A Method for the Synthesis of Liveness Enforcing Supervisors in Petri Nets

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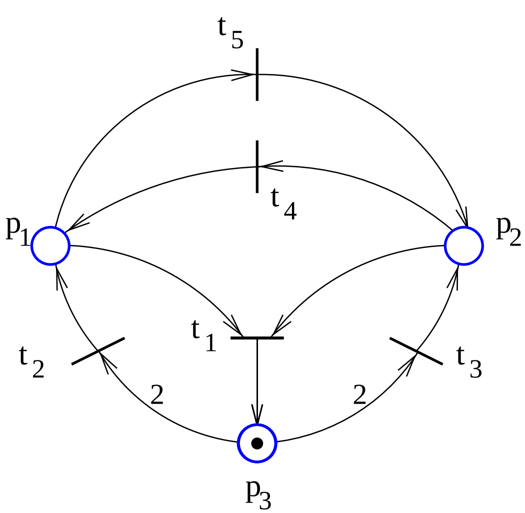
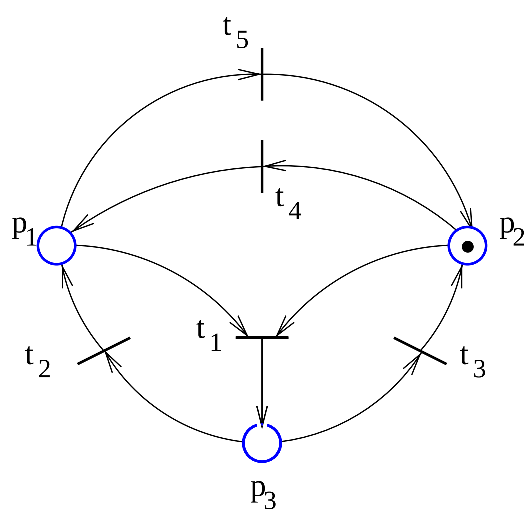
A Petri net is:

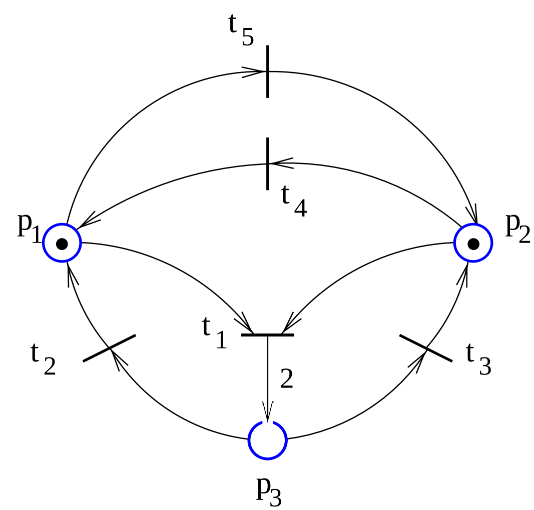
- **in deadlock** if no transition is enabled at the current marking.
- **deadlock-free** if for all reachable markings there is an enabled transition.
- **live** if for all reachable markings μ and for all transitions t there is a firing sequence σ containing t and enabled by μ.



Deadlock



Deadlock-free Petri net



Live Petri net

We present a *procedure* which generates liveness enforcing supervisors.

Given a Petri net structure, the procedure generates two sets of linear marking constraints:

$$L\mu \geq b \text{ and } L_0\mu \geq b_0$$

such that the Petri net supervised according to $L\mu \geq b$ is live for all initial markings μ_0 satisfying $L\mu_0 \geq b$ and $L_0\mu_0 \geq b_0$.

The constraints $L\mu \geq b$ are enforced using *supervision based on place invariants*.

The Petri net may have unobservable and/or uncontrollable transitions; the constraints $L\mu \geq b$ generated by the procedure are guaranteed to be *admissible*.

A **siphon** is a nonempty set of places S such that $\bullet S \subseteq S\bullet$. *The deadlock properties of PNs are related to siphons.*

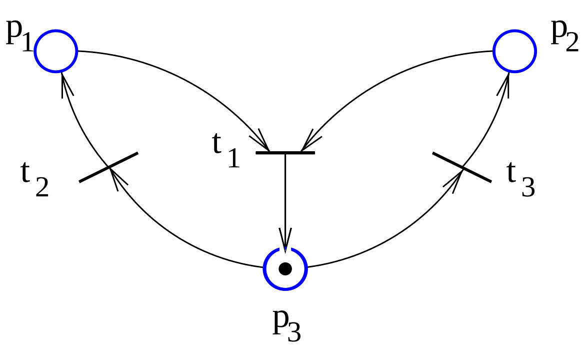
S is a **minimal siphon** if there is no siphon S' such that $S' \subset S$.

