EE4933–AUTOMATIC CONTROL SYSTEMS
Homework Sets Assigned in the Spring Semester of 2019

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EEGR4933 Automatic Control Systems

Homework Set 1

And whatever you do, do it heartily, as to the Lord and not to men, knowing that from the Lord you will receive the reward of the inheritance; for you serve the Lord Christ.
Colossians 3:23–24

Suggested Reading: Sections 3.1-3.4 cover the material discussed in class. Note the Matlab examples in the textbook. You may consult also Appendix A for matrix algebra and Appendix B for the Laplace transform.

1. Find an equivalent state-variable model for each of the following transfer functions.

   (a) $H(s) = \frac{2}{s^2+2s+1}$; (b) $H(s) = \frac{2}{2s^2+2s+1}$; (c) $H(s) = \frac{2s}{s^3+2}$; (d) $H(s) = \frac{s^2}{s^2+1}$.

2. A digital filter is described by the equation $y(n+1)-0.5y(n)-0.25y(n-1) = 10\,r(n)+r(n-1)$, where y is the output and r the input. Find a state-variable model of the filter assuming $x_1(n) = y(n)$ and $x_2(n) = -0.25y(n-1) - r(n-1)$.

3. A rotatory system is described by the equation $J\ddot{\theta} + b\dot{\theta} = \tau$. Assume that τ is the input and θ is the output.

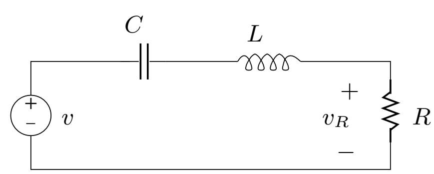
   (a) Determine the transfer function.

   (b) Determine a state-variable model.

   (c) Use Matlab to verify that the state-variable model gives the correct transfer function. Assume $J = 2$ and $b = 4$. You may want to use the Matlab functions `ss` and `tf`.

   (d) Assume now that the output is $b\dot{\theta} + k\theta$, while the system continues to be described by $J\ddot{\theta} + b\dot{\theta} = \tau$. Determine a state-variable model.

4. Assuming that v is the input and v_R is the output, find a state-variable model.



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Homework Set 2

For all the promises of God in Him are *Yes, and in Him Amen, to the glory of God through us.*
2 Corinthians 1:20

1. Indicate in each case whether the matrices correspond to stable systems. Assume continuous-time systems. You may use MATLAB (the `eig` function) for the 3 by 3 matrices.

   (a) $\begin{bmatrix} -38 & 38 & 20 \\ -2 & -31 & -20 \\ 26 & -19 & -49 \end{bmatrix}$; (b) $\begin{bmatrix} -4 & 49 & -36 \\ -27 & -3 & -41 \\ -16 & -13 & 3 \end{bmatrix}$; (c) $\begin{bmatrix} 5 & -2 \\ 15 & -6 \end{bmatrix}$.

2. Indicate in each case whether the matrices correspond to stable systems. Assume discrete-time systems. You may use MATLAB for the 3 by 3 matrices.

   (a) $\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & -1 \\ -1 & 0 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 1 \\ 4 & 3 & 0 \end{bmatrix}$; (c) $\begin{bmatrix} 5 & -2 \\ 15 & -6 \end{bmatrix}$.

3. (a) Example 3.3 of the textbook presents the following motor model:

   $$\begin{aligned} \dot{x} &= \begin{bmatrix} -\frac{B}{J} & \frac{K_\tau}{J} \\ -\frac{K_m}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{L_m} \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x. \end{aligned}$$

   Plot the response of the system for a unit step input. The MATLAB functions you need are `ss` and `step`. Assume all constants R_m, J, L_m, ... to be 1. Note that `step` assumes zero initial conditions: $x(0) = 0$.

   (b) Determine the matrix C such that the output of the system is

   $$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   For this value of C, plot y when u is the unit step.

   (c) Repeat (b) assuming the following nonzero initial state:

   $$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

   This time you may use `lsim`. Hint: The step input for `lsim` can be obtained by defining $t = 0 : 0.01 : 10$; and $u = ones(1, length(t))$;.

4. Given is the system

   $$\dot{x} = \begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

   Obtain the graphs of $x_1(t)$ and $x_2(t)$ assuming the initial state is $x(0) = [-1, 1]^T$. Verify that you obtain the same response to a step input using Simulink and the MATLAB function `lsim`. In Simulink, use the `state-space` block. Plot your Simulink diagram.

5. A series RLC circuit is supplied from a source of voltage $v(t)$.

   (a) Assuming $v(t)$ is the input, x_1 is the current, and x_2 is the capacitor voltage, find the matrix A.

   (b) Based on the eigenvalues of A show that the system is stable when $R \neq 0$.

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Homework Set 3

Let them praise the name of the LORD: for his name alone is exalted; his glory is above the earth and heaven. Psalms 148:13

Suggested Reading: Pages 407–428.

