EEGR3523 MECHATRONICS
Homework Sets Assigned in the Spring Semester of 2019

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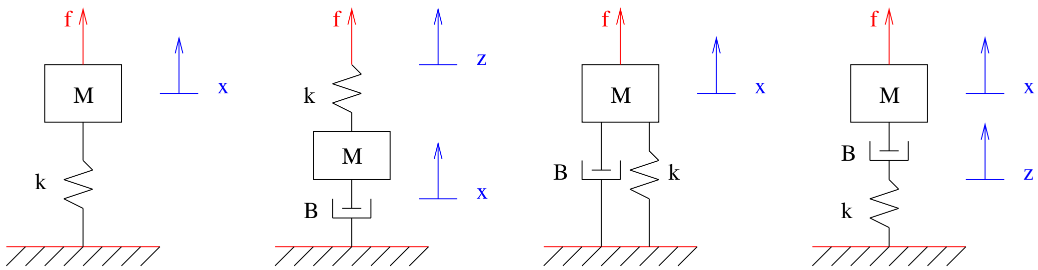
Homework Set 1

My son, forget not my law; but let thine heart keep my commandments: For length of days, and long life, and peace, shall they add to thee. Prov 3:1-2

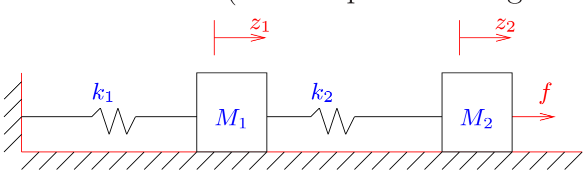
1. Write the differential equations describing the systems below, where x and z denote displacements. Be sure to include the Mg term in your equations.



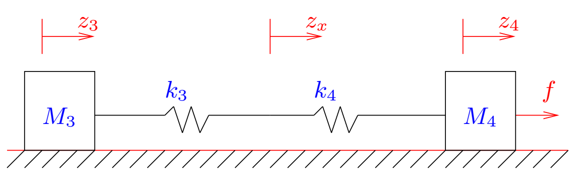
(a) (b) (c) (d)

Sample answers: (b) $f = k(z - x)$, $f = M\ddot{x} + B\dot{x} + k(x - z)$.

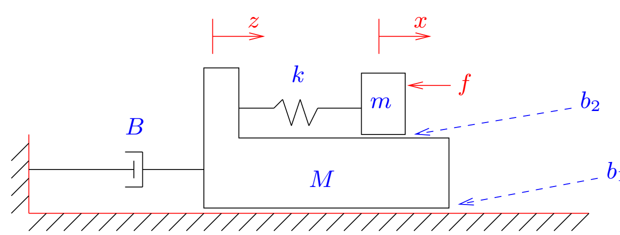
2. Unless otherwise mentioned, assume a viscous friction coefficient b. All displacement variables are absolute (with respect to the ground). Write the equations of motion in each case.



(a)



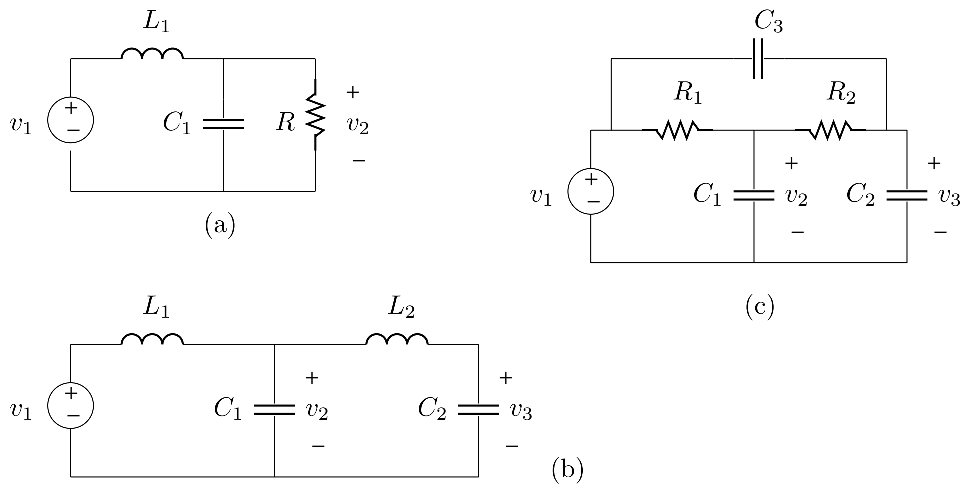
(b)



(c)

Sample answers: Two equations of part (b) are $0 = M_3\ddot{z}_3 + b\dot{z}_3 + k_3(z_3 - z_x)$ and $0 = k_3(z_x - z_3) + k_4(z_x - z_4)$. One equation of part (c) is $-f = m\ddot{x} + b_2(\dot{x} - \dot{z}) + k(x - z)$.

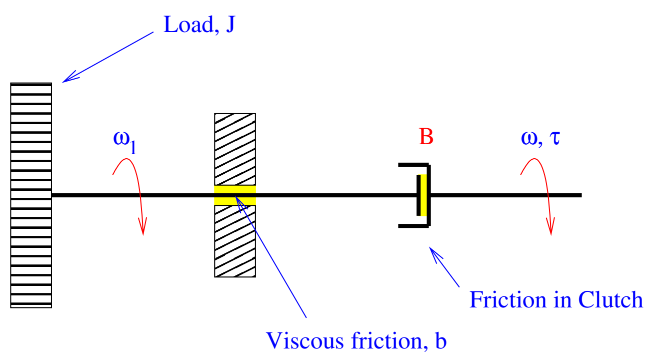
3. For each of the following circuits, write the differential and/or integral equations relating v_1, v_2, and v_3.



4. Write the equations of the circuit (c) of problem 3 in the Laplace domain. Assume nonzero initial conditions.

   Sample answers: One equation is $(C_2 + C_3)(sV_3 - v_3(-0)) + \frac{V_3}{R_2} - \frac{V_2}{R_2} - C_3(sV_1 - v_1(-0)) = 0$.

5. Consider the following model of a mechanical system. Note that τ denotes torque and ω speed. Assume nonzero initial conditions. Write the equations of motion (a) in the time domain; (b) in the Laplace domain.



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Homework Set 2

For the Scripture saith, Whosoever believeth on him shall not be ashamed. Romans 10:11

Suggested Reading: Laplace transform related material can be found at pages 1236-1248 and 130-131. For analogies, see pp. 21-24 and pp. 57-58. Frequency Domain Models are mentioned at pp. 157-159.

1. The file `solveexample.m` is a Matlab script that calculates $z_1(t)$ for the system shown in Figure 1(a) when $f(t) = f_0\mathcal{U}(t)$ and $z_2(0)$ is not necessarily zero. The script also plots $z_1(t)$ graphically for $M = 1\,\text{kg}$, $k = 55\,\text{N/m}$, $z_2(0) = 0.005\,\text{m}$, and $f_0 = 2\,\text{N}$. To run the script, open it first in Matlab from File/Open.... This will open the file into a new window. To run the program, select Run in the editor window. The results will be shown in the Command Window of Matlab.

   Modify the script such that it calculates and plots $z_1(t)$ for the system shown in Figure 1(b), where $m = 0.01\,\text{kg}$. Assume $z_1(0) = z_2(0)$. Document your changes and attach your results.

   The program requires the symbolic toolbox of Matlab. **Run the program on a Glaske computer** since the symbolic toolbox is installed in Glaske.

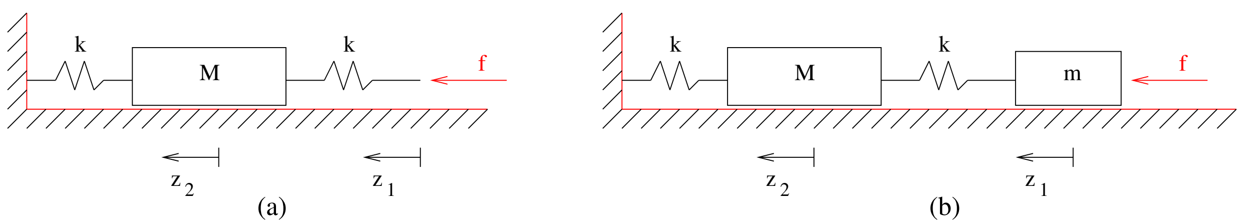
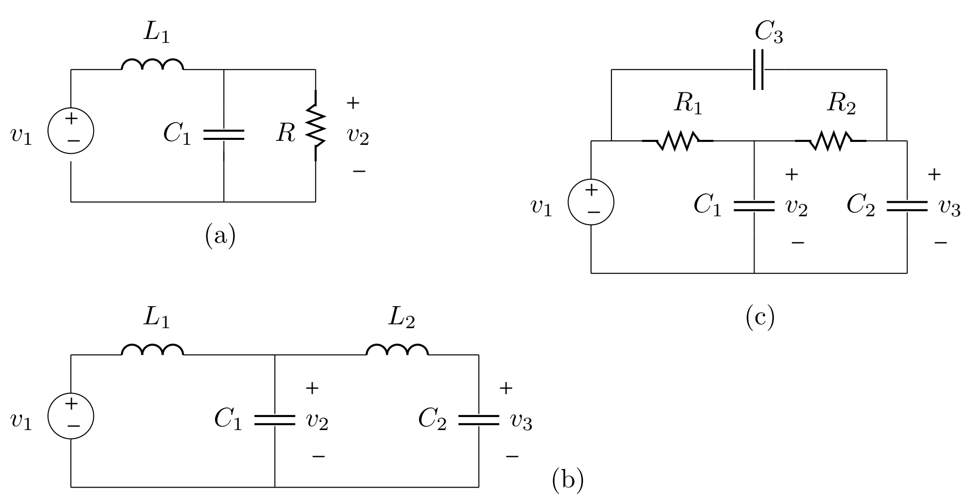


Figure 1: The systems of probem 1.

