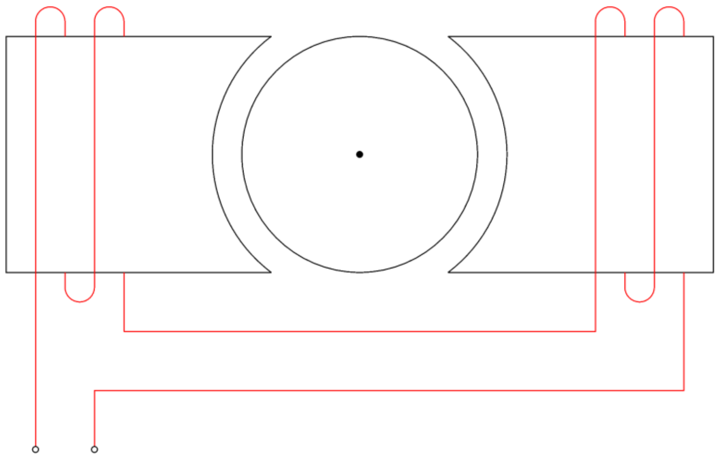
Single-Phase Induction Motors

M.V. Iordache, *EEGR3523 Mechatronics*, Spring 2020, LeTourneau University

Introduction

- AC motors rely on a rotating magnetic field.
  - The rotor of a synchronous motor follows the field at the same speed.
  - The rotor of an induction motor rotates because of the currents induced in it by the rotating field.
- In a single phase motor, however, the magnetic field of the stator does not rotate.
- For example, consider a single-phase motor with two stator poles.
- Depending on the direction of the current, the direction of the field is left-to-right or right-to-left.
- The field pulsates, but does not rotate!
- Additional stator poles connected to the same phase will not make the field rotate.



The Stator Field

- Consider a single-phase stator in which all poles are connected to the same phase.
- Let p be the number of poles.
- Let $\theta_0 = \frac{2\pi}{p}$ be the pole pitch angle (the angle between two adjacent poles).
- Consider the flux density B of the stator field inside of the rotor, near the surface of the rotor.
- The flux density is sinusoidal, because the stator current is sinusoidal.
- The coils of the stator poles are connected so that they produce fields in opposite directions.
- So if $B = B_m \cos(\omega t)$ near a stator pole, then $B = -B_m \cos(\omega t)$ near an adjacent stator pole.

The Stator Field

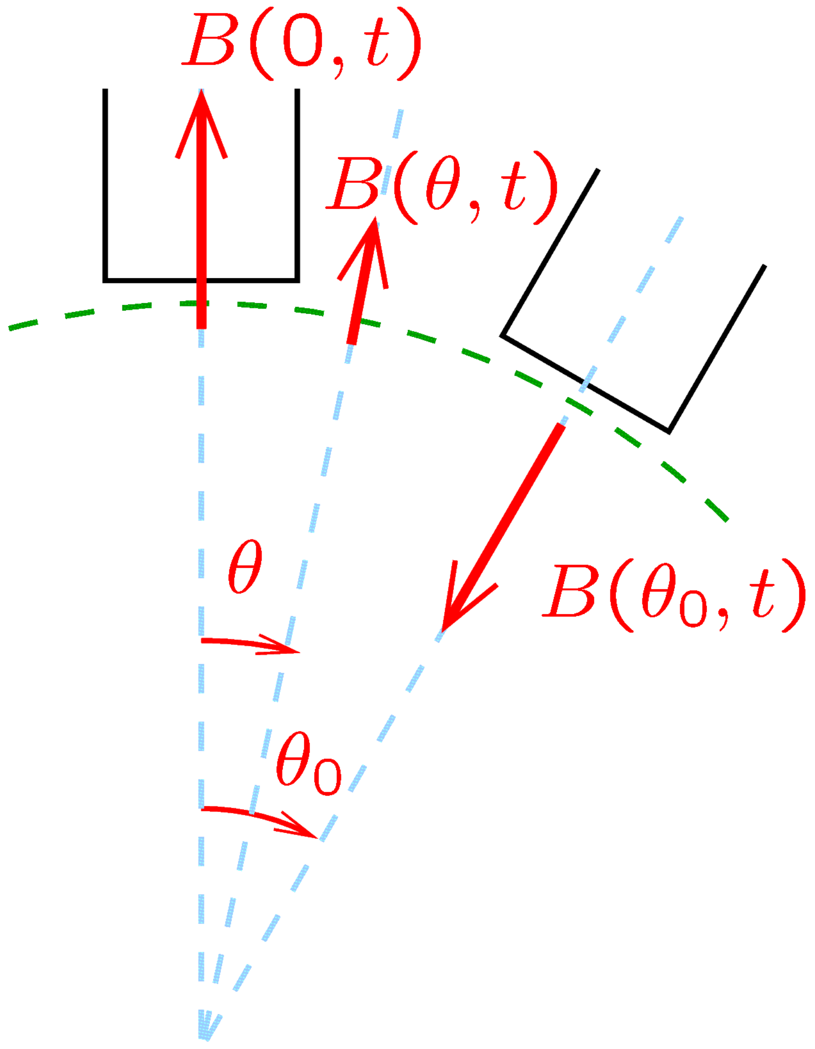
- The field at an angle θ could be approximated by

