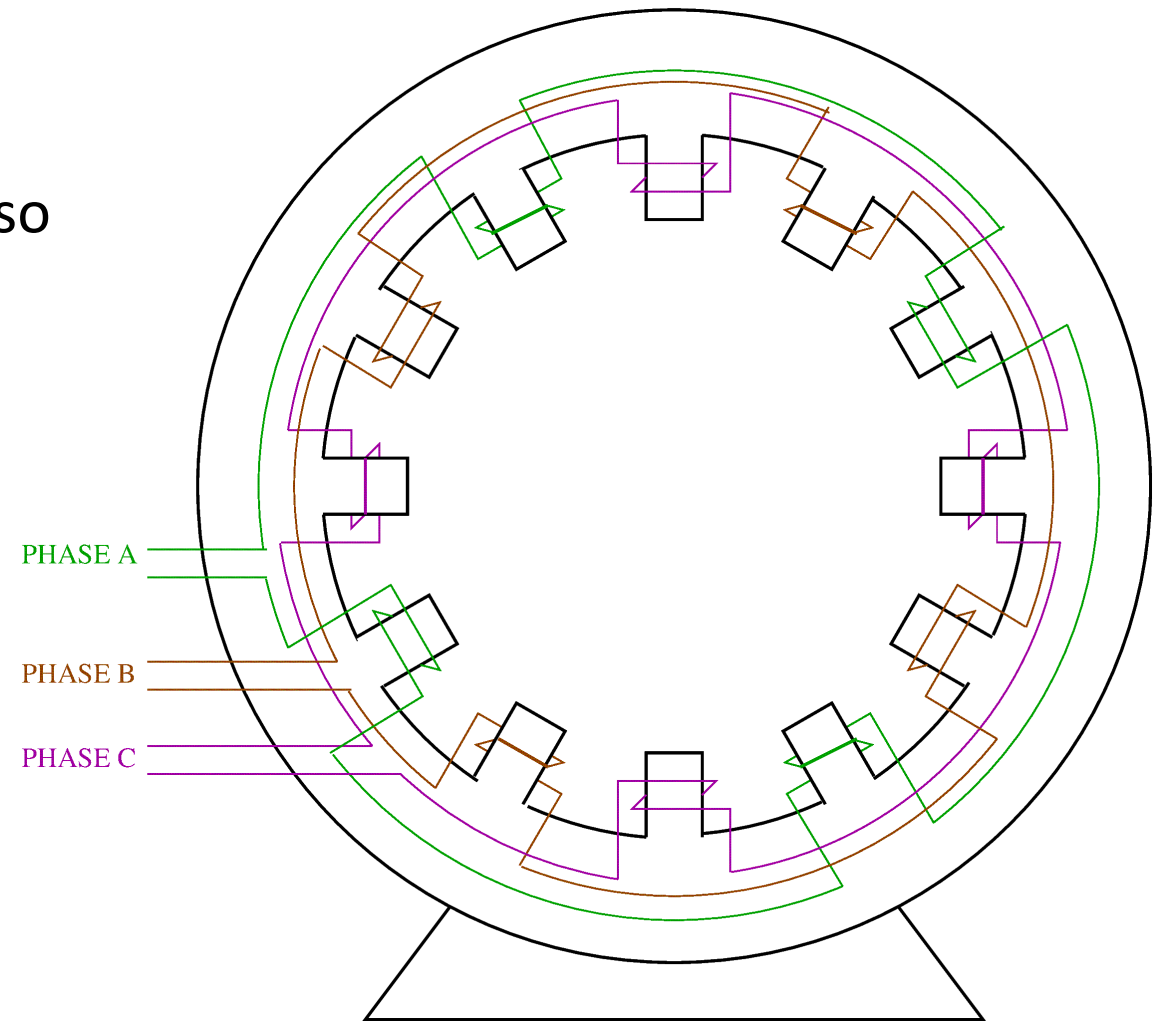


AC Machines

Principle. Synchronous Motors. Induction Motors.

Principle of Operation

- AC motors use a *rotating magnetic field*.
- A rotating magnetic field is obtained when the distribution of the magnetic field inside the motor changes in time, so as to give the appearance of rotation.
- To illustrate the rotating field, consider the figure showing the stator of an AC machine.
- The stator has 12 poles connected to three phases.
- There are 4 poles per phase.
- For this machine, the stator is the armature.