2. Draw the impedance diagrams of the following circuits assuming

   (a) zero initial conditions;

   (b) nonzero initial conditions.



3. Refer to page 7 of the motion-control handout. Assume a lead-screw mechanism operating at 85% efficiency. The pitch is $P_s = 10\,\text{rev/in}$, $\gamma = 0$, the mass of the load and of the moving parts is $M_L + M_T = 40\,\text{lb}$, and the inertia of the screw and of the coupling is $J_C + J_S = 15000\,\text{g}\cdot\text{cm}^2$. Assume no prelaod and neglect friction.

   (a) Find the motor inertia J_M in $\text{g}\cdot\text{cm}^2$ for which $J_{Total} = 5J_M$.

   (b) Find the motor torque in Nm when $F_P = 900\,\text{N}$ and the acceleration is zero.

   Sample answers: (a) $3759\,\text{g}\cdot\text{cm}^2$.

4. Refer to page 7 of the motion-control handout. Assume a conveyor mechanism operating at 85% efficiency. Objects of mass $M_L = 10\,\text{lb}$ should be transported, one at a time, at an incline of $\gamma = 30°$. The mass of the belt is $M_B = 4\,\text{lb}$. The three pulleys are identical, each with a diameter of 10 in and an inertia of $75000\,\text{g}\cdot\text{cm}^2$. Neglect friction.

   (a) Find the motor inertia J_M in $\text{g}\cdot\text{cm}^2$ for which $J_{Total} = 5J_M$.

   (b) Find the motor torque in Nm when the acceleration is zero.

5. An AGV uses gear transmission to reduce the motor speed ω_m by a factor of n, so that the angular velocity of the wheels is $\omega = \frac{\omega_m}{n}$. The efficiency of the transmission is e. The inertia of the motor is J_m, the inertia of the transmission (as seen by the motor) is J_r, the wheels have a radius of r, the total inertia of the wheels is J_w, and the vehicle has a total mass m. Find (a) The total inertia seen by the motor; (b) The motor torque required to climb an incline of angle γ at constant speed. Neglect friction.

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Homework Set 3

It is good for me that I have been afflicted; that I might learn thy statutes. Psalms 119:71

Suggested Reading: The transfer function, pages 130-131 and 1247–1248. Frequency domain models, sections 2.12.1 and 2.12.2.

1. Sketch the graph of the following functions and find their Laplace transform.
   (a) $\mathcal{U}(t-2) - \mathcal{U}(t-4)$;
   (b) $(t-2)\mathcal{U}(t-2) - 1.5(t-3)\mathcal{U}(t-3) + 0.5(t-5)\mathcal{U}(t-5)$.
2. Find the Laplace transform of the following
   (a) $\sin(t-1)$ Hint: $\sin(t-a) = \sin(t)\cos(a) - \cos(t)\sin(a)$
   (b) $\sin(t-1)e^{3t}$
   (c) $\sin(t-1)e^{3t}\mathcal{U}(t-1)$
   (d) $\sin(t)e^{3(t-1)}\mathcal{U}(t-1)$
3. Find the inverse Laplace transform of the following
   (a) $\dfrac{6s}{(s+2)(s-2)(s+1)(s-1)}$
   (b) $\dfrac{s+1}{s^2-4s+3}$
   (c) $\dfrac{2e^{-4s}}{s(s+2)}$
   (d) $\dfrac{e^{-(s-1)}}{s^2+4}$
   (e) $\dfrac{1}{(s+3)^2+9}$
4. Answer questions (a) and (b) of problem 2.49, and determine $y(t)$ when u is the unit step input. Sample answer: $y(t) = \frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}$.
5. Solve problem 2.65.
6. Solve parts (c) and (d) of problem 2.68.
7. What is a physically realizable system? (See section 2.11.1.) Solve problem 2.53(a).

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Homework Set 4

... all things are possible to him that believeth. Mark 9:23

Suggested Reading: The state space model, in section 2.3 at pages 29–37. Section 2.11.2 at pages 137–148 provides several methods of finding a minimal state-space representation of a system. Frequency Domain Models, pp. 157-159.

1. Let $f(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2+s-2}\right]$.
   (a) Calculate $\lim_{s\to 0} sF(s)$ and $\lim_{s\to\infty} sF(s)$.
   (b) Calculate $f(t)$, then $f(+0)$ and $\lim_{t\to\infty} f(t)$.
   (c) Can we use the initial and final value theorems to find $f(+0)$ and $\lim_{t\to\infty} f(t)$? Explain.

2. For all systems that follow, y is the output and r is the input.

$$\begin{array}{lll} (a)\ \ddot{y}+2ty=\dot{r}+3r & (b)\ y^{(3)}+3\ddot{y}+4\dot{y}+2y=r & (c)\ y^{(3)}+3y\dot{y}+2y=\dot{r} \\ (d)\ \dot{y}-y=\dot{r}+4r & (e)\ \ddot{y}+2\dot{y}+y=r & (f)\ \ddot{y}+2\dot{y}+y=r^2 \end{array}$$

   (a) Which systems are linear and time-invariant?
   (b) Write the transfer function of each LTI system.
   (c) For each LTI system for which the final value theorem can be applied, use the final value theorem to determine the steady state value of y when r is a unit step input.

3. Calculate the steady-state response of the system $H(s) = \frac{1}{s+7}$ to the input $r(t) = 3 + 2\sin(3t+2) - 0.1\cos(19t-3)$.

4. Solve problem 3.30 by following this approach.
   (a) Let τ_m be the input torque. Let Ω_m and Ω_l be the Laplace transforms of $\dot{\theta}_m$ and $\dot{\theta}_l$.
   (b) Let $\Omega = \Omega_m - \Omega_l$. Calculate $G(s) = \frac{\Omega(s)}{T_m(s)}$. (Assume zero initial conditions.)
   (c) Taking in account $\Omega(s) = \mathcal{L}\{\dot{\theta}\}$ and the formula for the Laplace transform of the derivative, derive $H(s) = \frac{\Theta(s)}{T_m(s)}$ from the expression of $G(s)$.
   (d) Write the differential equation represented by $H(s)$.

5. Solve part (b) of problem 2.52 using $x_1 = f_k$ and $x_2 = v$.

   Sample answers: $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ and $D = \frac{1}{m}$.

6. Find the unit-step response of the systems described by the following transfer functions: (a) $H_a(s) = \frac{s-2}{s+2}$; (b) $H_b(s) = 10$; (c) $H_c(s) = \frac{3}{s}$; (d) $H_d(s) = \frac{6}{(s+2)(s+3)}$.

   Sample answers: (b) 10; (c) $3t$.

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Homework Set 5

Teach me good judgment and knowledge: for I have believed thy commandments. Psalms 119:66

Suggested Reading: The state space model, in section 2.3 at pages 29–37. Section 2.11.2 at pages 137–148 provides several methods of finding a minimal state-space representation of a system. For block diagrams, see pages 137–139. Block diagram manipulation was addressed in class. In the textbook, it is addressed in Example 12.2 at page 1003. Note that in Example 12.2 lower case letters are used for Laplace domain variables, such as y for $Y(s)$ and u for $U(s)$. A description of Mason's gain formula is available on Blackboard in Course Documents.

1. Example 2.3 at pages 35–37 derives the matrices A and B of the state-space model of a prime mover.

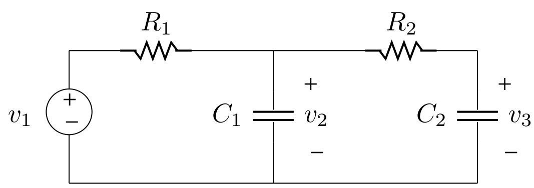
   (a) Write the numerical values of the matrices A, B, C, and D of the state-space model when $y = T$, $K = 1000\,\text{Nm/rad}$, $B_1 = 5000\,\text{Nsm/rad}$, $B_2 = 100\,\text{Nsm/rad}$, and $J = 250\,\text{kgm}^2$.

   (b) Plot the response of the system when $\Omega(t) = 30\,\mathcal{U}(t-1)\,\text{rad/s}$. (This input assumes the clutch is engaged beginning with time $t = 1\,\text{s}$ and that the engine maintains constant speed.) Assume zero initial conditions. Print and label your plots.

   Hint: See the attached examples in `state_intro.mdl` showing how to simulate state-space models in Simulink. (The file can be read when opened in Simulink.)