$$B(\theta, t) = B_m \cos\left(\frac{p\theta}{2}\right)\cos(\omega t)$$

- Let $k = p/2$.
- Note that

$$B(\theta, t) = \frac{B_m}{2}\cos(\omega t - k\theta) + \frac{B_m}{2}\cos(\omega t + k\theta)$$

- The first term, $\frac{B_m}{2}\cos(\omega t - k\theta)$, describes a field that rotates in the direction of θ (clockwise in the figure).
- The second term, $\frac{B_m}{2}\cos(\omega t + k\theta)$, describes a field rotating in the opposite direction (counterclockwise in the figure).



The Stator Field

- We conclude that the stator field of a single phase motor consists of two rotating fields rotating in opposite directions.
- The torques generated by the rotating fields are equal and in opposite directions.
- Unless we ensure that one of the rotating fields is stronger, the motor will not start.
- Once the rotor rotates, the torque produced by the field rotating in the same direction will be stronger, and the motor will run.

Single-Phase Induction Motors

- Single-phase induction motors, in order to be self-starting, create an additional phase.
- Split-phase motors have:
  - A main winding, that is always powered.
  - A starting winding, that is connected only when the motor starts.
- The starting winding is disconnected by a centrifugal switch when the rotor reaches a sufficiently high speed.
- The starting winding has a higher resistance and a different reactance/resistance ratio.
- In this way, the current of the starting winding is out of phase with the current of the main winding.
- This creates a second phase and in this way a predominant rotating field.