1. Determine whether the following systems are controllable, observable, and stable.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} u \tag{1}$$

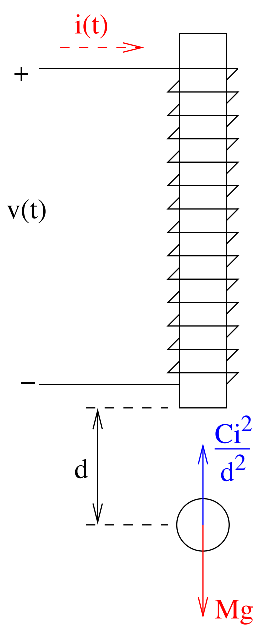
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{3}$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4}$$

For stability only, you may use MATLAB (see the function `eig`).

2. Problem 2.73 of "Linear Systems" by Antsaklis and Michel, presents the following linearized model of a magnetic ball suspension system



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{2ci_{eq}}{Md_{eq}^2} & \frac{2ci_{eq}^2}{Md_{eq}^3} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} u \tag{5}$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{6}$$

$R = 31.1\,\Omega$ and $L = 0.11\,\mathrm{H}$ are the resistance and inductance of the coil. $M = 0.011\,\mathrm{kg}$ is the mass of the ball. The current required to balance the ball at $d_{eq} = 0.01\,\mathrm{m}$ is $i_{eq} = 0.125\,\mathrm{A}$. $c = 6.59 \cdot 10^{-4}\mathrm{Nm}^2/\mathrm{A}^2$ is a constant. The state variables are defined relative to the equilibrium point of i_{eq} and d_{eq}: $x_1 = i - i_{eq}$ corresponds to the current, $x_2 = d - d_{eq}$ is the displacement of the ball from the equilibrium position, and x_3 is the velocity of the ball. The input is defined in terms of the voltage applied to the coil: $u = v - Ri_{eq}$.

(a) Intuitively, should the system be stable or unstable?
You may use MATLAB for the following questions.

(b) Find the eigenvalues of the system, and check whether the system is stable.

(c) Plot the state variables of the system assuming $u = 0$ and $x(0) = [0, 0.0025, 0]^T$. (Hint: use the identity matrix for C in `lsim`.)

(d) What is the transfer function?

(e) Check whether the system is controllable.

(f) Design a linear state feedback controller such that the poles of the controlled system are at -20, -50, and -200. Use Ackerman's formula (implemented by `acker` in MATLAB).

(g) Verify that the controlled system has the eigenvalues above.

(h) Write down the matrices $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ of the state-variable model of the controlled system. You may assume $K_r = 1$.

(i) Plot the state variables of the system assuming $r = 0$ and $x(0) = [0, 0.0025, 0]^T$. Compare your result to part 2c.

(j) Determine the transfer function of the controlled system.

3. Consider a system of transfer function $\dfrac{2}{s(s+5)}$.

(a) Let y denote the output and u the input. Write the differential equation relating y to u.

(b) If $x_1 = y$ and $x_2 = \dot{y}$, determine A, B, C, and D.

(c) Find a state feedback gain that ensures the closed-loop system is a critically damped second order system of natural frequency $\omega_n = 2$. Hint: second order systems are described in the textbook in section 4.2.

(d) Find a state feedback gain that ensures the closed-loop system is a second order system of natural frequency $\omega_n = 2\sqrt{2}$ and damping ratio $\zeta = \frac{1}{\sqrt{2}}$.

(e) Determine the A matrix of the closed-loop system in both cases (c) and (d).

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Homework Set 4

He stores up sound wisdom for the upright; He is a shield to those who walk uprightly; He guards the paths of justice, and preserves the way of His saints. Proverbs 2:7-8

Suggested Reading: Sections 10.1–10.4 and 10.7.

1. Use the attached MATLAB script `estim.m` and Simulink file `estimsim.mdl` to investigate the effect of measurement noise and modeling error on the state estimate.

   (a) What is the effect of measurement noise on the estimation error? Attach a few plots that prove your point. You may want to simulate the system for the following values of the variance parameter of the measurement noise block: 0, 0.1, 1, and 10.

   (b) Now assume zero noise. What is the effect of an inaccurate model on the estimation error? Attach a few plots that prove your point. You may want to add random numbers to the elements of the matrices A, B, C, D, *while keeping the estimator matrices unchanged.* The random numbers could be in the range -1.5 . . . 1.5. Consider the effect of small perturbations of the A, B, C, D matrices as well as of large perturbations.

2. Find the estimator gain G so that the estimator poles are at -10. The plant model is

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0.$$

3. Consider the following continuous-time system.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

   (a) Show that the system is unstable.

   (b) Design a state feedback controller that stabilizes the system. The controller should use for feedback the output of a state estimator.

   (c) Verify your design in Simulink. Assuming $u = r - K\hat{x}$ and that the reference r is a step input (such as $r = 1$), simulate the system and verify that the state vector reaches a constant steady state. Assume that the initial state of the plant is $x(0) = [1, -2]^T$ and the initial state of the estimator is $\hat{x}(0) = [0, 0]^T$. Include a plot of the Simulink diagram and a plot of the system response.

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Homework Set 5

Glory in His holy name, let the heart of those who seek the LORD rejoice. 1 Chronicles 16:10

Suggested Reading: Sections 10.1–10.4 and 10.7.

1. Given is the following system

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 0.$$

   Integral control is to be used to drive the output y to the reference r.

   (a) Determine the new state-variable model for integral control.

   (b) Design a state feedback controller that places all poles at -1. You may use the `acker` function of MATLAB.

