EEGR5973 SUPERVISORY CONTROL
Homework Sets Assigned in the Summer of 2013

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Homework Set 1

1. Sketch a Petri net with four places and five transitions. Some of the transitions should have multiple input placess and multiple output places. The transition weights may not be all one. The Petri net should have at least one self-loop.

   (a) Write the preset and postset of the places and transitions of the net.

   (b) Write the incidence matrix and the input and output matrices.

   (c) Write the inequalities describing the Petri net in terms of the Parikh vector v.

   (d) Write the H and C matrices.

2. Write an Octave function that returns 1 if the Petri net has one or more self-loops and 0 otherwise. See the examples in the spnbox toolbox.

3. Implement the Octave function of problem 2 in C according to the pattern of the functions in the supervisory control folder of the ACTS software. Notice the testing approach used for the functions of that folder and use the same approach to test your program.

He revealeth the deep and secret things: he knoweth what is in the darkness, and the light dwelleth with him. Dan 2:22

Special Topics–Supervisory Control

Homework Set 2

1. A Petri net has the incidence matrix

$$D = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

   (a) Assuming no self-loops, sketch the graphical representation of the Petri net.

   (b) Verify that $z = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is a place invariant.

   (c) Determine whether there are any other place invariants.

2. Assume the following incidence matrix and initial marking

$$D = \begin{bmatrix} 1 & -2 & 4 & 0 \\ 0 & 1 & -3 & -1 \end{bmatrix} \qquad \mu_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

   (a) Determine a supervisor enforcing $\mu_1 + \mu_2 \leq 5$.

   (b) Verify your result using the spnbox tools (see the function `linenf`).

   (c) Write the inequality constraint $\mu_1 + \mu_2 \leq 5$ in terms of the Parikh vector v only.

3. A Petri net has three transitions t_1, t_2, and t_3 and no places.

   (a) Determine a supervisor enforcing the constraints $v_1 + q_1 \leq 3 + v_2$ and $v_3 + 2q_2 \leq 1 + v_1$.

   (b) Verify your result using the spnbox tools (see the function `linenf`).

   (c) Indicate the transitions that should be controllable and the transitions that should be observable to the supervisor.

4. Consider a Petri net model of a single bidirectional track segment connecting two points A and B. The track is used by AGVs that go from A to B or from B to A. There are four transitions:

   - t_1 is fired when an AGV enters the track segment at the point A.
   - t_2 is fired when an AGV exits the track segment at the point B.
   - t_3 is fired when an AGV enters the track segment at the point B.
   - t_4 is fired when an AGV exits the track segment at the point A.

   (a) Design a supervisor that ensures that no more than two AGVs may use the track at the same time.

   (b) Enhance the supervisor so that it ensures that AGVs do not go in opposite directions at the same time on the track. The supervisor should still allow two AGVs at the same time on the track, provided they go in the same direction. Hint: You will need to use the q term of the inequality constraints.

(c) Verify your supervisors using the spnbox tools (see the function `linenf`).

5. Write an Octave function that verifies whether there is a directed path from a transition t to another transition t'. The function should return a nonzero value if a path has been found. Write the function according to the spnbox format. The function should have the following parameters:

    (a) The Petri net object.

    (b) The index of the transition t.

    (c) The index of the transition t'.

Beloved, if our heart condemn us not, then have we confidence toward God. And whatsoever we ask, we receive of him, because we keep his commandments, and do those things that are pleasing in his sight. 1Jn 3:21-22

Special Topics–Supervisory Control

Homework Set 3

1. Consider the Petri net components shown in Figure 1.

   (a) Write the inequality constraints describing the Petri nets in terms of the Parikh vector.

   (b) Assume the following transition synchronizations: t_a and t_1; t_b and t_2; t_c and t_2; t_d and t_3; t_a and t_3; t_b and t_4; t_c and t_5; t_d and t_6. Express these synchronizations by means of equality constraints in terms of the Parikh vector.

   (c) Sketch a Petri net representing the composition of the components of Figure 1.

   (d) Repeat part (b) assuming now that synchronizations are carried out according to the following labels: t_c, t_2, and t_5 have the label α, t_d, t_3, and t_6 have the label β, and all other transitions have distinct labels.

   (e) Sketch the parallel composition of the Petri net components at part (d).

   (f) Is the Petri net at part (c) bounded? Is the Petri net at part (e) bounded?