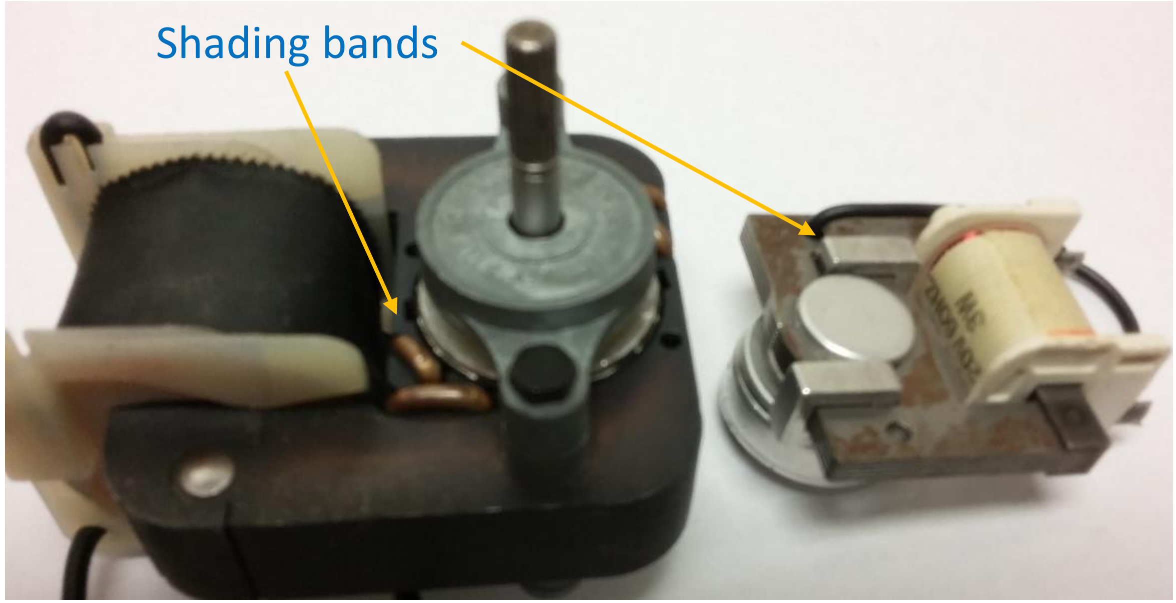
Single-Phase Induction Motors

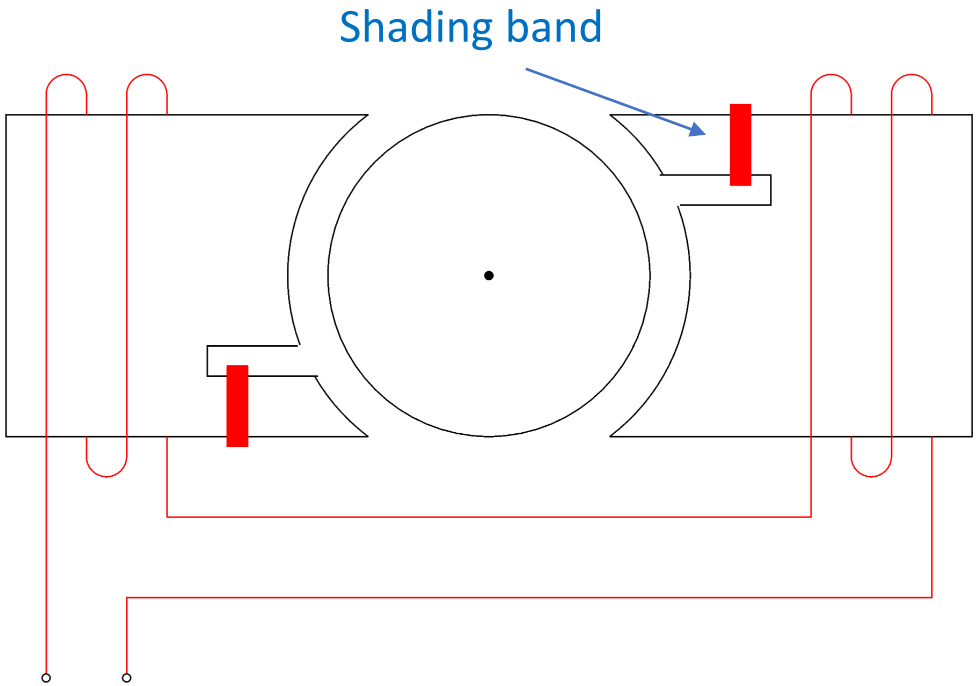
- Capacitor-start motors have:
  - A main winding, that is always powered.
  - A starting winding, connected in series with a capacitor.
- The capacitor causes the current of the starting winding to be out of phase with the current of the main winding.
- This creates a second phase and in this way a predominant rotating field.
- Some motors disconnect the starting winding after the rotor has reached a predetermined speed.

Spring-loaded parts that are shifted off-center by the centrifugal force. They can turn off a switch at high enough speed.

Single-Phase Induction Motors

- Shaded-pole motors have:
  - A short-circuited conductive strap (called shading band) wound on a portion of each pole.
  - This creates a lag, and in this way a predominant rotating field that turns from the main face of the pole towards the shaded portion of the pole.

