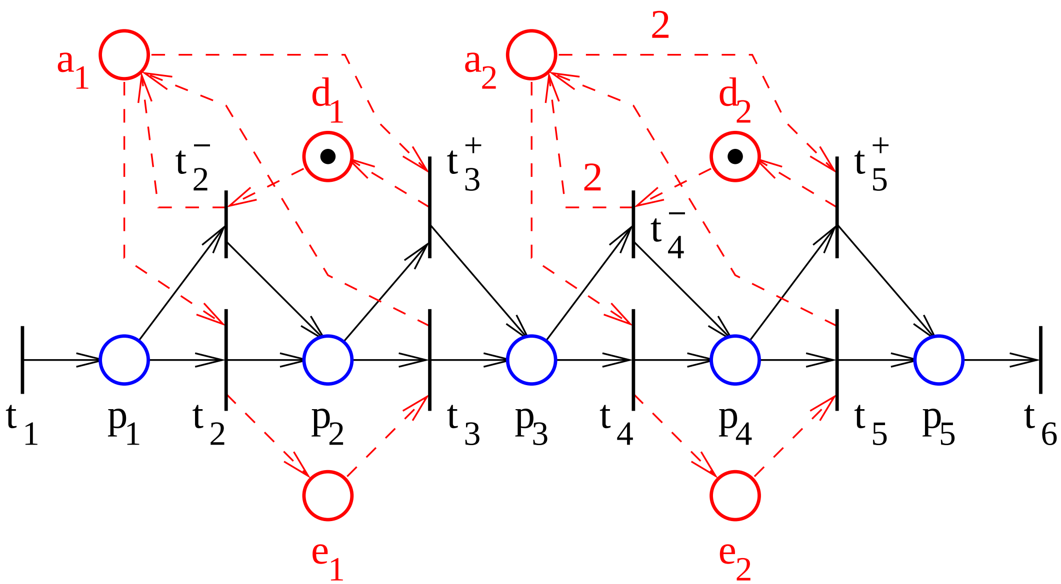
The second step is to add places d_i as input (output) places to T_i^- (T_i^+):



We intend to achieve $\delta_i = \mu(d_i)$.

The next step is to enforce the constraints involving the auxiliary variables.

Assume $m_1 = m_2 = 0$ and $M_1 = 2$ and $M_2 = 3$.



$$\begin{aligned} \mu_2 + 2\delta_1 &\leq 2 & [a_1] \\ \mu_2 + \delta_1 &\geq 1 & [e_1] \end{aligned}$$

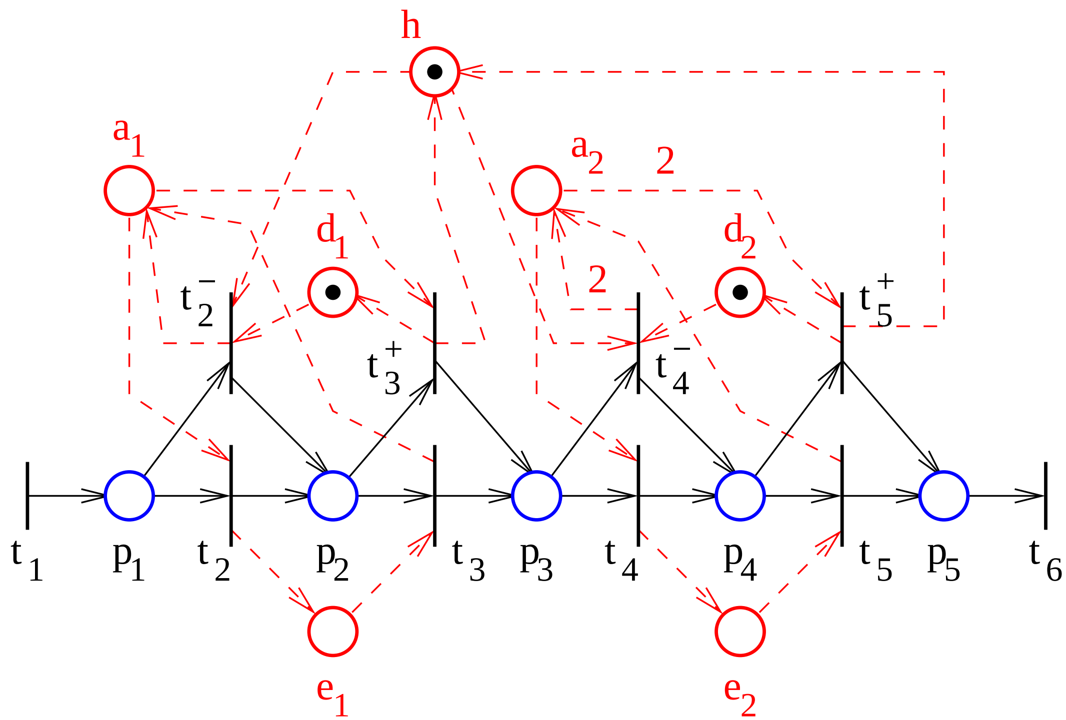
$$\begin{aligned} \mu_4 + 3\delta_2 &\leq 3 & [a_2] \\ \mu_4 + \delta_2 &\geq 1 & [e_2] \end{aligned}$$