Given an initial marking, S is said to be a **controlled siphon** if all reachable markings μ satisfy

$$\sum_{p \in S} \mu(p) \geq 1$$

S is an **empty siphon** w.r.t. the marking μ if $\mu(p) = 0$ $\forall p \in P$.

Siphon examples: $S_1 = \{p_1, p_3\}$, $S_2 = \{p_2, p_3\}$, and $S_3 = \{p_1, p_2, p_3\}$ are siphons.



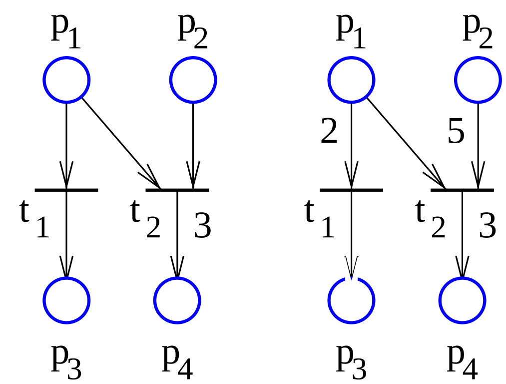
S_1 and S_2 are minimal; S_3 is not minimal.

S_3 is controlled; S_1 and S_2 are not controlled.

None of S_1, S_2 and S_3 is empty.

We call a PN PT-**ordinary** if $W(p,t) = 1$ for all $(p,t) \in F$.

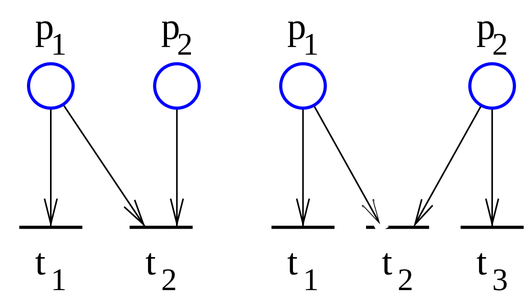
A deadlocked PT-ordinary PN contains an unmarked siphon.



PTO NOT PTO

A PN has **asymmetric choice** if for all places p_1 and p_2, if $p_1 \bullet \cap p_2 \bullet \neq \emptyset$ then $p_1 \bullet \subseteq p_2 \bullet$ or $p_2 \bullet \subseteq p_1 \bullet$.

A PT-ordinary PN with asymmetric choice is live if and only if all siphons are controlled.



AC NOT AC

Liveness is enforced by iteratively correcting the deadlock situations. This involves the following:

1. siphon control
2. transformations to PT-ordinary and asymmetric choice Petri nets

Each minimal siphon that is not controlled, is controlled by enforcing:

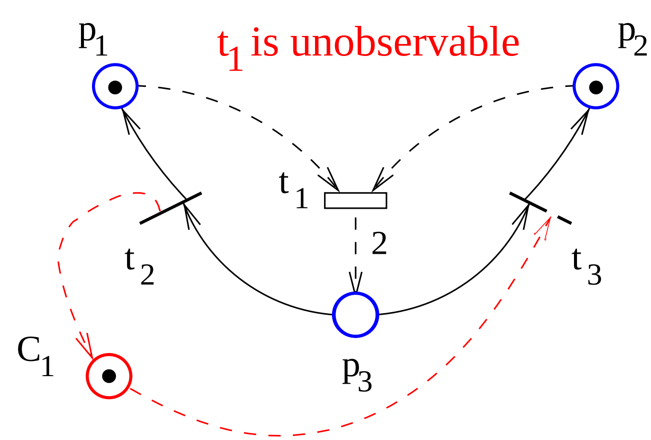
$$\sum_{p \in S} \mu(p) \geq 1 \tag{1}$$

When the constraint above is *inadmissible*, it is transformed to an admissible constraint

$$\sum_{p \in S} \alpha_p \mu(p) \geq 1 \tag{2}$$

where α_p are nonnegative integers.