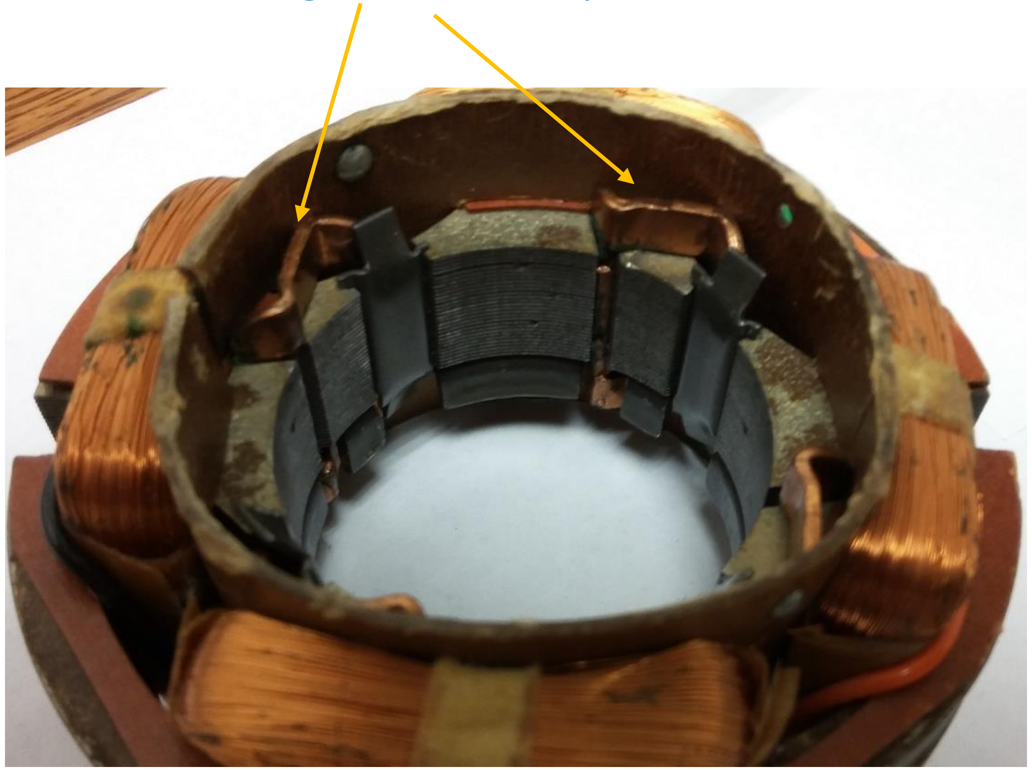




Single-Phase Induction Motors

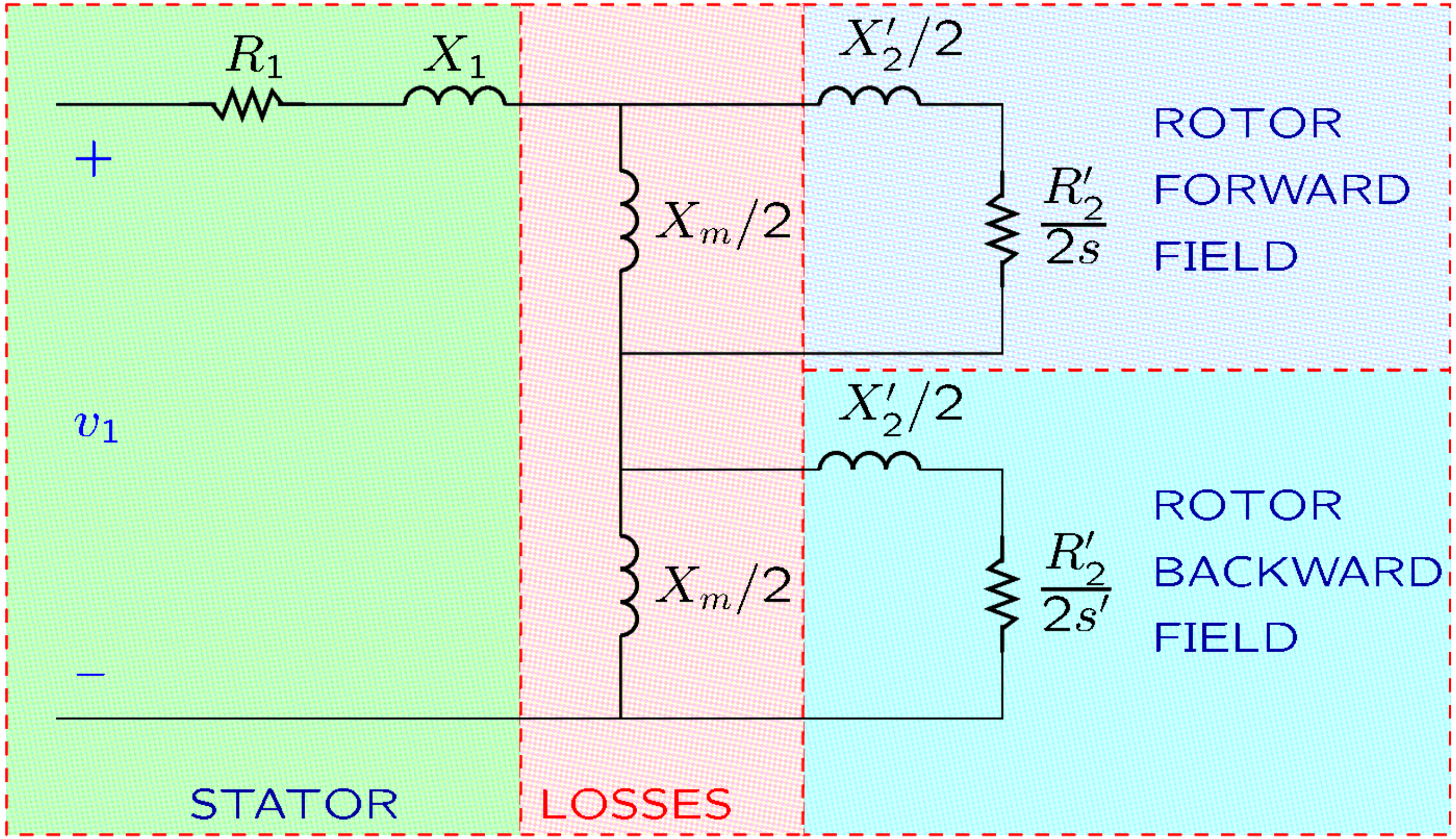
- Single-phase induction motors are typically for relatively low power applications.
- The rotor is typically of the squirrel-cage type.

Shading bands in a 4-pole stator.