Now, $[\mu(d_i) = 1] \leftrightarrow [l_i\mu \leq c_i]$ is enforced.

We only need to enforce $\mu(d_1) + \mu(d_2) \geq 1$ to finish our problem.

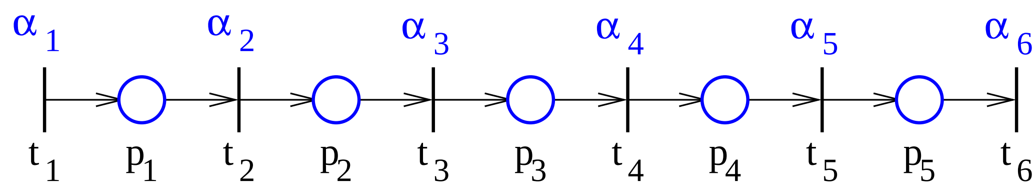
Since $[\mu(d_i) = 1] \leftrightarrow [l_i\mu \leq c_i]$ is already enforced, we only need to enforce

$$\mu(d_1) + \mu(d_2) \geq 1$$

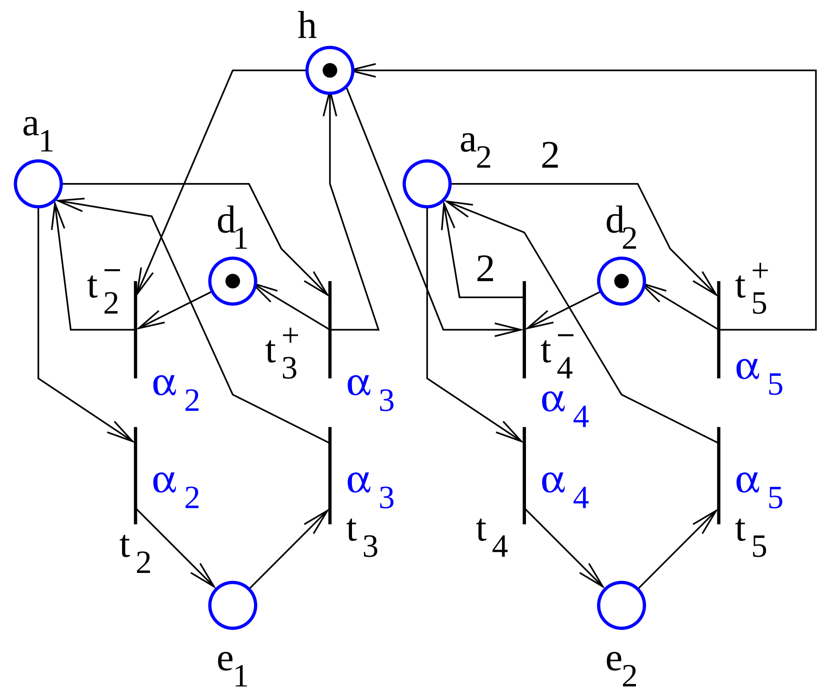


The place h is obtained.

Our previuos developments describe a closed-loop system corresponding to the composition of the plant



with the following supervisor:



Note that the supervisor is not free-labeled (though the plant is).

Final Remarks

Structural methods can design PN supervisors for a wide class of specifications:

- conjunctive linear constraints
- disjunctive linear constraints (under certain boundedness assumptions)
- P-language specifications

Structural methods promise computational gains.

The cost is that methods may be *suboptimal*, meaning:

- the supervisor may be overly restrictive
- solutions may exist even when no solutions are found.

The closed-loop may well deadlock. Additional methods need to be applied to ensure the closed-loop is live.