2. One of the design case studies of the Control Systems Toolbox of MATLAB models the head assembly of a hard disk by the equation $J\ddot{\theta} + b\dot{\theta} + k\theta = ai$, where $J = 0.01\,\text{g}\cdot\text{m}^2$, $b = 0.004\,\text{mN}\cdot\text{s}\cdot\text{m/rad}$, $k = 10\,\text{mN}\cdot\text{m/rad}$, and $a = 0.05\,\text{mN}\cdot\text{m/A}$ stand for the inertia, damping coefficient, spring constant of assembly, and the torque constant, respectively. The variable i is the current applied to the motor and θ is the angle of the head, indicating the position of the head.

   (a) Derive the transfer function $\dfrac{\Theta(s)}{I(s)}$.

   (b) Derive the state-variable model for $x_1 = \theta$, $x_2 = \dot{\theta}$, $u = i$ and $y = \theta$.

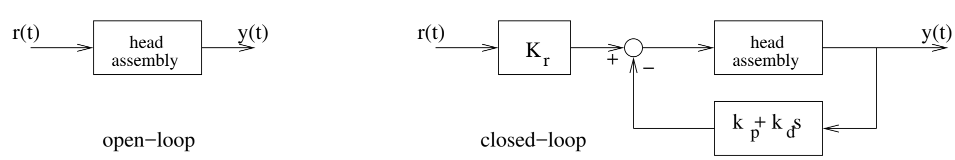
   (c) Plot the unit-step response of the system.

   (d) Apply the LQR method to find K so that the unit-step response of the closed-loop system is better than the response found at part (c). Plot the response and explain why it is better. Note that the MATLAB function `lqr` can be used to find K.

   Hint: Take $R = 1$, use a diagonal matrix for Q, and adjust the elements on the diagonal of Q until you are satisfied with the step response.

   (e) (Optional) Show that for the state-variable model found at part (b), linear state feedback is equivalent to output feedback with PD control. Find the relationship between the parameters k_p and k_d of the PD controller and the equivalent state feedback gain K.

   (f) (Optional) Express K_r in terms of k_p, a, and k_d so that the unit step response has the steady state observed at part (c). Hint: The transfer function of the plant was found at (a); the steady-state response could be found with the final-value theorem.



3. (a) Solve part (a) of problem 2.75 from *Linear Systems* by P. Antsaklis and A. Michel, McGraw Hill 1997. Note that there is a typo: y_3 should be $x_5 - x_3$.

   (b) Show that the entire state vector can be estimated by measuring $x_5 - x_3$ only.

   Hint: we can estimate the state of observable systems. Check whether this system is observable when the output is $y = x_5 - x_3$.

   (c) Assume that $x_5 - x_3$ is measured. Design a Kalman estimator that can recover the entire state vector x. Follow this approach:

      i. The system model is

$$\dot{x} = Ax + Bu + B_w w \quad \text{and} \quad y = x_5 - x_3 + w + v \tag{1}$$

      where A and B are the matrices from part (a), $B_w = B$, and w and v are the disturbance and noise inputs. Let $u_1 = [u, w]^T$. Determine B_1, C_1, and D_1 such that the equations (1) and (2) are equivalent.

$$\dot{x} = Ax + B_1 u_1 \quad \text{and} \quad y = C_1 x + D_1 u_1 + v \tag{2}$$

      ii. Using the MATLAB function `kalman` and (2), design a Kalman estimator. Assume $Q = R = 1$. Write the equations of the estimator.

      iii. (Optional) Implement the plant (2) and the Kalman estimator in Simulink. Assume the initial state of the plant is $x(0) = [0, 0, 0.005, 0, 0.01, 0]^T$ and the initial state of the estimator is 0. For v and w, use two random number sources with variance 0.005, sample time 0.001, and different initial seed numbers. Before applying y to the estimator, add the noise term v to it. You can create the input u_1 from u and w, by using a multiplexer with three inputs. Note that the function `kalman` gives you the estimator in the state-variable form with input $[u, y]^T$. Assume the input $u = \mathcal{U}(t) - \mathcal{U}(t-2)$ is applied, where $\mathcal{U}(t)$ is the unit step input. Plot the estimation error for the states x_1, x_3, and x_5. Does the error decrease in time? Plot also your Simulink diagram. It could have a structure similar to the block diagram of Figure 1.