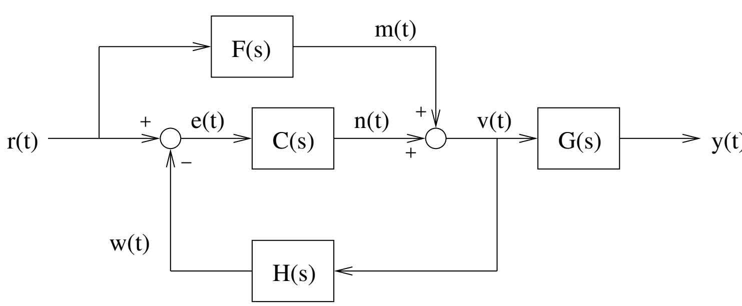
2. Determine a state variable model for the circuit shown below. (That is, determine the matrices A, B, C, and D.) Assume that v_1 is the input and v_3 is the output. Assume also zero initial conditions.

   Sample answers: If $x_1 = v_2$ and $x_2 = v_3$, the first element of A is $A_{11} = -\frac{1}{R_1 \| R_2 C_1}$ and $D = 0$.



3. Solve problem 2.73. Hint: Let $G(s)$ be the transfer function $\frac{X_1(s)}{F(s)}$. Determine the frequency ω for which $G(j\omega) = 0$.

4. Write the equations relating $E(s)$, $M(s)$, $N(s)$, $R(s)$, $V(s)$, $W(s)$ and $Y(s)$. Then, determine the transfer function $Y(s)/R(s)$.



5. [From D. Shetty and R. Kolk, *Mechatronics Systems Design*, PWS Publishing Company, 1997] Determine the transfer function of the block diagrams of Figure 1.

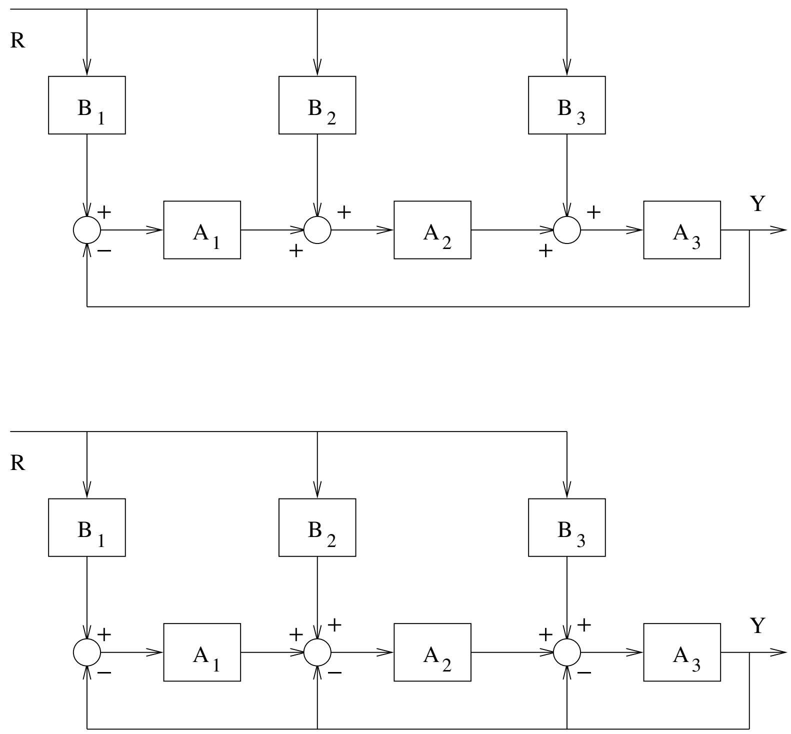


Figure 1: Block diagrams of problem 5.

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Homework Set 6

O give thanks unto the God of heaven: for his mercy endureth forever. Psalm 136:26

Suggested Reading: Pages 1006–1007 and 1009–1018. The Routh-Hurwitz criterion is discussed at pages 1038–1044.

1. Given are the following systems

$$H_1(s) = \frac{s}{(s+1)(s+2)}, \quad H_2(s) = \frac{s-1}{s(s+1)}, \quad H_3(s) = \frac{s+2}{s-10},$$
$$H_4(s) = \frac{s^2+2s+6}{s^4+4s^3+6s^2+4s+1}, \quad H_5(s) = \frac{s-1}{s^2+4s+4}, \quad H_6(s) = \frac{s+5}{s^2-s+2}$$

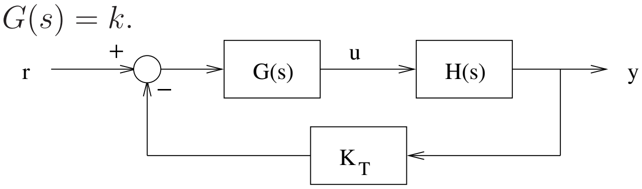
   (a) Determine the poles and the zeros of each transfer function. When necessary, you may use a computer or calculator (for MATLAB, you could use the function `roots` or `pole` and `zero`.)

   (b) Which of the systems is stable, and why?

2. Using the Routh-Hurwitz criterion, determine which of the following systems are stable.

$$H_1(s) = \frac{s}{s^2+3s+10}, \quad H_2(s) = \frac{s-1}{s^3+s^2+s+2}, \quad H_3(s) = \frac{s+2}{s^2-s-10},$$
$$H_4(s) = \frac{s+6}{s^2-s+1}, \quad H_5(s) = \frac{s-1}{s^3+0.1s^2+4s+4}, \quad H_6(s) = \frac{s+5}{s^3+8s^2+s+4}$$

3. Assume that $K_T = 1$ and $G(s) = k$.



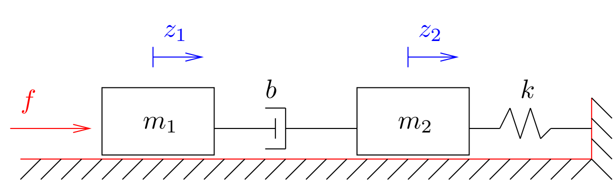
   (a) Determine for which of the following systems there is k such that the closed-loop is stable.

$$\text{(i) } H(s) = \frac{s-1}{s^2+3s+10} \quad \text{(ii) } H(s) = \frac{s+1}{s^3+s^2-s} \quad \text{(iii) } H(s) = \frac{4}{s^4+2s^3+s^2+s}$$

   (b) In each case determine the range of k for which the closed-loop is stable.

   Sample answers: At part (b), one of the answers is $0 < k < 1/16$.

4. Given the system below:



   (a) Write the differential equations describing the system.

(b) Find the matrices A, B, C, and D of the state space model. Assume that $x = [\dot{z}_1 \quad z_2 \quad \dot{z}_2]^T$, $y = z_2$, and the input is $r = f$.

(c) Assuming $m_1 = m_2 = 1\,\text{kg}$, $k = 100\,\text{N/m}$, $b = 1000\,\text{Ns/m}$, $f(t) = 30\,\mathcal{U}(t)\,\text{N}$, and zero initial conditions, simulate the system in Simulink. Attach a plot of z_2.

5. Determine the transfer function of the systems of Fig. P12.34.

6. Given the block diagram of Figure 1, determine the transfer function Y/R.

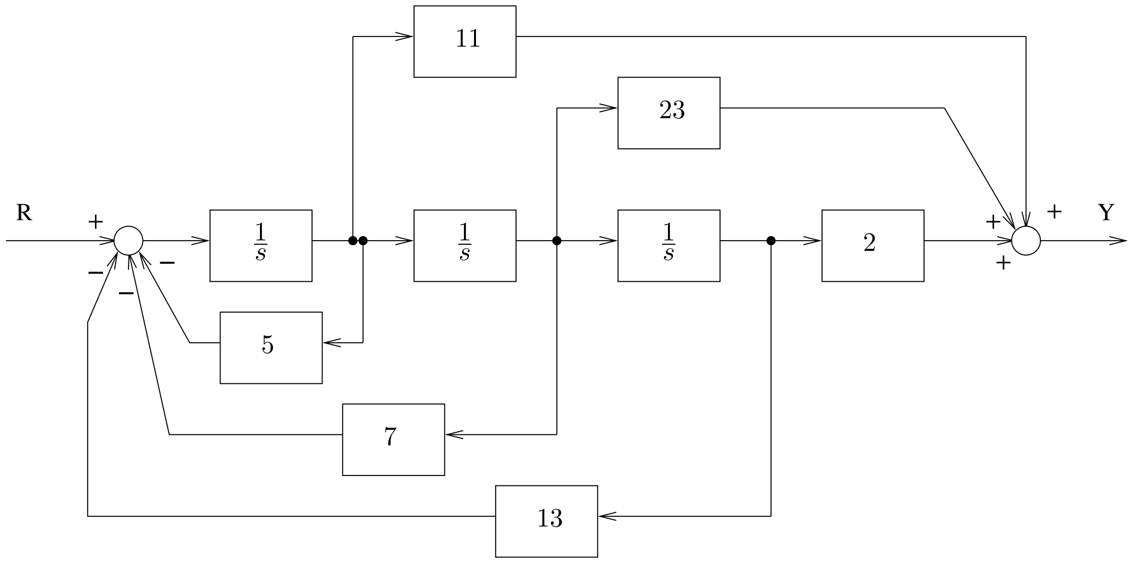


Figure 1: Block diagram for Problem 6.