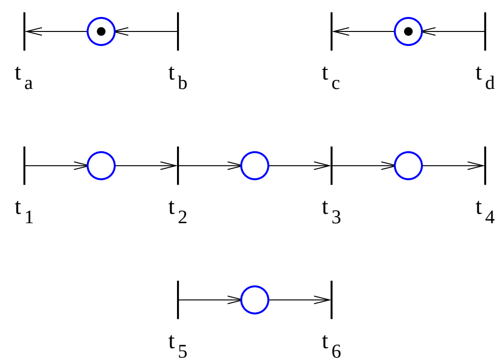


Figure 1: Petri net components for problem 1.

2. Assume the following incidence matrix and initial marking.

$$D = \begin{bmatrix} -1 & 0 & 1 & -1 \\ 1 & -1 & 0 & -1 \\ 0 & 1 & -1 & 1 \end{bmatrix} \qquad \mu_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

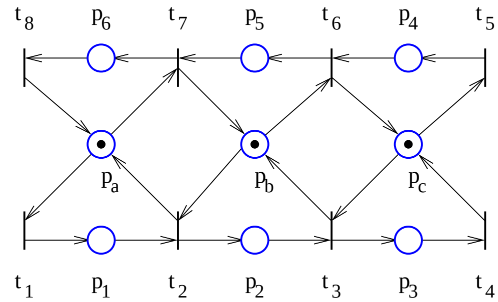
   (a) Draw the reachability graph.

   (b) Verify your result in Ocatave using spnbox (see the `pncgraph` and `disgraph` functions).

   (c) How many strongly connected components has the graph?

   (d) Indicate whether the Petri net is live, deadlock-free, (partially) repetitive, or bounded.

3. Assume the following Petri net.

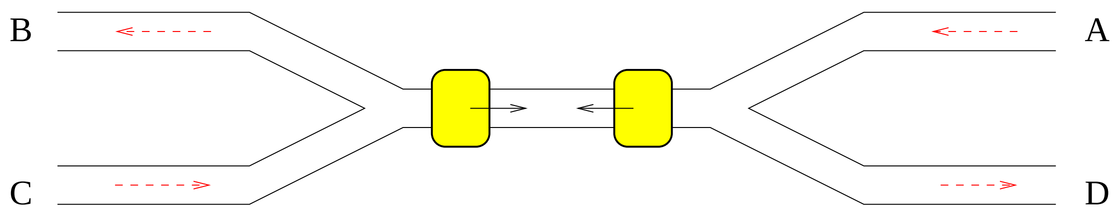


   (a) Determine all siphons.

   (b) Verify your answer using the spnbox function `asiph`. Note that this function looks for *minimal* siphons.

   (c) Indicate whether the Petri net is live, deadlock-free, (partially) repetitive, or bounded.

4. One vehicle moves from A to B and one from C to D. Assuming that the vehicles can stop but cannot reverse their direction, deadlock will occur if the two vehicles are in the middle section at the same time. Obtain a Petri net model of this system and show how it can deadlock. Hint: Divide the course of a vehicle into stages and use one place per stage.



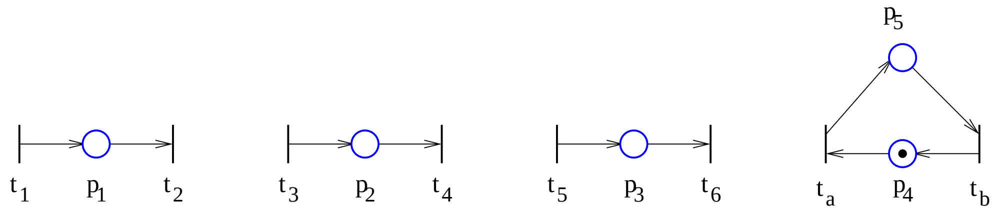
5. Write a C program that determines all source and sink transitions of a Petri net. (t is a source transition if $\bullet t = \emptyset$ and a sink transition if $t\bullet = \emptyset$.) Follow the pattern of the functions in the supervisory control folder of the ACTS software.

The LORD of hosts hath sworn, saying, Surely as I have thought, so shall it come to pass; and as I have purposed, so shall it stand. Is 14:24

Special Topics–Supervisory Control

Homework Set 4

1. Consider the Petri net components shown below.



   (a) Assume the following synchronizations: t_1 and t_a; t_3 and t_a; t_5 and t_a; t_6 and t_b. Express these synchronizations by means of equality constraints in terms of the Parikh vector.