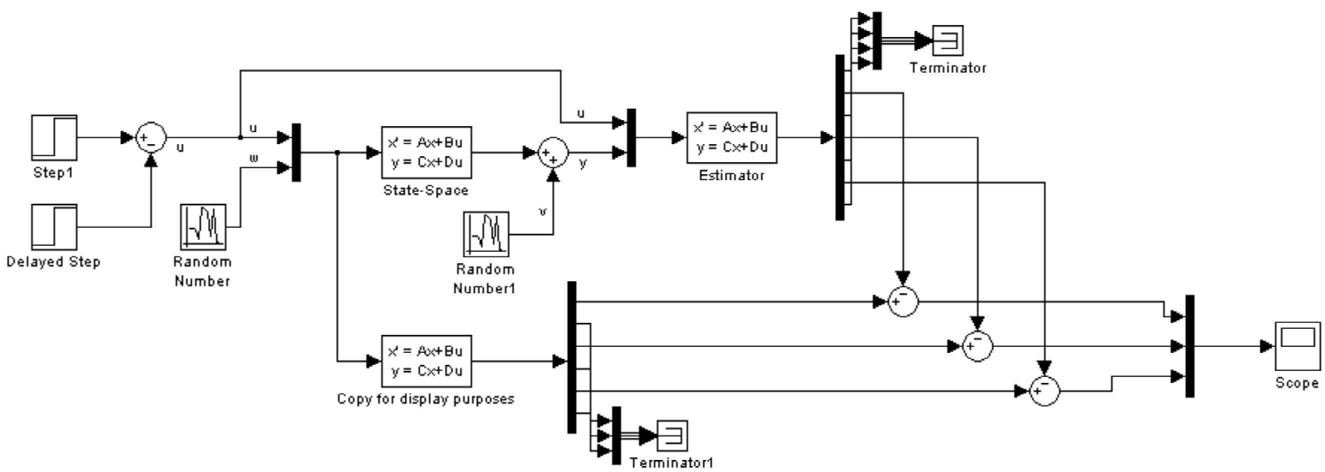


Figure 1: Suggested Simulink diagram for Problem 3(c)-iii.

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Homework Set 6

Show me thy ways, O LORD; teach me thy paths. Psalms 25:4

1. In the following system u is the control input, d is the disturbance, and v is the measurement noise.

$$\begin{aligned}\dot{x}_1 &= x_1 + x_2 + 0.1d \\ \dot{x}_2 &= x_2 + 0.1d + 0.2u \\ y &= 2x_1 + u + v\end{aligned}$$

To account for the fact that the disturbance is a low frequency signal, the following equation is added to the model, implementing a low pass filter that filters a white noise signal w:

$$\dot{d} = -d + w$$

The goal of this problem is to design a control system using the LQG method and MATLAB.

(a) Find the state feedback gain K using the LQR method. Select Q and R so as to reduce the control effort (u should be small). Hint: Ignore d, w, and v for this part of the problem.

(b) Find a Kalman estimator. First, modify the matrices A, B, C, D to account for a third state variable $x_3 = d$. Select Q and R under the assumption that w affects more the performance of the system than v.

2. Given is a plant modeled by a transfer function $G(s) = \dfrac{0.01}{s^2 + s + 0.1} e^{-4s}$.

(a) Design a control system using a Smith predictor that ensures the closed-loop has the response of a second order system with $\zeta = 0.707$ (that is, approximately 5 % overshoot). Hint: A constant gain controller is sufficient for the delay compensated plant (Figure 1).

(b) Simulate the system in Simulink, plot the response, and show that it is the correct response. Use the `Transport delay` block to implement the e^{-sT} delays. Your diagram could be similar to the one of Figure 1.

(c) How is the response changed if the delay of the plant is greater than the delay used in the controller? Include a plot.

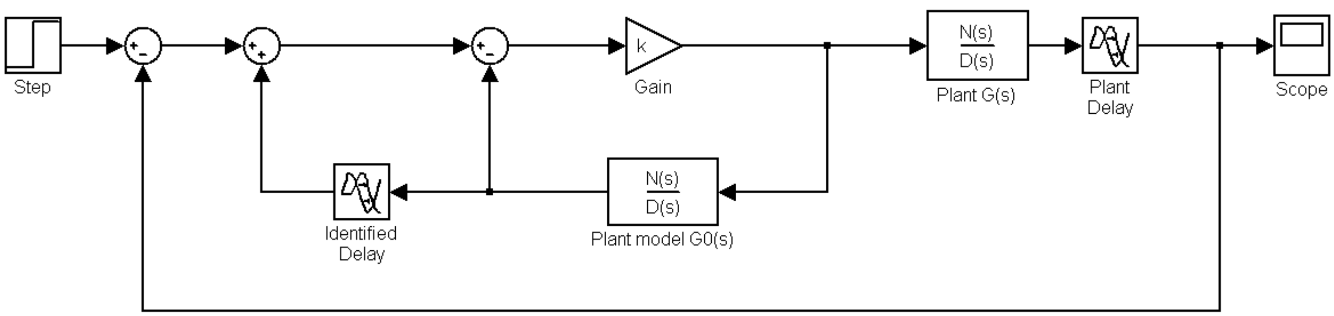

Figure 1: Simulink diagram for Problem 2.

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Homework Set 7

Praise ye the LORD. Blessed is the man that feareth the LORD, that delighteth greatly in his commandments. Ps 112:1

Suggested Reading: Lecture notes. Linearization is treated also in the book in sections 14.9 and 14.10.

1. Assume $\dot{x}_1 = (x_1 + 1)x_1x_2$ and $\dot{x}_2 = x_1 - x_2$.
   (a) Find the equilibrium points
   (b) Linearize the system about each equilibrium point.

2. Given is the system

$$\begin{aligned}\dot{x}_1 &= (2 - x_1^2)u + x_1x_2 \\ \dot{x}_2 &= -x_1 - x_2 \\ y_1 &= 3x_1 + x_1u \\ y_2 &= x_2.\end{aligned}$$

   (a) Determine the setpoints (x, u) that satisfy $u = 1$.
   (b) Linearize the system at the setpoints found at (a).