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Homework Set 7

If the iron be blunt, and he do not whet the edge, then must he put to more strength: but wisdom is profitable to direct. Ecclesiastes 10:10

Suggested Reading: Steady-state error is discussed in sections 12.3.1–12.3.7. Please note the textbook notation. In sections 12.3.1–12.3.3 u is a constant input, so $U(s) = u/s$. In the remaining sections u, e, and y are in the Laplace domain. That is, (12.21) means $e(s) = u(s) - H(s)y(s)$, (12.22) means $y(s) = G(s)e(s)$, and so on. Laplace domain variables are not capitalized through much of section 12. In the lecture notes e is always $e(t)$, E is always $E(s)$, and so on.

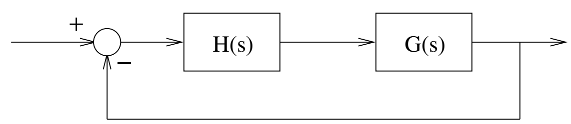
1. Assume a control system with unity feedback (Fig. 12.11).

   (a) What is the difference between the loop transfer function and the closed-loop transfer function?

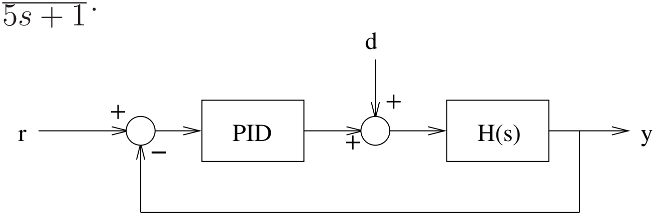
   (b) Solve problem 12.18, under the assumption that stability can be ensured in every case. Note that loop, not closed-loop, transfer functions are given.

   Sample answers: (d) $k = 95.95$; (e) any $k > 0$ ensures zero steady state error.

2. Given $G(s) = \dfrac{1}{s^2 + 9s - 10}$, determine a controller $H(s)$ that ensures stability and zero steady state error to a step input.



3. In the following system r is a unit step and d is an unknown constant disturbance satisfying $|d| < 2$. Assume $H(s) = \dfrac{0.1}{5s + 1}$.



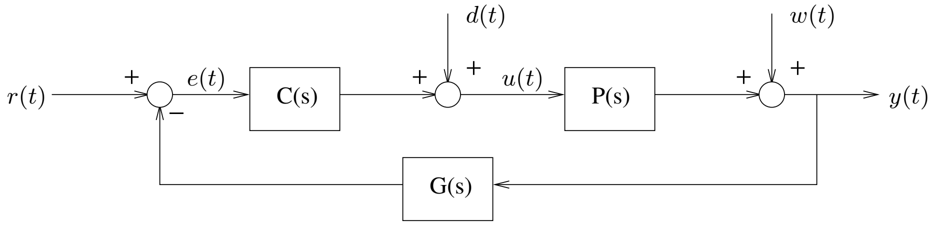
   (a) Determine a proportional controller (P-type controller) that ensures the steady-state error does not exceed 5%.

   (b) Determine a PI controller that ensures zero steady-state error.

4. Assume stability, $r = 2$, $P(0) = 0.1$, $G(0) = 0.5$, and constant disturbances $d = 0.4$ and $w = 0.3$.

   (a) Find the steady state value of e, u, and y when $C(s)$ is a proportional controller with $k = 100$.

   (b) Find the steady state value of e, u, and y when $C(s)$ is a PI controller.



5. Solve Problem 12.21(a)–(b).

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Homework Set 8

Teach me thy way, O LORD; I will walk in thy truth: unite my heart to fear thy name. Psalm 86:11

Suggested Reading: Pages 1009–1018.

1. Consider a system of transfer function $G_p(s) = \frac{1}{s+1}$. Note that $G_p(s)$ could be the transfer function of a DC motor.

   (a) Determine the 2 % settling-time of the response of $G_p(s)$ to a step input.

   For parts (b)–(d), assume the system is in the unity feedback configuration of Figure 12-11. Assume that $G_c(s) = k$ and that the input is a step input.

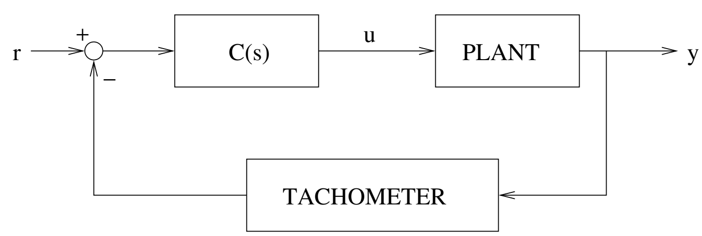
   (b) Determine the values of k for which the 2 % settling time is 0.1 s, 1 ms, and 1 μs.

   (c) Verify that the maximum value of the control signal $c(t)$ occurs at $t = 0$ and calculate $c(0)$ for the values of the settling time given at (b).

   (d) Based on the results at parts (b) and (c), discuss practical limitations in making a system (such as a motor) move its output instantaneously to the final value.

   Sample answers: For a 0.1 s settling time, $k = 39$ and $c(0) = 39$.

2. A motion control system involves an electronic controller $C(s)$, a tachometer used as a velocity sensor, and a plant consisting of a motor, a motor drive, and a gearbox. When the plant is applied a control input $u = 5$ V, the steady-state value of the output y is 200 rpm. The tachometer outputs 100 mV when the output $y = 100$ rpm. Assume that all components are modeled by transfer functions. Assume also stability.

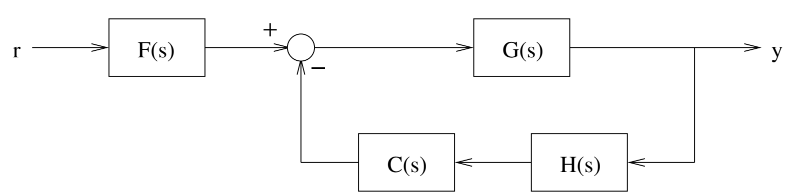


   (a) Assuming a PI controller, what should be the reference input for a steady-state output of 500 rpm?

   (b) Assuming $C(s) = 250$, what should be the reference input for a steady-state output of 500 rpm?

   Sample answers: (a) 500 mV.

3. Consider the system below for $G = \frac{1}{s^2+1}$, $H = 2s$, and $F = 1$.



(a) Calculate the closed-loop transfer function for $C(s) = \zeta$.

(b) Enter the following commands in the command window of Matlab:

```
s = tf('s');
Gm = 1/(s^2+1)
Hm = 2*s
sisotool
```

(The commands define the variable s, the transfer functions $F(s)$, $G(s)$, and $H(s)$, and open the `sisotool`. A description of the `sisotool` can be found in its help menu or in appendix B.4 at pp. 1270-1279.)

(c) Click on the *Edit Architecture* button of the *Control System* tab. This will open the *Edit Architecture* window. In the *Edit Architecture* window:

   i. Select the first control architecture.

   ii. In the *Blocks* tab select `F` = `1`, `G` = `Gm`, and `H` = `Hm`.

   iii. Press the *OK* button to close the *Edit Architecture* window.

(d) Locate the step response plot.

   i. Right click on the white area of the plot and select *Full View*.

   ii. Right click on the white area of the plot, select *Characteristics* and then *Settling Time*.

(e) Click on C in the *Data Browser/Controllers and Fixed Blocks*. This will display the value of C in the *Preview* area of the *Data Browser*. At the end of this step the `sisotool` window should be similar to Figure 1.

(f) There are several ways to change the value of C.

   i. Drag up or down the magnitude Bode plot.

   ii. Drag the purple squares of the *Root Locus* plot.

   iii. Open the *Edit Architecture* window, write the desired value of C, and then close the *Edit Architecture* window.

(g) Vary $C(s)$ over the interval $0 \ldots 100$. Let $\zeta = C(s)$.

   i. Does the rise time of y decrease or increase as ζ increases?

   ii. How does the 2% settling time change when ζ is varied?

   iii. How does the overshoot change as ζ is varied?

(h) Measure the settling time for $\zeta = 0.2$, $\zeta = 0.5$, $\zeta = 0.7$, $\zeta = 1$, and $\zeta = 5$. Note: You may right click the curve and then drag the black square to the point of the curve that you want to measure.

(i) The formula $t_s = \frac{4}{\zeta\omega_n}$ can be used to estimate the 2% settling time only when $0 < \zeta < 1$. Are your results consistent with this statement?

(j) According to the formula $t_s = \frac{4}{\zeta\omega_n}$, the response is faster for $\zeta = 1$ than for $\zeta < 1$. However, the formula $t_s = \frac{4}{\zeta\omega_n}$ is an approximation. Use the sisotool to estimate ζ that minimizes the settling-time.