   (b) Sketch a Petri net representing the composition of the PN components.

   (c) Repeat part (a) assuming now that synchronizations are carried out according to the following labels: t_a, t_1, t_3, and t_5 have the label α, t_b and t_6 have the label β, and all other transitions have distinct labels.

   (d) Sketch the parallel composition of the PN components at part (c).

   (e) Is the Petri net at part (b) live? Is the Petri net at part (d) live?

2. Figure 1 shows a PN model of a system in which resources (such as the cores of a microprocessor) can be allocated to two different types of jobs.

   (a) Show how the PN could be obtained as a composition of three state machines, one involving the places p_1 and p_2, one involving p_a, and one involving p_3 and p_4.

   (b) How can the PN deadlock?

   (c) Determine all minimal siphons.

   (d) Add control places to all minimal siphons that are not controlled.

   (e) Are there any new siphons involving the control places? Are they controlled?

   (f) Write the inequalities that the marking must satisfy for a deadlock-free operation (under adequate supervision).

   (g) Use the function `asiph` of SPNBOX to verify the siphons determined at parts (c) and (e).

3. A textbook illustration of deadlock is shown in Figure 2. The vehicles can stop but cannot reverse direction. Obtain a PN model that predicts this deadlock state. Explain your work. Hint: The areas A, B, C, and D could be seen as resources. For example, a vehicle has to acquire both C and D in order to move from left to right. The position of each vehicle could be represented by a state machine and a global PN model can be obtained by combining the state machines with the resource places.

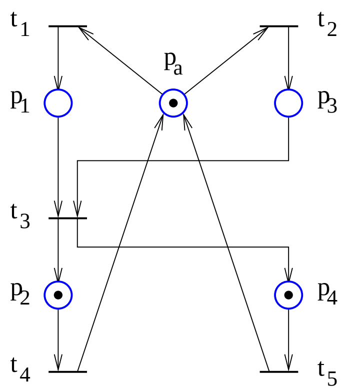


Figure 1: Figure of problem 2.

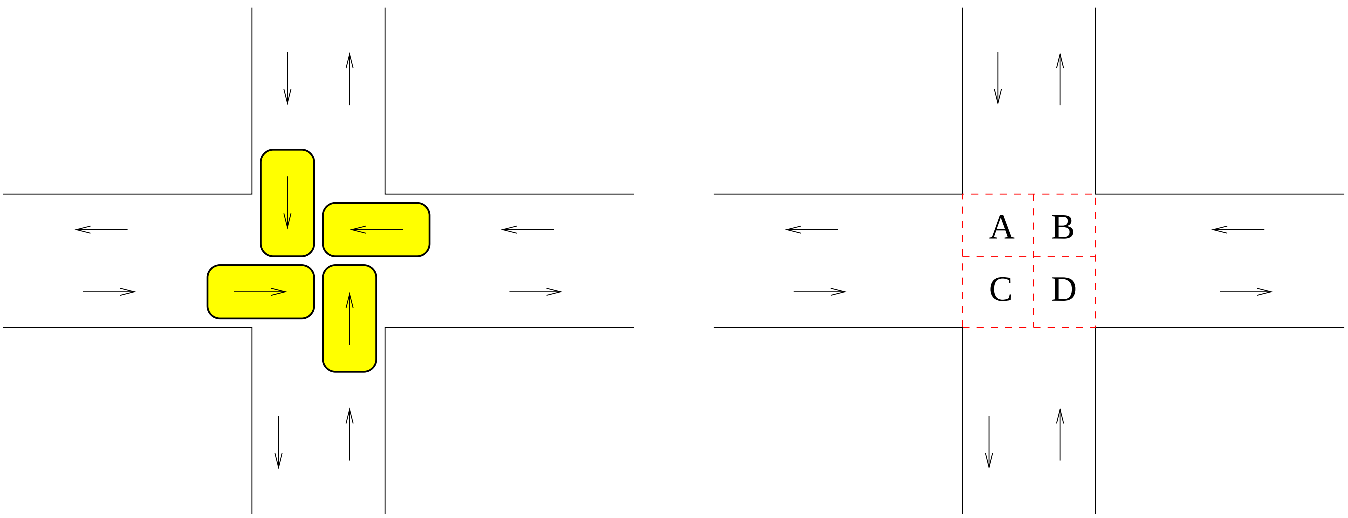


Figure 2: Figure of problem 3.