3. The following system appears in Example 11.6 of *Nonlinear Systems* by. H. Khalil, Prentice Hall, 1996.

$$\dot{x}_1 = \tan(x_1) + x_2 \quad (1)$$
$$\dot{x}_2 = x_1 + u \quad (2)$$
$$y = x_2 \quad (3)$$

   For a scheduling variable $\alpha = y_R$, the setpoint is $x_{1e}(\alpha) = -\tan^{-1}(\alpha)$, $x_{2e}(\alpha) = \alpha$, and $u_0(\alpha) = \tan^{-1}(\alpha)$. The linearized system has

$$A = \begin{bmatrix} 1+\alpha^2 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = 0 \quad (4)$$

   An example showing how MATLAB can calculate the state feedback gain $K(\alpha)$ is shown in the file `tansys2.m`. Enhance the program to calculate also the state estimator gain $G(\alpha)$. The estimator poles should be $-5 \pm (5\sqrt{3})j$. Write down the expression of $G(\alpha)$ and attach your program.

4. Use the same approach as in problem 3 to design a gain scheduled state feedback gain $K(v)$ for the system

$$\dot{x}_1 = x_2 \quad (5)$$
$$\dot{x}_2 = \frac{\alpha_1}{v}x_2 + \frac{\beta_1}{v}x_3 \quad (6)$$
$$\dot{x}_3 = -ax_3 + ax_4 \quad (7)$$
$$\dot{x}_4 = u \quad (8)$$

Choose the poles such that the system is stable. Assume $\alpha_1 = \beta_1 = a = 1$.

This model has been used for the design of wheel-slip controllers for anti-lock braking systems. The model describes the dynamics of a slipping wheel. The scheduling variable v is the velocity of the vehicle and x_2 is the slip error. Since x_1 is the integral of x_2, it is clear that an integral approach is used to minimize the error. For more information, see "Gain-Scheduled Wheel Slip Control in Automative Brake Systems" by T. Johansen et al., in *IEEE Transactions on Control Systems Technology*, 11(6), 2003.

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Homework Set 8

And it is easier for heaven and earth to pass, than one tittle of the law to fail. Luke 16:17

1. The membership functions of a Sugeno fuzzy controller are shown in Figure 1. The rules are:

   (r1) IF A AND NOT N THEN $u = 10$.

   (r2) IF B OR M THEN $u = -10$.

   (r3) IF A AND M AND N THEN $u = 0$.

   Probabilistic AND and OR operations are used in the rules. Determine the control output u when (a) $v = 0$ and $w = 0$; (b) $v = 0.5$ and $w = -30$.

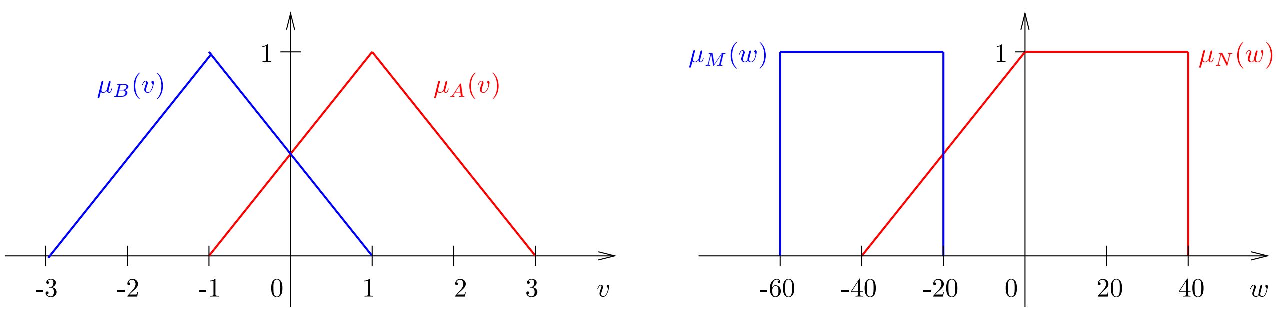
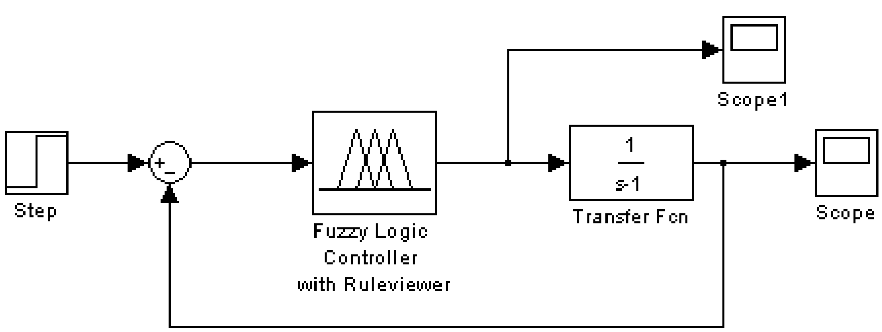


Figure 1: Membership functions for Problem 1.

2. Design a fuzzy controller that can stabilize the system $\dfrac{1}{s-1}$. Use the following control configuration:



   Assume the input is a step function. MATLAB has a very detailed description of the fuzzy toolbox. You may want to follow these steps:

   (a) Type `fuzzy` in the MATLAB command window. This will open the FIS editor (FIS–fuzzy inference system).