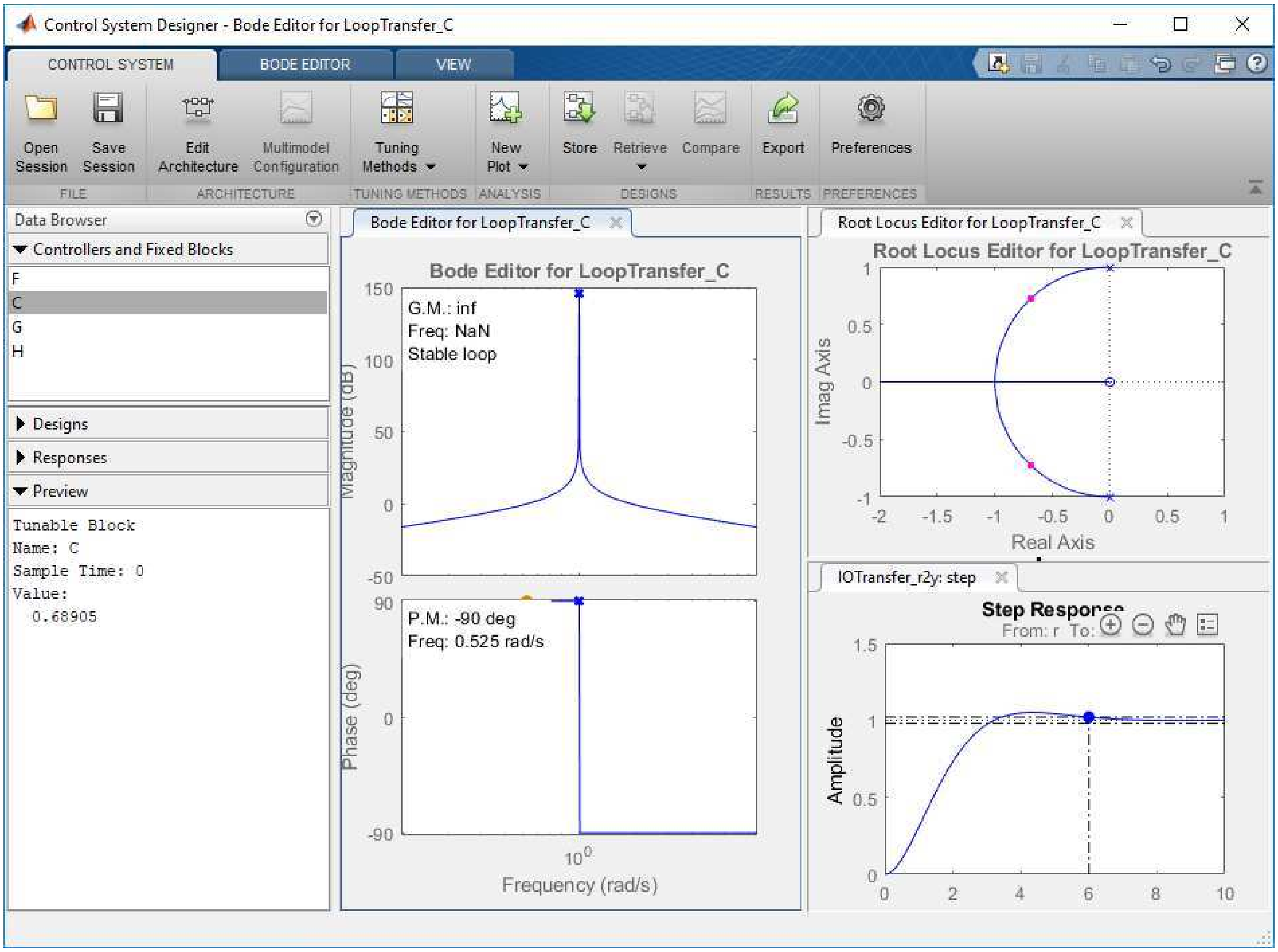
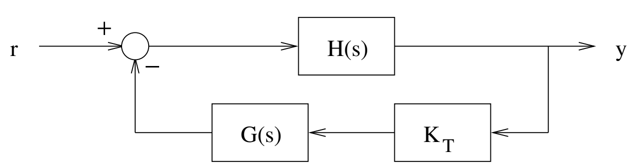


Figure 1: The `sisotool` window in problem 3.

4. Consider the response to a step input of the following system. Assume $K_T = 1$, $G(s) = k_p + k_d s$, and $H(s) = \dfrac{1}{s(s+1)}$. Determine k_p and k_d for 7 % overshoot and 10 ms settling time.



5. A rear-wheel drive vehicle is actuated by a DC motor. The gear ratio is $n = \frac{\omega}{\omega_w}$, where ω is the speed of the motor and ω_w the angular velocity of the wheels. The radius of the wheels is ρ and the mass of the vehicle is M. The motor satisfies the equations $v = ri + k\omega$ and $\tau = ki$, where i is the electrical current, r is the internal resistance of the armature, v is the voltage,

τ is the torque, and k is the torque constant. Determine the transfer function $\dfrac{N(s)}{V(s)}$, where N is the Laplace transform of the velocity ν of the vehicle, and V is the Laplace transform of the voltage v.

Sample answers: If the answer is written in the form $\dfrac{a}{s+b}$, the numerator is $a = \dfrac{kn}{Mr\rho}$.

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Homework Set 9

Thy hands have made me and fashioned me: give me understanding, that I may learn thy commandments. Psalm 119:73

Suggested Reading: The root locus, at pages 1044–1059. One interesting application of the root locus is to the design of lead/lag compensators (not discussed in class): pages 1104-1116.

1. Find a first order or a second order approximation of the following systems:

$$\frac{180(s+20)}{(s+2)(s^2+30s+900)};\quad \frac{50}{(s^2+2s+2)(s+5)};\quad \frac{1200}{s^4+32s^3+263s^2+490s+600}$$

   Hint: For the third transfer function you may find the poles with a calculator or with software (such as the `roots` function of Matlab).

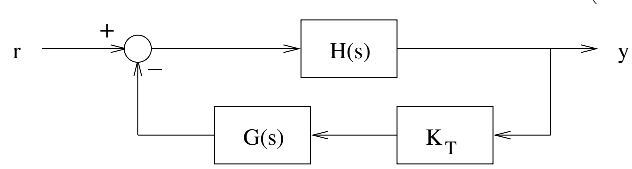
2. (a) Assuming $k > 0$ and $F(s) = \dfrac{(s+5)(s+6)}{(s+3)(s+4)(s+7)}$, plot the root locus. You may use either `rlocus` or `sisotool` (the latter recommended).

   (b) Indicate: (i) the number of closed-loop poles; (ii) the number of asymptotes; (iii) The position of the closed-loop poles when $k \to 0$; (iv) The position of the closed-loop poles when $k \to \infty$.

   (c) Assuming $k > 0$ and $F(s) = \dfrac{(s+5)(s+6)}{(s+1)(s+2)(s+3)(s+4)(s+7)}$, plot the root locus.

   (d) Indicate: (i) the number of closed-loop poles; (ii) the number of asymptotes; (iii) The position of the closed-loop poles when $k = 10$. (In `sisotool` you may change k in `Edit Architecture` or by moving with the mouse one of the purple squares of the root locus. You can read a pole value at the bottom of the window when you click and hold the corresponding purple square of the root locus plot.)

3. In the following system $G(s) = k_p + k_d s$, $K_T = 1$, and $H(s) = \dfrac{2(s+10)}{(s+20)(s^2+s+1)}$.



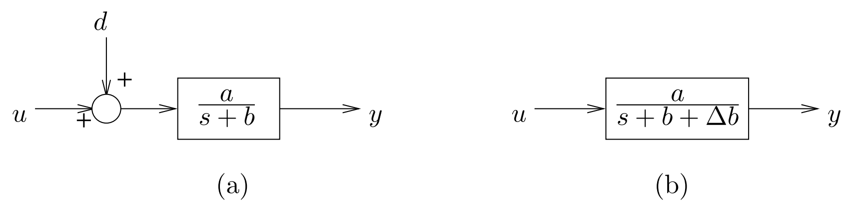
   (a) Using the Routh array determine the constraints that k_p and k_d must satisfy such that the closed-loop is stable.

   (b) Determine a second order approximation of $H(s)$.

   (c) Based on the second order approximation find k_p and k_d so that the step response has a 5 % overshoot and a settling time $t_s = 1$ sec.

(d) Simulate the closed-loop system using the precise value of $H(s)$ and compare the real overshoot and settling time to the intended values.

4. A rear-wheel drive vehicle is actuated by a DC motor. The gear ratio is $n = \frac{\omega}{\omega_w}$, where ω is the speed of the motor and ω_w the angular velocity of the wheels. The radius of the wheels is ρ and the mass of the vehicle is M. The motor satisfies $v = ri + k\omega$ and $\tau = ki$, where i is the electrical current, r is the armature resistance, v is the voltage, τ is the torque, and k is the torque constant. The traction force satisfies $f = M\dot{\nu} + B\nu + f_d$, where ν is the velocity of the vehicle and f_d is a disturbance term that is due to the condition of the road (such as the percent grade of an incline). Note that n, M, ρ, r, k, and B are given.



(a) By eliminating all unnecessary variables from equations, show that the speed ν is related to the voltage v and disturbance f_d by an equation of the form $\dot{\nu} + b\nu = av + cf_d$. Determine also a, b, and c.