4. Examine the example programs of the ACTS software and write a different program involving at least three concurrent threads.

And Jesus said unto them, Because of your unbelief: for verily I say unto you, If ye have faith as a grain of mustard seed, ye shall say unto this mountain, Remove hence to yonder place; and it shall remove; and nothing shall be impossible unto you. Mt 17:20

Homework Set 5

1. Examine the example programs of the ACTS software and especially `inteq.sp`. Write a program that does the following.

   (a) Prompts the user to type a number n between 2 and 20.

   (b) Generates a random matrix with n rows and 20 columns. The value of each matrix element must be 0, 1, or 2. Use `rand()%3` to initialize the matrix elements.

   (c) The program starts n identical threads: thread 0, thread 1, thread 2, …, thread n-1.

   (d) Thread number i should write to $vect[i]$ the number of nonzero elements of the row i of the matrix, where $vect$ is a global vector of integers.

   (e) After all threads terminate the program should display the matrix and the result (the vector $vect$).

2. (a) Find a regular expression describing $\mathcal{L}(G_1)$.

   (b) Find a regular expression describing $\mathcal{L}_m(G_1)$.

   (c) Draw the automaton $G_1 \times G_2$ (the parallel composition of G_1 and G_2).

   (d) Draw the automaton $G_2 \times G_3$.

   (e) Draw the automaton $G_1 \times G_2 \times G_3$.

   (f) Draw $G_1 \times G_2 \times G_3$ assuming G_1, G_2, and G_3 are labeled Petri nets, using the parallel composition of Petri nets.

   (g) Compare the results of parts (e) and (f).

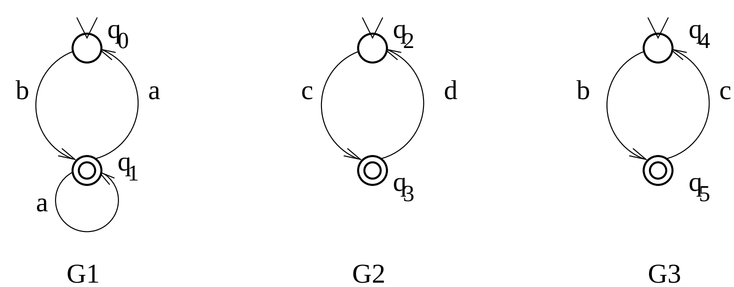


Figure 1: Figure of problem 2.

3. The automata G_1 and G_2 have the alphabets $\Sigma_1 = \{a, b\}$ and $\Sigma_2 = \{b, c\}$. Let G be the parallel composition of G_1 and G_2.

   (a) Determine the projection $P_1(aabcba)$.

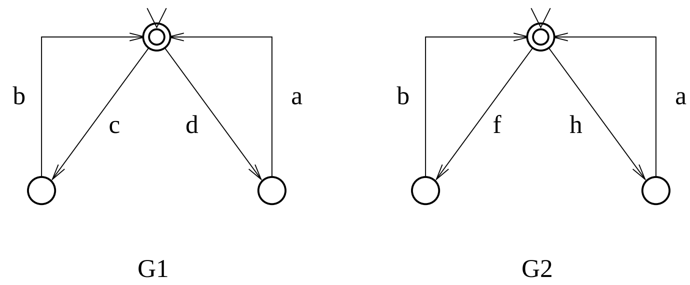
   (b) Write a regular expression expressing $P_2^{-1}(bcb)$.

Commit thy way unto the LORD; trust also in him; and he shall bring *it* to pass. Ps 37:5

Special Topics–Supervisory Control

Homework Set 6

1. Consider two tasks that synchronize themselves on two pairs of transitions.



   (a) Show that the system can deadlock.

   (b) Sketch the parallel composition $G_1 \times G_2$.

   (c) How is the deadlock possibility indicated by $G_1 \times G_2$?

   (d) Assume now that G_1 and G_2 are labeled Petri nets and sketch their parallel composition.

   (e) Determine all minimal siphons.