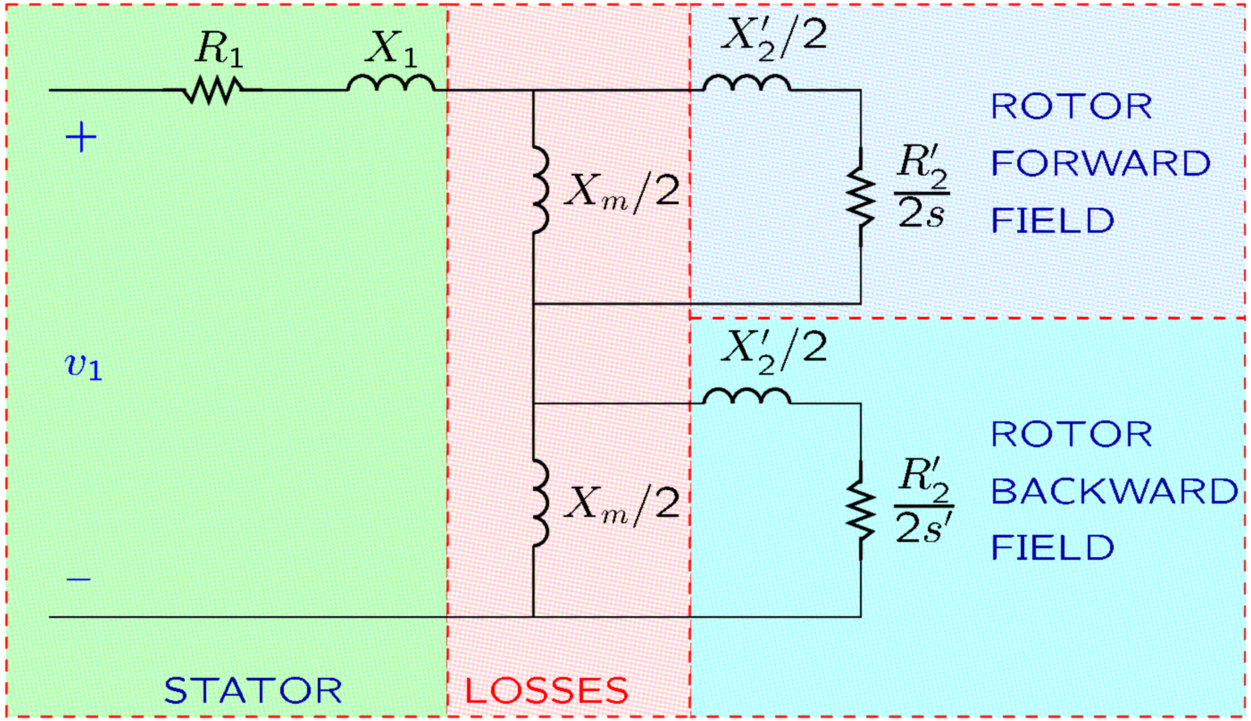
Electric Model of Single-Phase Induction Motors

- The model resembles the per-phase model of polyphase induction motors.
- There are differences because single phase motors have two rotating fields.



Electric Model of Single-Phase Induction Motors

- Let n be the speed of the motor.
- The slip with respect to the forward field is $s = \frac{n_s - n}{n_s}$.
- The slip with respect to the backward field is $s' = \frac{-n_s - n}{-n_s} = 2 - s$.
- R_1 and X_1 are the stator resistance and reactance, respectively.
- R_2' and X_2' correspond to the resistance and reactance, respectively, of the rotor.
- X_m models that the magnetic coupling of the rotor and stator is imperfect.