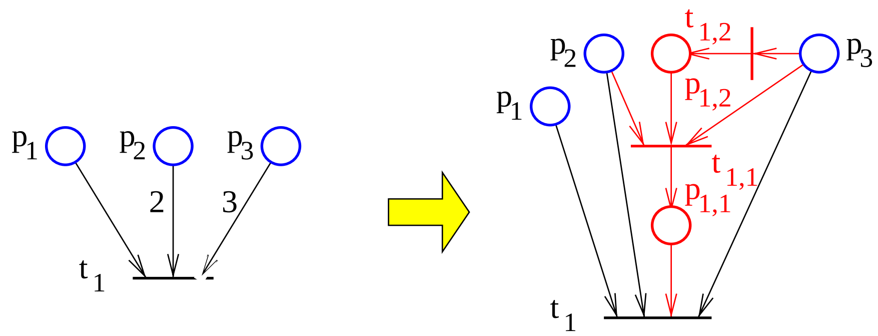
Example: The constraint (1) for $S = \{p_1, p_3\}$ is not admissible. It can be transformed to the admissible constraint $2\mu(p_1) + \mu(p_3) \geq 1$, of the form (2); enforcing it yields the control place C_1.



Transformation to PT-ordinary PNs

In the example, any inequalities on the original PN are changed as follows:

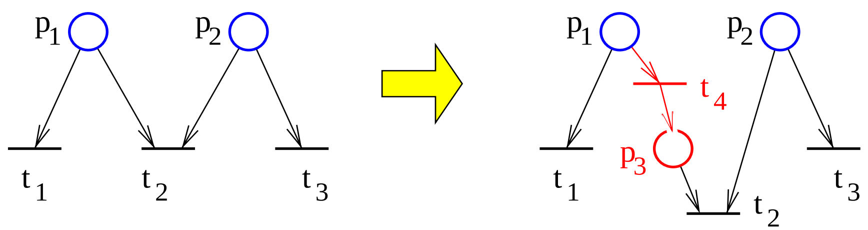
$\mu(p_1) \longrightarrow \mu(p_1)$
$\mu(p_2) \longrightarrow \mu(p_2) + \mu(p_{1,1})$
$\mu(p_3) \longrightarrow \mu(p_3) + \mu(p_{1,2}) + 2\mu(p_{1,1})$



Transformation to AC nets

In the example, any inequalities on the original PN are changed as follows:

$\mu(p_1) \longrightarrow \mu(p_1) + \mu(p_3)$
$\mu(p_2) \longrightarrow \mu(p_2)$



In general: $\mu(p_i) \longrightarrow \mu(p_i) + \sum_j k_j \mu(p_{i,j})$

Input: *The target Petri net* $\mathcal{N}_0$
Output: *Two sets of constraints* (L, b) *and* (L_0, b_0)

repeat

1. Transform the current net to a PT-ordinary Petri net with asymmetric choice.

2. **For** *every uncontrolled minimal siphon* S **do**

If S *needs to be controlled with a* $control\ place$ **then**
add control place to Petri net and inequality in (L, b).
Else
add inequality to (L_0, b_0).

In case of siphon control failure (which may occur when uncontrollable and unobservable transitions are present), exit the procedure and declare failure.

until *no uncontrolled minimal siphon is found at 2.*

Restrict the constraints (L, b) *and* (L_0, b_0) *to the places of* $\mathcal{N}_0$.

Liveness is enforced for all initial markings μ_0 such that $L\mu_0 \geq b$ and $L_0\mu_0 \geq b_0$, by supervising $\mathcal{N}_0$ with $L\mu \geq b$.

Case 1: No Uncontrollable and Unobservable Transitions

Theorem 1. *Given a Petri net* $\mathcal{N}_0$*, assume that the procedure terminates. If* (L, b) *and* (L_0, b_0) *are the two sets of constraints generated by the procedure, then* $\mathcal{N}_0$ *supervised according to* $L\mu \geq b$ *is live for all initial markings* μ_0 *which satisfy* $L_0\mu_0 \geq b_0$*.*

Theorem 2. *The supervisors generated by the procedure are least restrictive.*

Case 2: Uncontrollable and/or Unobservable Transitions are Present

Theorem 3. *Given a Petri net* $\mathcal{N}_0$*, assume that the procedure terminates without any siphon control failures. If* (L, b) *and* (L_0, b_0) *are the two sets of constraints generated by the procedure, then* $\mathcal{N}_0$ *supervised according to* $L\mu \geq b$ *is live for all initial markings* μ_0 *which satisfy* $L_0\mu_0 \geq b_0$*.*

Theorem 4. *Whenever the procedure terminates without siphon control failures and generates only admissible constraints* $\sum_{p \in S} \alpha_p \mu(p) \geq 1$ *with all* α_p *nonzero, it generates least restrictive supervisors.*

To guarantee termination, the siphon control method is modified: instead of enforcing $\sum_{p \in S} \mu(p) \geq 1$, we enforce $\sum_{p \in S \cap R} \mu(p) \geq 1$, where R is the set of places which are not places of $\mathcal{N}_0$ or control places.