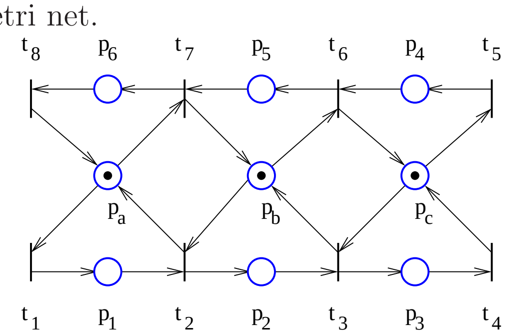
   (f) Add control places to all minimal siphons that are not controlled.

   (g) Are there any new siphons involving the control places? Are they controlled?

   (h) Write the inequalities that the marking must satisfy for a deadlock-free operation (under adequate supervision).

   (i) Use the function `asiph` of SPNBOX to verify the siphons determined at parts (e) and (g).

2. Assume the following Petri net.



   (a) Sketch the reachability graph.

   (b) Does the graph correspond to a deterministic or nondeterministic automaton?

   (c) Show that the Petri net can be obtained as the parallel composition of three components: one component involving p_1, p_2, and p_3, one component involving p_4, p_5, and p_6, and one component involving p_a, p_b, and p_c.

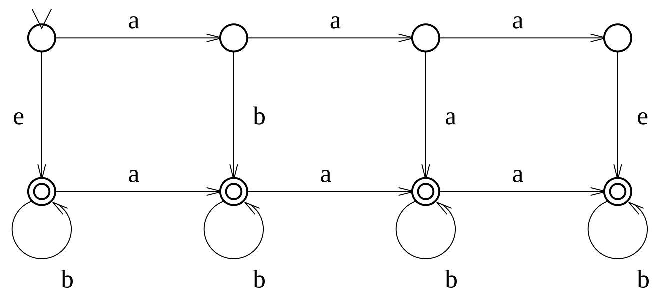
   (d) Can any of the three components be represented by a finite automaton? Why or why not?

3. Consider the expression $a(aba)^*$.

   (a) Obtain the equivalence classes.

   (b) Sketch a deterministic finite automaton G for which $\mathcal{L}_m(G)$ is described by the expression $a(aba)^*$.

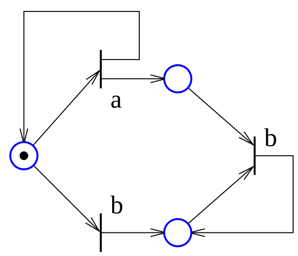
4. Convert the following automaton to a deterministic automaton.

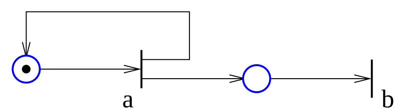


Special Topics–Supervisory Control

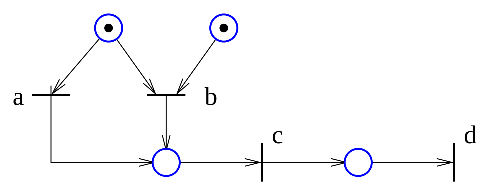
Homework Set 7

1. Consider the language consisting of all possible sequences of events generated by a Petri net. For each of the Petri nets shown below indicate whether they generate a regular language. Explain your answer.





2. Consider the language consisting of all possible sequences of events generated by a Petri net.



   (a) Sketch the reachability graph.

   (b) Convert the reachability graph to a deterministic automaton.

   (c) Minimize the number of states of the automaton.

3. Consider the following C function.

   (a) Sketch the automaton that is simulated.

   (b) Minimize the number of states of the automaton.

   (c) Write a simpler function that performs the same task.

```
int CheckString(char *str) {
  int state, i;

  for(i = 0, state = 0; str[i]; i++) {
    switch(state) {
      case 0:
        if(str[i] == 'a') state = 1;
        else if(str[i] == 'b') state = 2;
        else state = 5;
        break;
      case 1:
        if(str[i] == 'b') state = 3;
```

```
        else state = 5;
        break;
      case 2:
        if(str[i] == 'a') state = 4;
        else state = 5;
        break;
      case 3:
        if(str[i] == 'a') state = 1;
        else state = 5;
        break;
      case 4:
        if(str[i] == 'b') state = 2;
        else state = 5;
    }
  }

  if(state == 1 || state == 2 || state == 3 || state == 4)
    return ACCEPT;
  return REJECT;
}
```

4. Sketch automata implementing the following languages.

   (a) $\mathcal{L}_{m1} = aba^* \cup a(ba)^*a^*$;

   (b) $\mathcal{L}_{m2} = \{a, b\}^* \setminus a^*b^*$;

   (c) $\mathcal{L}_{m3} = \mathcal{L}_{m1} \cap \mathcal{L}_{m2}$.