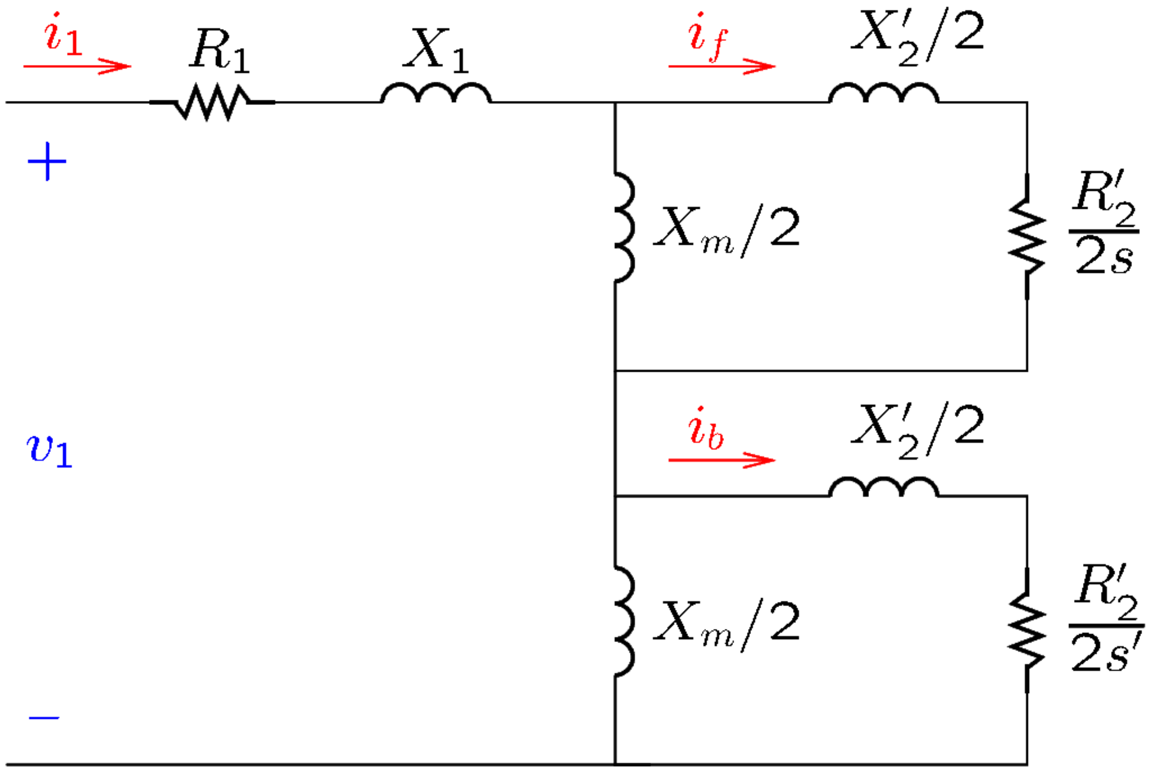


Electric Model of Single-Phase Induction Motors

- Let I_f, I_b, I_1, and V_1 be the rms values of i_f, i_b, i_1, and v_1.
- The input average power is $P_1 = V_1 I_1 \cos(\alpha_v - \alpha_i)$, where α_v and α_i denote the phase angles of v_1 and i_1, respectively.
- Recalling that reactances do not dissipate power,

$$P_1 = I_1^2 R_1 + I_f^2 R_2'/2s + I_b^2 R_2'/2s'$$

- $I_1^2 R_1$ is the stator copper loss.
- $P_g = I_f^2 R_2'/2s + I_b^2 R_2'/2s'$ is the rotor power.
- $I_f^2 R_2' + I_b^2 R_2'$ is the rotor copper loss.
- $P_d = P_g - (I_f^2 R_2' + I_b^2 R_2')$ is the developed power.
- Thus, the torque satisfies $T\omega = P_d$.



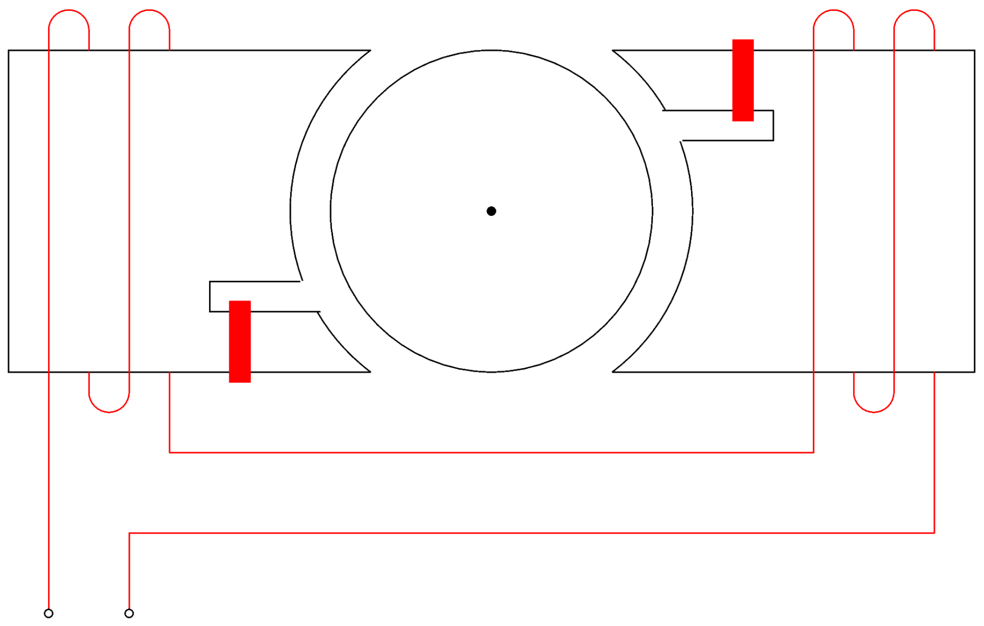
Example 1

The motor is powered at $120\ V$ rms and $60\ Hz$. What is the maximum speed of the motor? Can the direction of rotation be reversed?