(b) Show that the system can be modeled as in figure (a) for $u = v$ and $y = \nu$. Moreover, find d in terms of f_d.

(c) Assume that f_d can be measured using a sensor that produces a voltage $v_s = \gamma f_d$, where γ is a constant. Design a feed-forward control system that eliminates the effect of f_d on y. The inputs of the control system should be u and v_s. Hint: Refer to Figure 12.7(a). There is no feedback, so $H(s) = 0$. You only have to find $G_f(s)$ and $G_c(s)$.

(d) Referring to the block diagram (a), assume that $f_d = 0$ and b changes to $b + \Delta b$ (figure (b)). Calculate $D(s)$ in terms of $U(s)$, b, and Δb such that the model of figure (a) is equivalent to the model of figure (b).

Sample answers: (a) $a = \frac{nk}{Mr\rho}$; (b) $d = -\frac{r\rho}{nk} f_d$.

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Homework Set 10

Love worketh no ill to his neighbor: therefore love is the fulfilling of the law. Romans 13:10

Suggested Reading: The root locus, at pages 1044–1059. Bode diagrams, at pages 1065–1069. Nyquist diagrams, pages 1069–1080.

1. Consider the root locus of $F(s) = \dfrac{s(s+10)}{(s+1)(s+2)(s+5)(s+8)(s+15)}$.

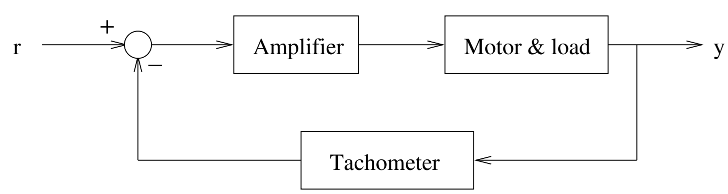
   (a) What is the number of branches?

   (b) What is the number of asymptotes?

   (c) Indicate the position and the angle of the asymptotes in the case $k > 0$.

   (d) Indicate the value $k > 0$ for which there is one closed-loop pole at $-2 + 6.28i$.

2. The following figure shows a basic speed-control system. Typically, the motor and the load can be modeled by a first order system $G(s) = \dfrac{a}{s+b}$. For simplicity, assume $G(s) = \dfrac{1}{s+1}$. Real amplifiers have a frequency dependent gain. Assume the amplifier model is $A(s) = \dfrac{k}{10^{-3}s+1}$. Tachometers could be modeled as constant gains. For simplicity, assume the tachometer has a transfer function $T(s) = 1$. Answer the following questions based on equations and/or the root locus.



   (a) What is the effect of k on the steady state error to a step input?

   (b) What is the value of k for a damping ratio $\zeta = 0.2$?

   (c) How is the step response affected as $k \to \infty$? (Is the error better or worse? Is the response more or less damped?)

   (d) Repeat part (c) when an additional pole is incorporated in the model of the amplifier: $A(s) = \dfrac{k}{(10^{-3}s+1)(10^{-5}s+1)}$.

3. For each root locus of Figure 1 indicate which of the following applies.

   - The system is stable for all $k > 0$.
   - The system is unstable for all $k > 0$.
   - The system is stable for some $k > 0$.

   If there is some $k > 0$ such that a system is stable, indicate also whether k should be large or small. (In other words, is the system stable as $k \to \infty$ or as $k \to 0$?)

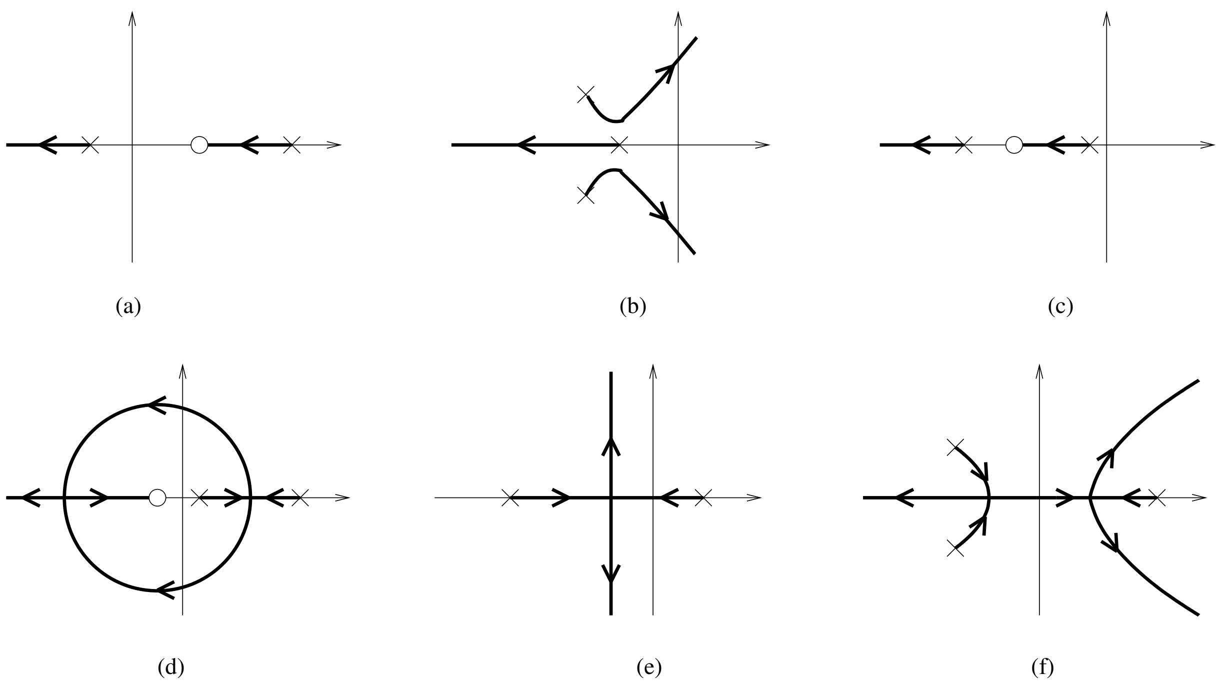


Figure 1: Root loci for Problem 3.

4. Sketch the root locus assuming a loop transfer function $kF(s)$, $k > 0$, and

$$\text{(a) } F(s) = \frac{s-3}{(s+2)(s+1)};\ \text{(b) } F(s) = \frac{1}{(s+1)(s+4)(s+5)};\ \text{(c) } F(s) = \frac{s+3}{(s+1)^2(s+4)}$$

Hint: *At part (c), sketch first the root locus for* $F(s) = \dfrac{s+3}{(s+1)(s+1+\varepsilon)(s+4)}$ *for some small* $\varepsilon > 0$, *and then let* $\varepsilon \to 0$.

5. (a) Verify your results at problem 4 by using MATLAB. You may use either `rlocus` or `sisotool` (the latter recommended). For credit, plot the root loci.

   (b) In MATLAB, you can click the root locus plots and drag the poles along the curves. The value of the gain k is displayed for each pole position. Use this feature to estimate the range of k for which the systems of problem 4 are stabilized. (Of course, it is possible to determine precisely the range of k using the Routh-Hurwitz criterion.)

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Homework Set 11

And ye shall seek me, and find me, when ye shall search for me with all your heart. Jeremiah 29:13

Suggested Reading: Operational amplifiers are discussed in section 4.3. Implementation issues are discussed in the handout "Troubleshooting Operational Amplifier Circuits", in Course Documents, on Blackboard.

1. Figure 1 shows the Bode diagrams of the loop transfer function of four systems.
   - (i) Indicate the cases (a)–(d) in which the closed-loop is unstable, if any.
   - (ii) Determine the phase margin and the gain margin in each case in which the closed-loop is stable.
   - (iii) If any cases of instability are found, estimate the gain of a proportional controller that should multiply the loop transfer function such that a phase margin of 60° is achieved.

2. Figure 2 shows the Nyquist diagrams of the loop transfer function of two systems. The system (a) has two poles in the RHP and the system (b) has no poles in the RHP.
   - (i) Indicate the cases in which the closed-loop is unstable, if any.
   - (ii) If any cases of instability are found, estimate the gain of a proportional controller that should multiply the loop transfer function in order to stabilize the closed-loop system. Hint: An appropriate gain will shrink/magnify the plot such as to change the number of encirclements of the point (-1,0). Determine the factor by which the graph should be shrunk/magnified.

3. The Bode plots of a system are shown for two frequency ranges in Figure 3. Sketch the Nyquist plot.

4. The data sheet of the amplifier OPA541 produced by Texas Instruments includes the Bode plots shown in Figure 4.
   - (a) Determine the phase margin in the case of a 3.3 nF load.
   - (b) Determine the phase margin in the case of a 2 kΩ load.
   - (c) If the loop gain is multiplied by $k = 0.01$, what will be the phase margin?
   - (d) Assuming the curve for the 2 kΩ load, let $H(s)$ be the transfer function of the amplifier. If two identical amplifiers are cascaded so that the loop transfer function is changed from H to $H \cdot H = H^2$, what will be the phase margin? Will the system be stable (when used with unity feedback)?