   (d) Two automata are equivalent if they generate the same language. This can be checked by converting each automaton to a DFA with minimum number of states. If the DFAs are identical, the automata are equivalent. Use this procedure to determine whether $\mathcal{L}_{m1}$ and $\mathcal{L}_{m2}$ are identical.

Special Topics–Supervisory Control

Homework Set 8

1. (Adapted from example 8.3 of *Linear and Nonlinear Programming* by S. Nash and A. Sofer, McGraw-Hill 1996) A total of 9000 identical parts should be delivered from locations A and B to the locations M and N. Location A has 3000 parts and location B has 6000 parts. Moreover, 2000 parts should be delivered to M and 7000 to N. The shipping cost for one part from A to M is 1, from A to N is 3, from B to M is 2, and from B to N is 1.5.

   (a) Formulate a linear program that minimizes the total shipping cost.

   (b) Solve the linear program using `ip_solve` and indicate the minimum cost and how many parts should be sent from each of A and B to M and N.

2. Formulate linear programs that estimate the minimum and the maximum possible values of a term $l\mu$, where l is a $1 \times n$ vector and n is the number of places. Use the equation $\mu = \mu_0 + Dv$ and impose the condition $\mu \geq 0$. Find the numerical values of the minimum and the maximum when $l = [1,\ 2,\ 0]$, $\mu_0 = [1,\ 1,\ 0]^T$, and the incidence matrix is

$$D = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & -1 & -1 \end{bmatrix}.$$

3. Assume that x_1 and x_2 are integers constrained to $-20 \leq x_1 \leq 30$, $-40 \leq x_2 \leq 50$, and $[-x_1 + 2x_2 \geq -2] \vee [2x_2 + x_1 \geq 4]$.

   (a) Define auxiliary variables allowing to express the constraints as a conjunction of linear inequalities.

   (b) Use `ip_solve` to find the minimum value of $x_1 + x_2$.

4. Consider the logic expression $E = (\overline{x}_1 \vee x_2) \wedge (x_3 \vee x_4)$. Express the condition that E be true by means of linear inequalities.

5. Assume a program involving the tasks A, B, C, D, E, F, and G. Task A takes 4 seconds, task B takes 2 seconds, and the remaining tasks take each 1 second.

   - Task C may start only after task B is done.
   - Task D may start only after task B is done.
   - Task E may start only after task D is done.
   - Task F may start only after task E is done.
   - Task G may start only after all other tasks are done.

   Use the ACTS software to implement a program that executes the tasks concurrently. The ACTS specification should prescribe the order of the tasks so that the overall program takes a minimum amount of time. For each task use the function `sleep` to determine its execution time. The program should also measure the total execution time (see the functions in `time.h`.)

I will praise *thee*, O LORD, with my whole heart; I will shew forth all thy marvellous works. Ps 9:1

Homework Set 9

1. Assume that x_1 and x_2 are integers constrained to $-5 \leq x_1 \leq 5$, $-5 \leq x_2 \leq 5$, and $[-x_1+x_2 \leq 0 \wedge x_1 + 2x_2 \leq -1] \vee [x_2 - x_1 \leq -8]$.
   (a) Define auxiliary variables allowing to express the constraints as a conjunction of linear inequalities.
   (b) Use `ip_solve` to find the maximum value of $x_1 + x_2$.
   (c) Verify your result by solving two separate integer linear programs:
      i. One maximizing x_1+x_2 subject to $-5 \leq x_1, x_2 \leq 5$, $-x_1+x_2 \leq 0$, and $x_1+2x_2 \leq -1$.
      ii. One maximizing $x_1 + x_2$ subject to $-5 \leq x_1, x_2 \leq 5$ and $x_2 - x_1 \leq -8$.

2. A Petri net can be seen as a directed graph with two types of nodes: transitions and places. Look up and describe algorithms for each of the following problems.
   (a) Determine the cycles of a directed graph.
   (b) Determine the connected components of an undirected graph. (A component consists of connected nodes.)

And because ye are sons, God hath sent forth the Spirit of his Son into your hearts, crying, Abba, Father. Ga 4:6