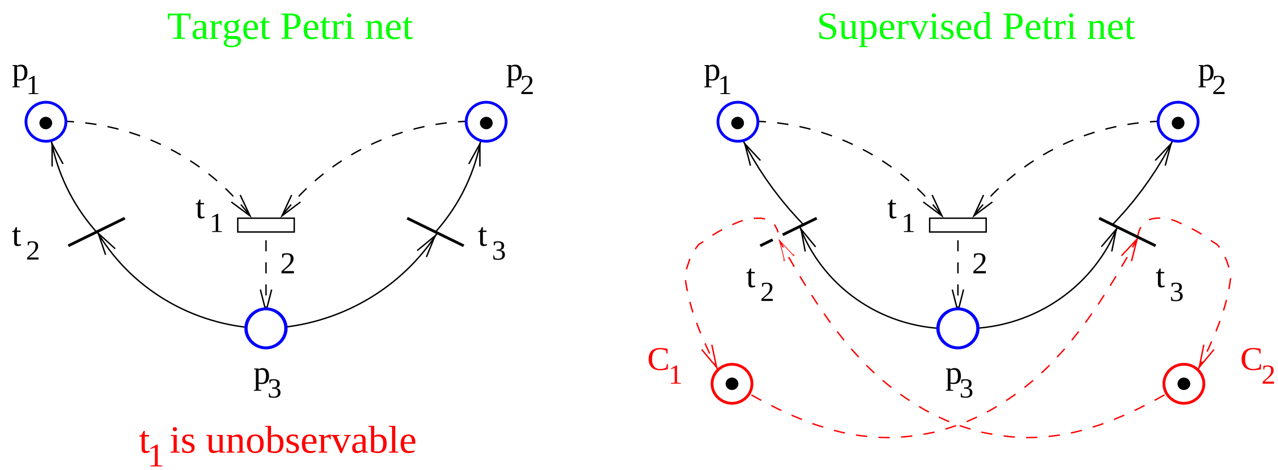
Theorem 5. *Let $\mathcal{N}_0$ be a Petri net and (L_i, b_i) be a set of marking constraints with bounded feasible region. The modified procedure terminates if started with initial constraints (L_0, b_0) equaling (L_i, b_i).*

The usage of the modified procedure is as follows:

- Let $\mathcal{M}_I$ be the set of initial markings of interest. Find a set of marking constraints $L_i \mu \geq b_i$ of bounded feasible set F such that $\forall \mu_0 \in \mathcal{M}_I$: $\mathcal{R}(\mathcal{N}, \mu_0) \subseteq F$.
- Use the modified procedure with initial constraints (L_0, b_0) which equal (L_i, b_i).
- The generated supervisor can be used for all initial markings $\mu_0 \in \mathcal{M}_I$ which also satisfy $L\mu_0 \geq b$ and $L_0 \mu_0 \geq b_0$.

Examples

C_1 controls $\{p_2, p_3\}$ and C_2 controls $\{p_1, p_3\}$. The siphon $\{C_1, C_2\}$ does not need control place enforcement, so its control inequality is included in (L_0, b_0).



$$L = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} 2 & 2 & 2 \end{bmatrix} \qquad b_0 = \begin{bmatrix} 3 \end{bmatrix}$$