   (b) Develop the controller in the editor. Export your work as a `.fis` file and also as a variable in the workspace. Enter that variable as a parameter in the Simulink block of the fuzzy controller.

(c) You do not need to make any changes in the types of methods mentioned there (for AND, OR, implication, aggregation, and defuzzification). The default settings are those we have used in class. You only need to define the input and output membership functions. For instance, you could use four input membership functions and two output membership functions.

(d) The membership functions do not need to be triangles. You may choose "zmf" and "smf" for the input membership functions used at the ends of the input interval.

(e) Note that the system cannot be stabilized unless the output is large enough compared to the error signal (indeed, a constant gain would stabilize the system only if it amplifies the error signal.)

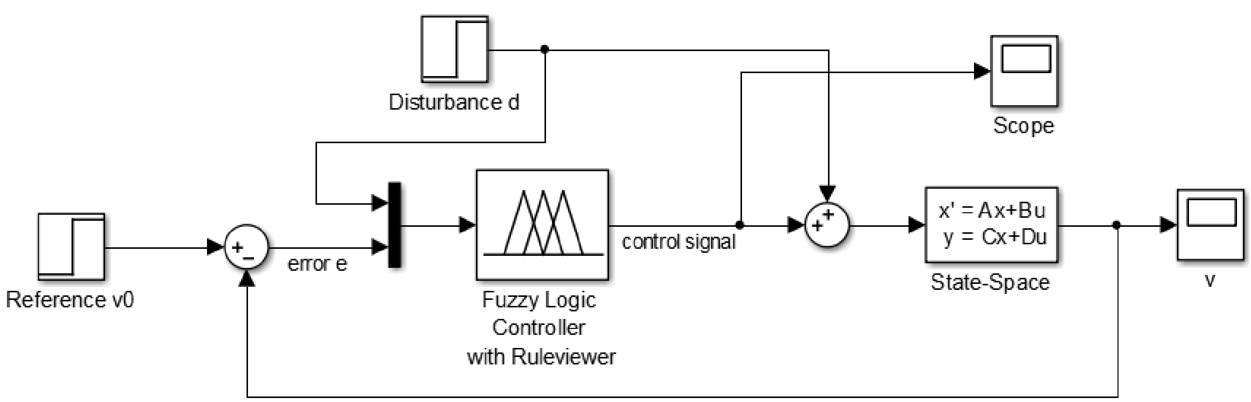
(f) An example is provided in the files fz2ord.slx and fz2order.fis.

Submit a description of your solution (Simulink block diagram, membership functions, rules, a plot of the step response, ...).

3. (a) Design a gain scheduled controller that stabilizes the system $\dot{x}_1 = |x_1 - 3| + u$, $y = x_1$. (Note that $|x|$ denotes the absolute value of x.)

   (b) For extra credit, verify your solution in Simulink. Use $4\sin(\omega t)$ for the reference input. Note that ω should be low enough to give enough time to the plant to follow the reference. Attach the block diagram and the plots.

4. A vehicle is described by the equation $\dot{v} = -0.25v + u + d$, $y = v$, where v is the velocity, u is the input applied to the vehicle, and d is a disturbance input modeling the effect of the incline of the road. The disturbance d can be measured. Implement a fuzzy control system that regulates the speed of the vehicle subject to the following constraints.



(a) The architecture shown in the figure should be used.

(b) The disturbance range is $-15 \dots 15$.

(c) The controller output range should be limited to $-20 \dots 20$ in the absence of a considerable disturbance (to save energy), but it should be extended to $-40 \dots 40$ if a large disturbance is present.

Test your design for a reference input of 50 and various disturbance levels. Describe the rules of the fuzzy controller and the membership functions. Attach plots.

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Homework Set 9

Righteous art thou, O LORD, and upright are thy judgments. (Psalm 119:137)

Suggested Reading: Sections 11.1-5.

1. Solve problem 11.1.

2. Solve problem 11.8.

3. Find the inverse z-transform of $E_1(z) = \dfrac{z}{(z-1)(z-0.8)}$ and $E_2(z) = \dfrac{z(z+1)}{(z-1)(z-0.8)}$.

4. (a) Write the difference equation relating the input and the output of a system of transfer function $G(z) = \dfrac{z+1}{z^2+0.1z+2}$.

   (b) Write the transfer function $G(z) = \dfrac{Y(z)}{R(z)}$ if $y(k+2) - 3y(k) = 2r(k+1) - r(k)$.

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Homework Set 10

Order my steps in thy word: and let not any iniquity have dominion over me. (Psalm 119:137)

Suggested Reading: Chapter 12.

1. Solve problem 11.21(b)–(e).
2. (a) Find the pulse transfer function $G(z)$ of $G(s) = \frac{1}{s+1}e^{-sT}$.
   (b) Find the pulse transfer function $G(z)$ of $G(s) = \frac{1}{s+1}e^{-1.3sT}$. (Note that e^{-nsT} may be factored out of the starred transform only when n is an integer. A possible way to solve this exercise is to factor out e^{-sT}, find the inverse Laplace transform of $\frac{1}{s+1}e^{-0.3sT}$, and then the Z-transform.)
3. Solve problem 12.18(a).
4. Solve problem 12.20(a).
5. Solve problem 12.22.