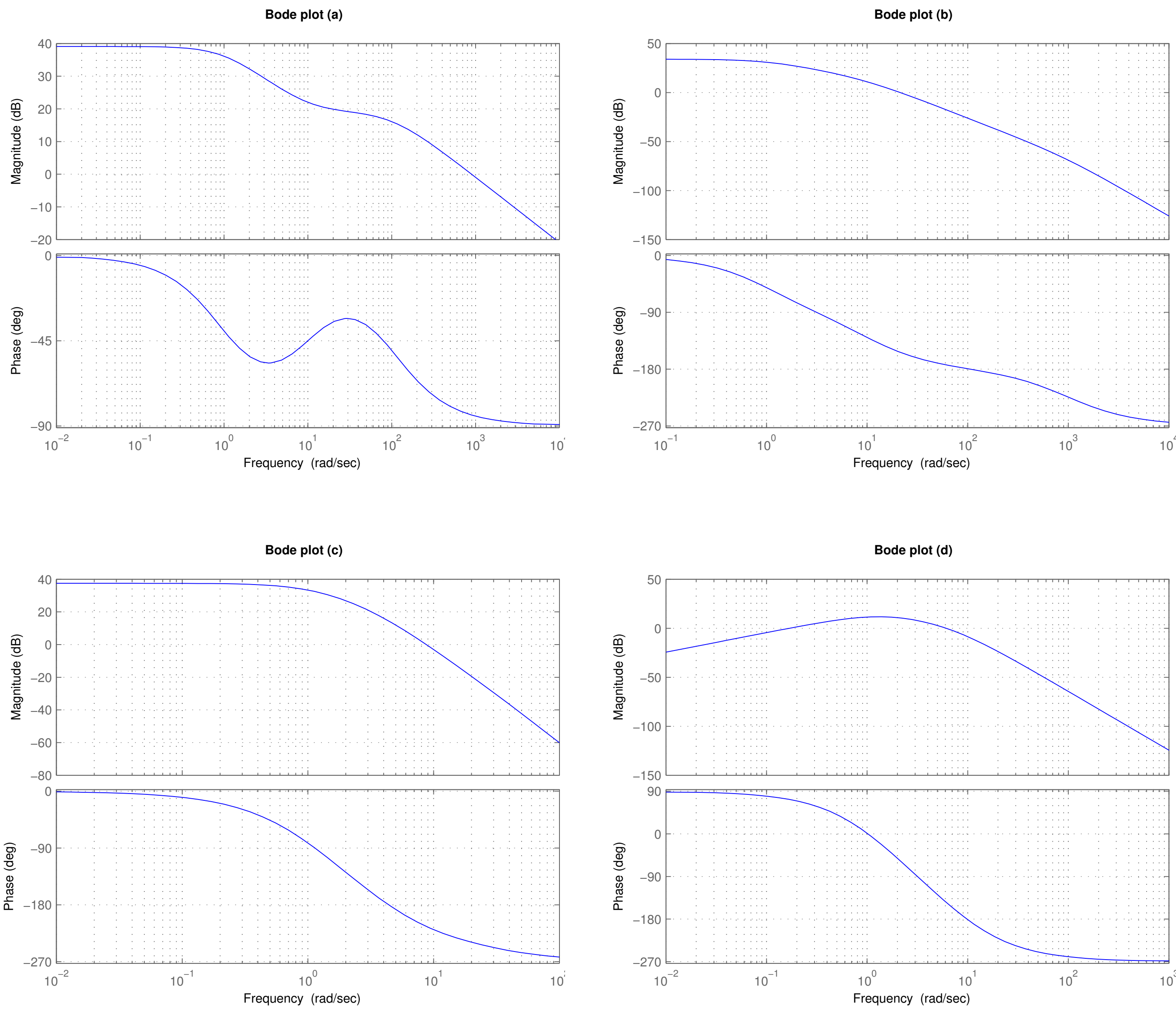

Figure 1: Bode diagrans for problem 1.

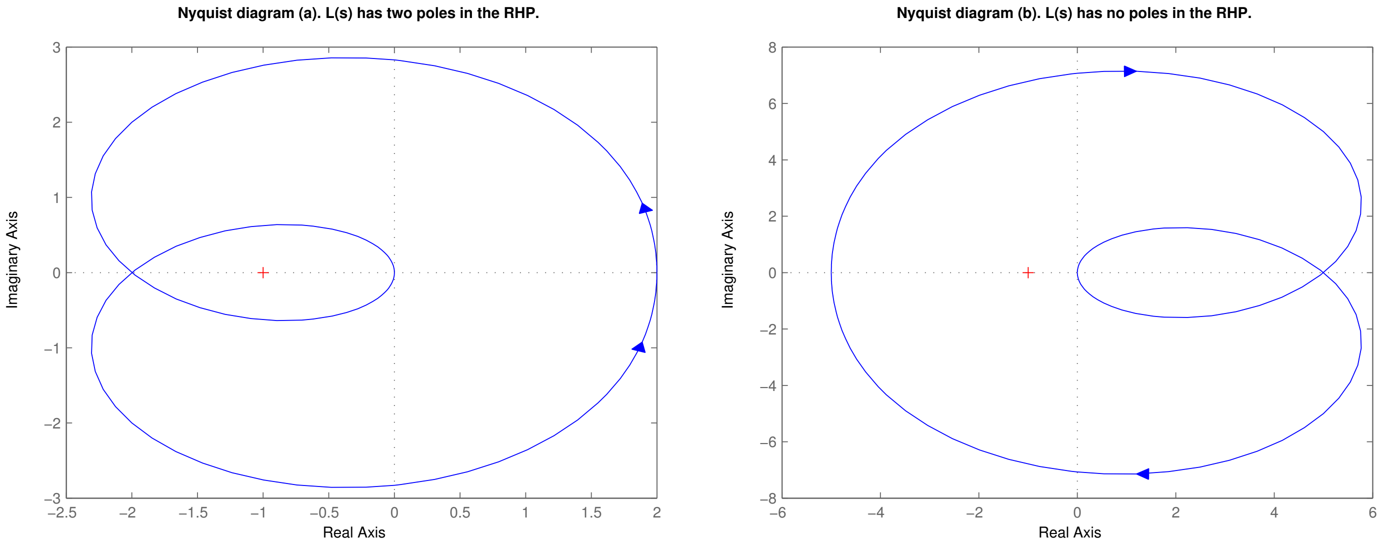

Figure 2: Nyquist diagrans for problem 2.

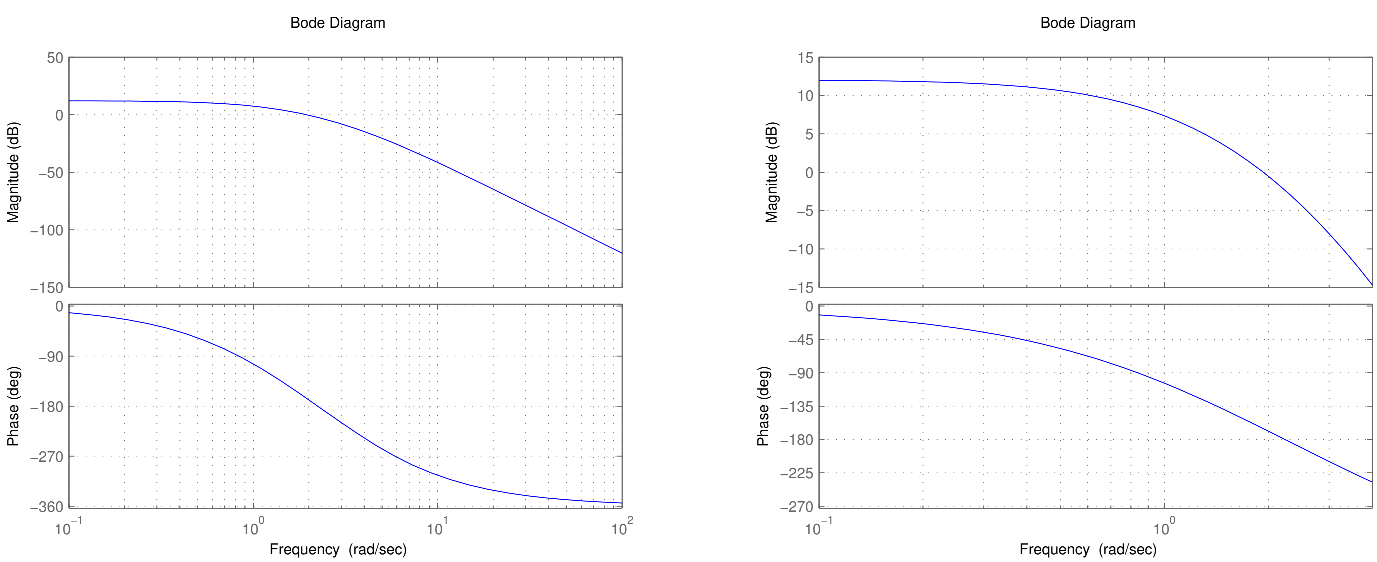

Figure 3: Bode plots for problem 3.