- *The synchronous speed is*

$$n_s = \frac{120f}{p} = \frac{120 \cdot 60}{2} = 3600\ rpm$$

- *The motor speed is* $n = n_s(1 - s) < n_s$.
- *The maximum speed is* $n_s = 3600\ rpm$.
- *Practical motors cannot reach the synchronous speed because the torque is zero at the synchronous speed and the motor has to overcome friction.*
- *This motor turns counterclockwise and its direction cannot be reversed.*



Example 2

A single phase induction motor has 4 poles, is powered at $V_1 = 120\ V$ *rms and* $60\ Hz$*, has* $R_1 = 2\ \Omega$*,* $R_2' = 1\ \Omega$*,* $X_1 = 2\ \Omega$*,* $X_2' = 2\ \Omega$*, and* $X_m = 200\ \Omega$*. Find the torque and the efficiency when the motor has a slip of* 5%*.*

- *The slip of the rotor with respect to the backward field is* $s' = 2 - s = 1.95$.
- *Let*

$$\boldsymbol{Z_f} = \left(\frac{jX_m}{2}\right) || \left(\frac{jX_2'}{2} + \frac{R_2'}{2s}\right) = 9.9\angle 11.37^\circ\ \Omega$$

$$\boldsymbol{Z_b} = \left(\frac{jX_m}{2}\right) || \left(\frac{jX_2'}{2} + \frac{R_2'}{2s'}\right) = 1.02\angle 75.76^\circ\ \Omega$$

- *The total impedance of the motor is*

$$\boldsymbol{Z} = R_1 + jX_1 + \boldsymbol{Z_e} + \boldsymbol{Z_f} = 12.94\angle 22.45^\circ\ \Omega$$

- *The input current is* $I_1 = \frac{V_1}{|\boldsymbol{Z}|} = 9.27\ A\ rms.$

Example 2

- *The total rotor power is*

$$P_g = I_1^2 \cdot Real\left(\boldsymbol{Z_f}\right) + I_1^2 \cdot Real(\boldsymbol{Z_b})$$

- *The developed power is*

$$P_d = P_g - I_1^2 \cdot Real\left(\boldsymbol{Z_f}\right) \cdot s - I_1^2 \cdot Real(\boldsymbol{Z_b}) \cdot s'$$

$$\Rightarrow P_d = I_1^2 (1 - s) \left(Real\left(\boldsymbol{Z_f}\right) - Real(\boldsymbol{Z_b})\right)$$

- *The torque is*

$$T = \frac{P_d}{\omega} = \frac{P_d}{\omega_s (1 - s)}$$

where $\omega_s = \frac{2\pi}{60} \cdot \frac{120f}{p} = 188.5\ rad/s$.

- *Thus,*

$$T = \frac{I_1^2 \left(Real\left(\boldsymbol{Z_f}\right) - Real(\boldsymbol{Z_b})\right)}{\omega_s} = 4.31\ Nm$$

Example 2

- *The efficiency is*

$$\eta = \frac{P_d}{P_1} = \frac{I_1^2(1-s)\left(Real\left(\boldsymbol{Z_f}\right) - Real(\boldsymbol{Z_b})\right)}{I_1^2 Real(\boldsymbol{Z})}$$

$$\eta = \frac{(1-s)\left(Real\left(\boldsymbol{Z_f}\right) - Real(\boldsymbol{Z_b})\right)}{Real(\boldsymbol{Z})} = 75.12\%$$