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Homework Set 11

Order my steps in thy word: and let not any iniquity have dominion over me. (Psalm 119:137)

Suggested Reading: Sections 13.1–13.6.

1. A DC motor system satisfies the following equations:

$$\begin{aligned} v &= k\omega + (r + R_s)i \\ J\dot{\omega} &= ki \\ \dot{v}_m &= \frac{1}{RC}(iR_s - v_m) \end{aligned}$$

   (a) Find the state-variable model of the system, assuming $x_c = [\omega, v_m]^T$, $y_c = v_m$ and $u_c = v$.

   (b) Using MATLAB, find the equivalent discrete-time model. Assume the following numerical values: $T = 10\,\text{ms}$, $R = 100\,\text{k}\Omega$, $C = 1\,\mu\text{F}$, $R_s = 1\,\Omega$, $r = 34\,\Omega$, $k = 13.8\,\text{mN·m/A}$, $J = 10^{-5}\,\text{kgm}^2$. The MATLAB function is `c2d`, where the third argument is `'zoh'`.

2. You may use MATLAB to solve this problem.

   (a) Find a state-variable model of $G(z) = \dfrac{z+1}{(z+2)(z+0.1)}$ (see equations (3-12) and (3-13) in the textbook).

   (b) Is the system stable?

   (c) Design a state feedback controller. Assume $T = 0.01\,\text{sec}$, $z = e^{sT}$, and that the desired poles z of the closed-loop should correspond to $s = -1 \pm j$.

   (d) What would be adequate values of the poles of a state estimator and why?

   (e) Design a state estimator.

3. Given is the following system

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 0.$$

   Integral control is to be used to drive the output $y(k)$ to the reference $r(k)$.

   (a) Determine the new state-variable model for integral control.

   (b) Design a state feedback controller that places all poles at -0.5. You may use the `acker` function of MATLAB.

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Homework Set 12

And the LORD, he it is that doth go before thee; he will be with thee, he will not fail thee, neither forsake thee: fear not, neither be dismayed. (Deu 31:8)

1. Find a state-variable model of $G(z) = \dfrac{z+1}{(z-1)(z-0.1)}$. Then, design a state-feedback controller that ensures deatbeat response. You may use MATLAB. (Hint: place all poles at zero; the `acker` MATLAB function could be used.)

2. Determine a controller $C(z)$ that approximates the PID controller $C(s) = 5 + 0.1s + \frac{10}{s}$ for a sampling period of $T = 0.1\,\text{sec}$.

3. Consider a system relating the output y to the input r according to the differential equation $\dot{y} + 100y = r$.

   (a) Is the system stable?

   (b) Using the approximation $\dot{y}(kT) \simeq \dfrac{y((k+1)T) - y(kT)}{T}$, find an equivalent discrete-time system.

   (c) Find the range of T for which the discrete-time system is stable.

4. Solve parts (a) and (b) of problem 13.5.

5. In active cruise control, the speed of the vehicle is adjusted to maintain a constant (and safe) distance to the vehicle ahead. This problem investigates the benefits of a Kalman filter used to estimate the relative position and speed of the vehicle ahead. You may use MATLAB to answer the following questions.

   (a) Find a continuous-time state-variable model of the vehicle ahead, based on the equation $\ddot{d} = a$, where d is the distance between the two vehicles and a is the relative acceleration. The state variable should be $x_c = [d, v]^T$, where v is the relative velocity.

   (b) Find a discrete-time state-variable model, assuming a sampling period of T and the zero-order hold method.

   (c) The relative acceleration will be assumed to be a random variable. Since there is an obvious correlation between $a(k)$ and the previous values $a(k-1)$, $a(k-2)$, ..., the variable $a(k)$ will be modeled as the output of a low pass filter driven by zero-mean white noise: $a(k+1) = 0.95a(k) + w(k)$.

   Find A and B_n in the state-variable model below

$$\begin{bmatrix} d(k+1) \\ v(k+1) \\ a(k+1) \end{bmatrix} = A \cdot \begin{bmatrix} d(k) \\ v(k) \\ a(k) \end{bmatrix} + B_n \cdot w(k) \qquad\qquad y(k) = d(k) + n(k)$$

   where $n(k)$ is the measurement noise.

   (d) Assuming $T = 1\,\text{ms}$, design a Kalman estimator of d and v. Assume that w has a larger variance than n.

(e) Consider the model of part (b) with $a(t) = 0.5u(t) + 2u(t-5) - 2.5u(t-6)$. Simulate the system from $t = 0$ to $t = 10\,\mathrm{s}$ and compare the real distance d to its measurement y and estimate $\hat{d}$. Assume $d(0) = 20\,\mathrm{m}$ and $v(0) = -4\,\mathrm{m/s}$ and $\hat{d}(0) = 15\,\mathrm{m}$ and $\hat{v}(0) = 0\,\mathrm{m/s}$. Note that $n(k)$ is simulated by a `Random Number` block with variance R and sampling time T. Attach plots.

(f) Compare also the real relative velocity v to its measurement estimate $v_e(k) = (y(k) - y(k-1))/T$ and Kalman estimate $\hat{v}$. Attach plots.