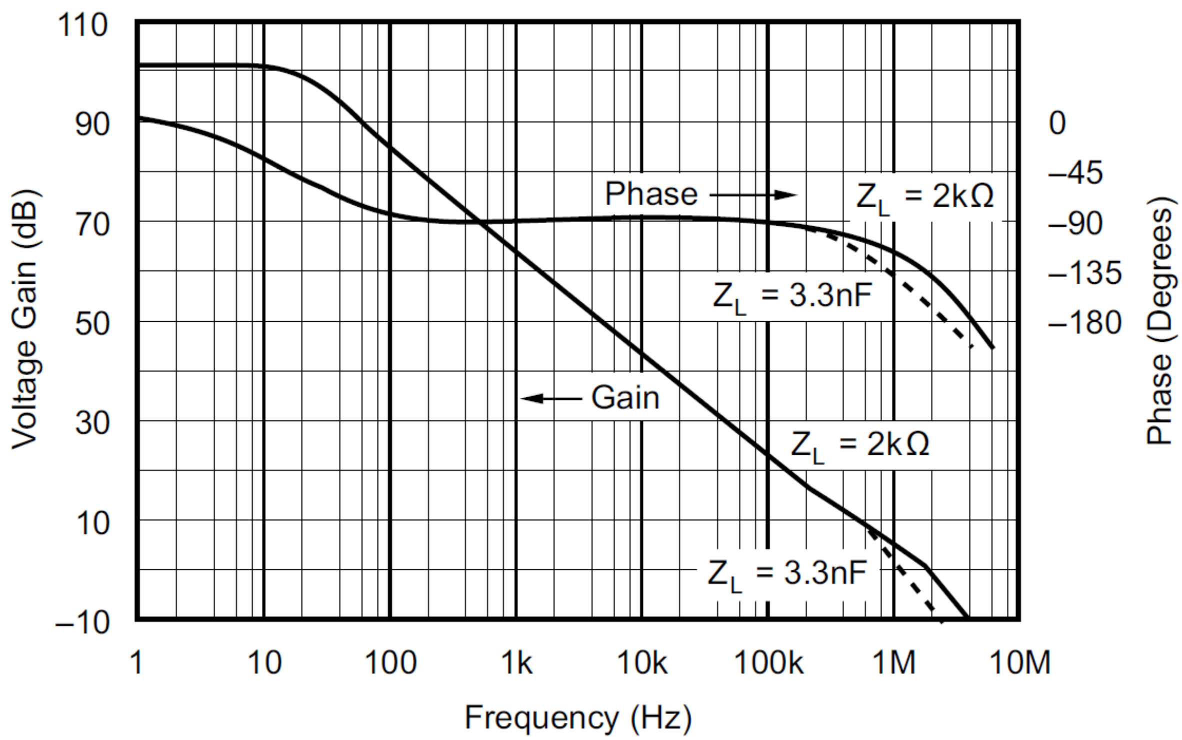

Figure 4: Open-loop gain and phase plots of OPA541 (Problem 4) for a 3.3 nF load and for a 2 kΩ load. The curves of the two loads overlap at low frequency, but separate at high frequency. The dashed line curve is for the 3.3 nF load. Read the gain values using the vertical axis marked in decibels at the left of the graph, and the phase values using the vertical axis marked in degrees at the right of the graph.

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Homework Set 12

I will delight myself in thy statutes: I will not forget thy word. Psalm 119:16

Suggested Reading: Additional detail to the material presented in class can be found in chapters 4 (DC Machines) and 5 (Polyphase Induction Motors) of the second textbook by Nasar. Note that the unsolved problems of the textbook are similar to the solved problems that precede them.

1. For all problems below, use resistors in the range $100\,\Omega \ldots 1\,\text{M}\Omega$ and capacitors in the range $100\,\text{pF} \ldots 1\,\mu\text{F}$. Show your work.
   (a) Draw a circuit that implements $v_o = v_1 - v_2 - v_3 - v_4$, where $v_1, \ldots v_4$ are four input voltages.
   (b) Draw a circuit that implements $v_o = v_1 + 2v_2 - v_3 - 0.5v_4$, where $v_1, \ldots v_4$ are four input voltages.
   (c) Draw a circuit that implements the transfer function $H(s) = 30 + \frac{0.01}{s}$.
   (d) Draw a circuit that implements the transfer function $H(s) = 10\frac{s+100}{s+1000}$.
   (e) Draw a circuit that implements $H(s) = 1000\frac{s}{s+100}$.
2. The following data is known about two permanent magnet (PM) motors. In each case, estimate k_m and r (the armature resistance).
   (a) (Motor 1) 380 mA at 9 V and 11000 rpm, 400 mA at 12 V and 15200 rpm, and 430 mA at 18 V and 24000 rpm.
   (b) (Motor 2) 35 mA at 12 V and 7500 rpm, and 350 mA at 12 V when the motor is stalled.
3. Consider a PM DC motor of resistance r and torque constant k_m. The supply voltage v is given. Calculate the no-load speed ω_n (the speed when $\tau = 0$) and the stall torque τ_s (the torque when $\omega = 0$). Then, draw the graph of τ versus ω.
4. Solve problem 4.41 from the textbook by S. Nasar.
5. Determine the torque required to turn the shaft of a PM DC generator at speed ω, when its terminals are shortcircuited ($v = 0$). The torque constant k_m and the armature resistance r are given.
6. Solve problem 4.65 from the textbook by S. Nasar. Moreover, assuming a field resistance $R_f = 50\,\Omega$, a field voltage $V_f = 110\,\text{V}$, and an armature resistance $R_a = 0.2\,\Omega$, determine the efficiency of the motor for an armature current of 35 A.

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Homework Set 13

If ye abide in me, and my words abide in you, ye shall ask what ye will, and it shall be done unto you. John 15:7

Suggested Reading: Additional detail to the material presented in class can be found in chapters 4 (DC Machines) and 5 (Polyphase Induction Motors) of the second textbook by Nasar. Note that the unsolved problems of the textbook are similar to the solved problems that precede them.

1. Solve problem 4.47 from the textbook by S. Nasar. Assume core and mechanical losses stay constant.

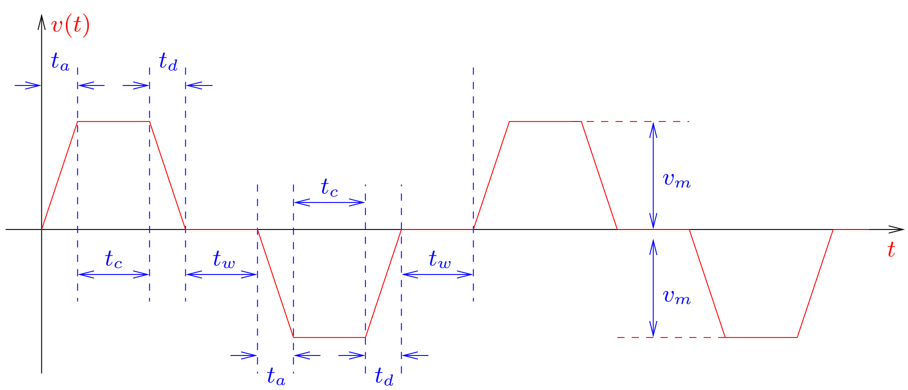
2. Solve problem 4.54 from the textbook by S. Nasar.

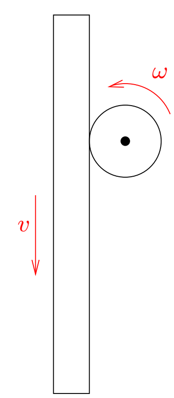
3. Solve problem 4.62 from the textbook by S. Nasar.

4. Solve problem 4.63 from the textbook by S. Nasar.

5. Consider a rack and pinion mechanism in which the pinion is connected to a motor by means of a 10 : 1 gear transmission. Assume that the velocity v of the rack has the profile shown in the figure with the following parameters: $v_m = 1\,\text{m/s}$, $t_a = t_d = 0.1\,\text{s}$, $t_c = 0.3\,\text{s}$, and $t_w = 0.3\,\text{s}$. The mass of the rack is $m = 3\,\text{kg}$, the pinion radius is $\rho = 0.05\,\text{m}$, and the inertia of the motor is $J_m = 8 \cdot 10^{-4}\,\text{kgm}^2$. Use $g = 9.81\,\text{m/s}^2$ for the gravity. Find (a) the load inertia reflected to the motor (the load inertia as seen by the motor); (b) the peak torque; (c) the rms torque; (d) Is the motor inertia appropriate for this system?

   Sample answers: $T_{rms} = 0.89\,\text{Nm}$.





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Homework Set 14

O LORD, how manifold are thy works! in wisdom hast thou made them all: the earth is full of thy riches. Psalm 104:24

Suggested Reading: Additional detail to the material presented in class can be found in chapters 5 (Polyphase Induction Motors), 6 (Synchronous Machines), 7 (Single-Phase Motors) of the textbook by Nasar and in chapter 8 (Stepper Motors) of the main textbook.

1. Solve problem 5.37 from the textbook by S. Nasar.
2. Solve problem 5.38 from the textbook by S. Nasar.
3. Solve problem 5.47 from the textbook by S. Nasar.
4. Refer to the main textbook by C. de Silva (not the textbook by S. Nasar). Solve problem 8.1.
5. Refer to the main textbook by C. de Silva (not the textbook by S. Nasar). Solve problem 8.9.