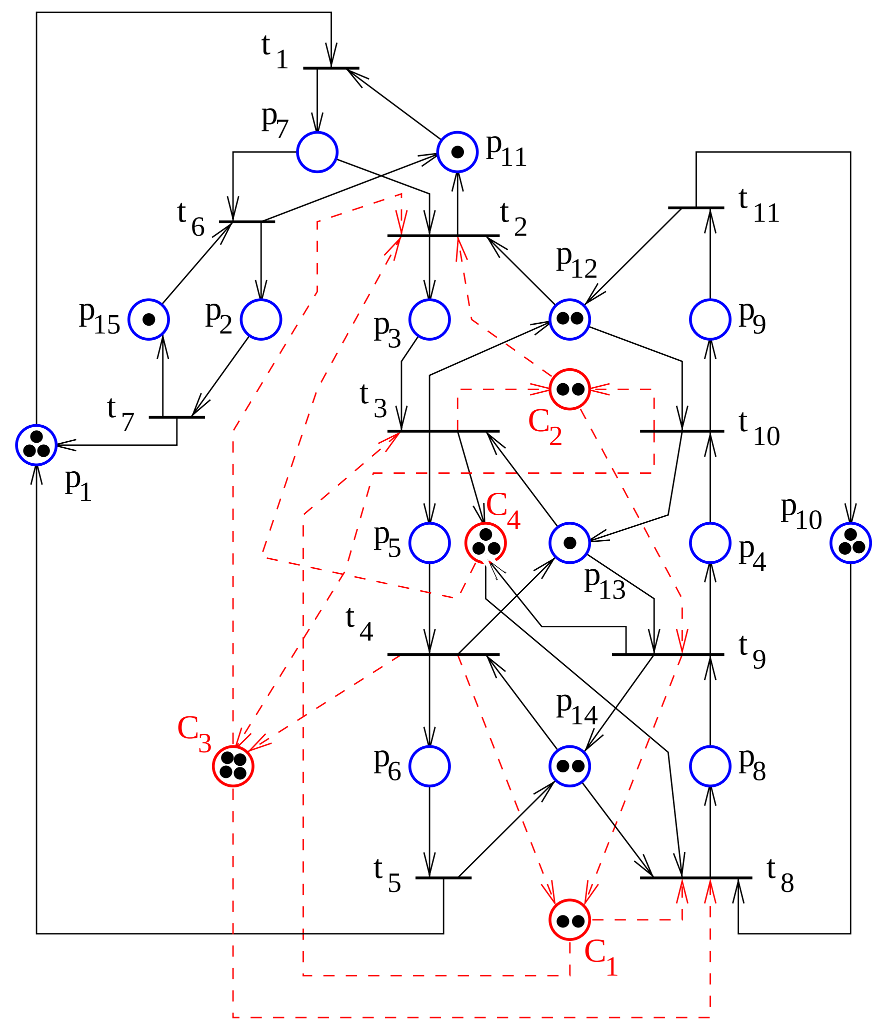
Examples

Constraints of (L, b):

$$\mu_4 + \mu_6 + \mu_{13} + \mu_{14} \geq 1$$
$$\mu_5 + \mu_9 + \mu_{12} + \mu_{13} \geq 1$$
$$\mu_6 + \mu_9 + \mu_{12} + \mu_{13} + \mu_{14} \geq 1$$
$$2\mu_4 + 2\mu_5 + \mu_6 + \mu_9 + \mu_{12} + 2\mu_{13} + \mu_{14} \geq 3$$

Constraints of (L_0, b_0):

$$\mu_{0,6} + \mu_{0,8} + \mu_{0,14} \geq 1$$
$$\mu_{0,2} + \mu_{0,15} \geq 1$$
$$\mu_{0,1} + \mu_{0,2} + \mu_{0,3} + \mu_{0,5} + \mu_{0,6} + \mu_{0,7} \geq 1$$
$$\mu_{0,4} + \mu_{0,5} + \mu_{0,13} \geq 1$$
$$\mu_{0,3} + \mu_{0,9} + \mu_{0,12} \geq 1$$
$$\mu_{0,7} + \mu_{0,11} \geq 1$$
$$\mu_{0,4} + \mu_{0,8} + \mu_{0,9} + \mu_{0,10} \geq 1$$



Performance

+ The procedure makes no assumption on the PN structure; it is applicable to PNs which may be unbounded, generalized, and with uncontrollable and unobservable transitions.

+ When no uncontrollable and unobservable transitions are present, if the procedure terminates, the procedure provides the least restrictive liveness enforcing supervisor.

+ When uncontrollable and/or unobservable transitions are present, the procedure terminates, and no siphon control failures occur, the procedure provides liveness enforcing supervisors which are guaranteed to be least restrictive under a (rather restrictive) condition.

+ The procedure does not assume a given initial marking, but rather provides the constraints that the initial markings must satisfy for the supervisor to be effective.

− Procedure termination is not guaranteed.

+ The procedure can be modified for guaranteed termination.

− The modified procedure is useful only for structurally bounded Petri nets, and the results guaranteeing a least restrictive liveness supervisor may not apply.

Performance

– The procedure may perform in each iteration computationally expensive operations (checking whether a siphon is uncontrolled may involve solving integer programs; finding the minimal siphons of a PN may also be computationally complex).

\+ All computations are performed off-line. Very little computation is required to run a supervisor on-line.

\+ The procedure allows fully automated computer implementation (and we have implemented it).

\+ When the procedure generates least restrictive liveness enforcing supervisors, for all initial markings for which the supervisors are not defined, liveness enforcing is impossible.

\+ In flexible manufacturing systems, presenting the set of acceptable initial markings as a set of linear markings constraints has the advantage that we can easily solve minimum resource number problems via integer programming.