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Homework Set 13

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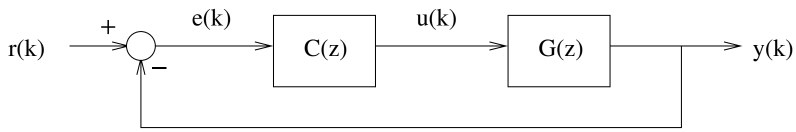
Suggested Reading: Sections 11.7–8, 12.9, 13.14.

1. Solve problem 13.8.

2. Assume a stable control system. The input is a unit step input.



   (a) Assuming $C(z) = k_p + \dfrac{k_i}{1 - z^{-1}}$ (a PI controller), determine the steady state value of $e(k)$, $u(k)$, and $y(k)$.

   (b) Assuming $C(z) = k_p$ (a proportional controller), determine the steady state value of $e(k)$, $u(k)$, and $y(k)$.

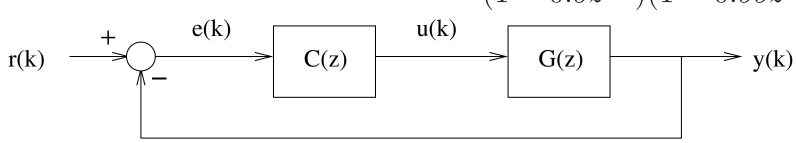
3. Let $G(z) = \dfrac{z}{z - 0.9}$ and $H(z) = \dfrac{z}{z + 0.9}$. Assume a sampling period $T = 0.01\,\text{s}$.

   (a) Plot the step response of each transfer function.

   (b) Explain the difference between the aspect of the two graphs.

   (c) Are the settling times equal? Are the steady-state values equal? Should they be equal?

4. Assume $T = 0.01\,\text{sec}$, $C(z) = k > 0$, and $G(z) = \dfrac{0.01}{(1 - 0.9z^{-1})(1 - 0.99z^{-1})}$.



   (a) Determine the range of k for stability.

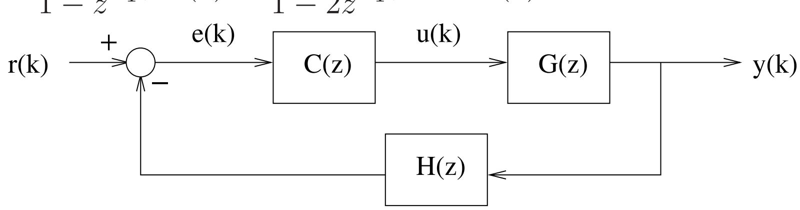
   (b) Find a first-order approximation of $G(z)$.

   (c) Using a first-order approximation, find k for a $2\,\%$ settling time of $1\,\text{sec}$.

   (d) Find the steady-state error to a unit step input in terms of k.

   (e) Simulate the system using the exact transfer function model of G. Verify the steady-state error and the settling time. Attach a plot.

5. Assume $C(z) = \dfrac{k_i z^{-1}}{1 - z^{-1}}$, $G(z) = \dfrac{1}{1 - 2z^{-1}}$, and $H(z) = k - z^{-1}$.



(a) Assuming stability and a unit step input, determine the steady-state value of the output.

(b) Assuming $T = 0.01\,\text{sec}$, what should be the poles of the system for a second order response with $\zeta = 0.7$ and $\omega_n = 4\,\text{rad/s}$?

(c) Find k and k_i so that the system has the poles calculated at part (b).

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Homework Set 14

And the LORD, he it is that doth go before thee; he will be with thee, he will not fail thee, neither forsake thee: fear not, neither be dismayed. (Deu 31:8)

1. A feedforward neural network has two layers. The network has 5 inputs and 7 outputs. The hidden layer has 20 neurons.
   (a) How many neurons are in the output layer?
   (b) What is the number of (scalar) parameters tuned in the training process?

   Explain your answers.

2. The file `nnexample.m` shows an example of a feedforward neural network.
   (a) Run the example when the number of neurons in the hudden layer is: (a) $S = 5$; (b) $S = 20$; (c) $S = 100$; (d) $S = 200$.
   (b) Attach plots for each case.
   (c) What happens when the number of neurons is too small? What happens when the number of neurons is too large?
   (d) Select a different signal that should be approximated by the neural network. (Not a sine nor a cosine.) Find a neural network that can approximate it and attach a plot.

3. Go to `http://www.cybosoft.com/technologies/sisomfa.html` and read about the SISO MFA controller. Then describe it in your own words. (Your description should use 100 words or more.)

4. (a) Consider the system $y(k+1) = -0.5y(k) - 0.4y(k-1) + 0.6r(k) + 0.3r(k-1)$. Write the transfer function $Y(z)/R(z)$.
   (b) From this point on, assume the nonlinear system $y(k+1) = -0.5y(k) - 0.4y(k-1) + 0.6e^{u(k)} + 0.3e^{u(k-1)}$ obtained by replacing r with e^u.
   (c) Create a data set of about 1000 points, listing the response of the nonlinear system to a random input u.
   (d) Train a neural network to approximate the nonlinear system. Use `ntstool` in MATLAB. Export the resulting neural network to Simulink.
   (e) Compare the response of the actual system to the response of the neural network. Use another random input. Plot the graph of the error.