Principle of Operation

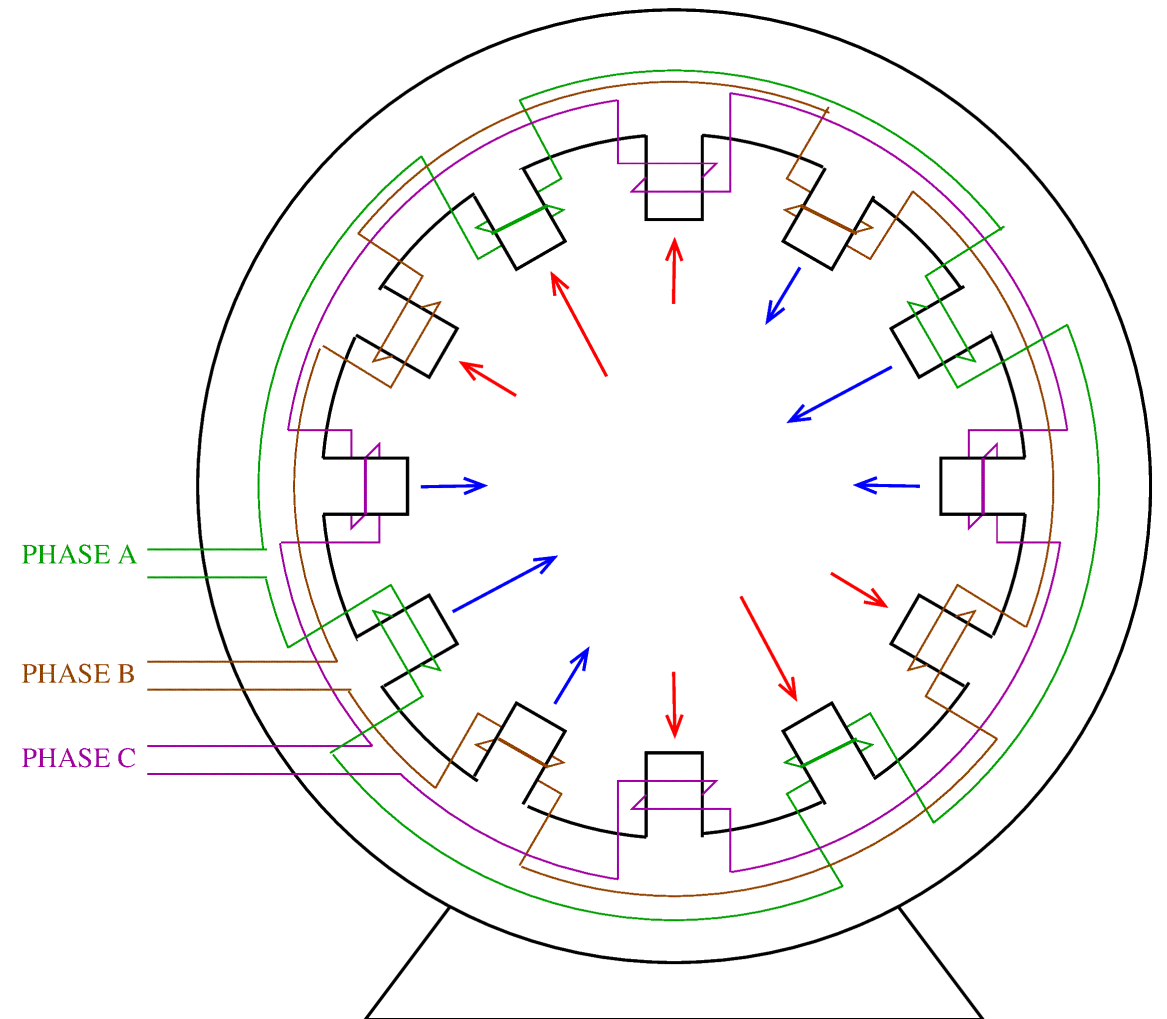
- Assume that the stator is connected to a three-phase source of voltage:

$$v_A = V_m \cos(\omega t)$$

$$v_B = V_m \cos(\omega t - 120^\circ)$$

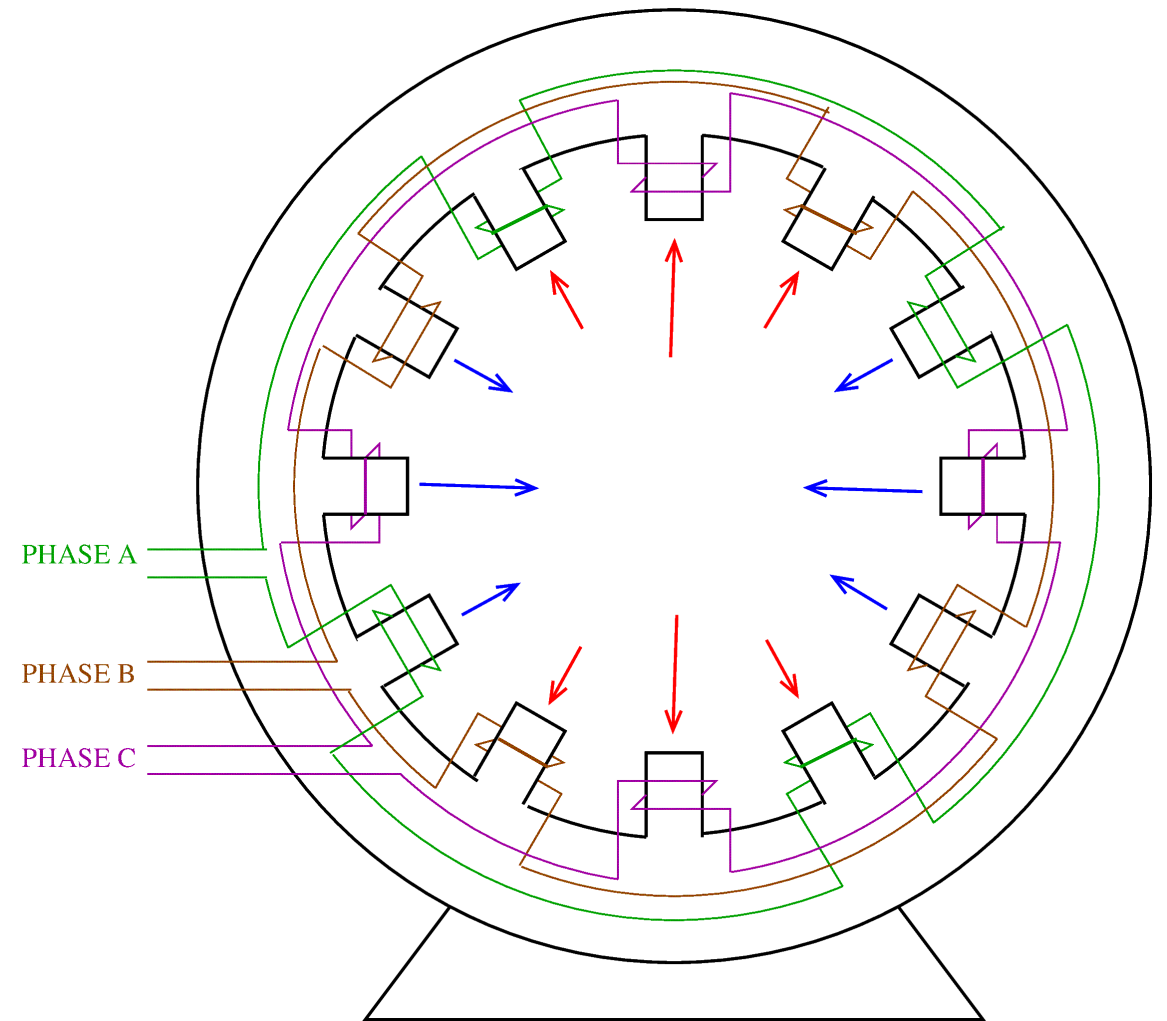
$$v_C = V_m \cos(\omega t + 120^\circ)$$

- Suppose that at time t_0 the coils connected to the phase A have maximum current.
- The direction of the field near each pole is shown in the figure.
- A long arrow indicates a large value of the flux density, and a short arrow a smaller value.



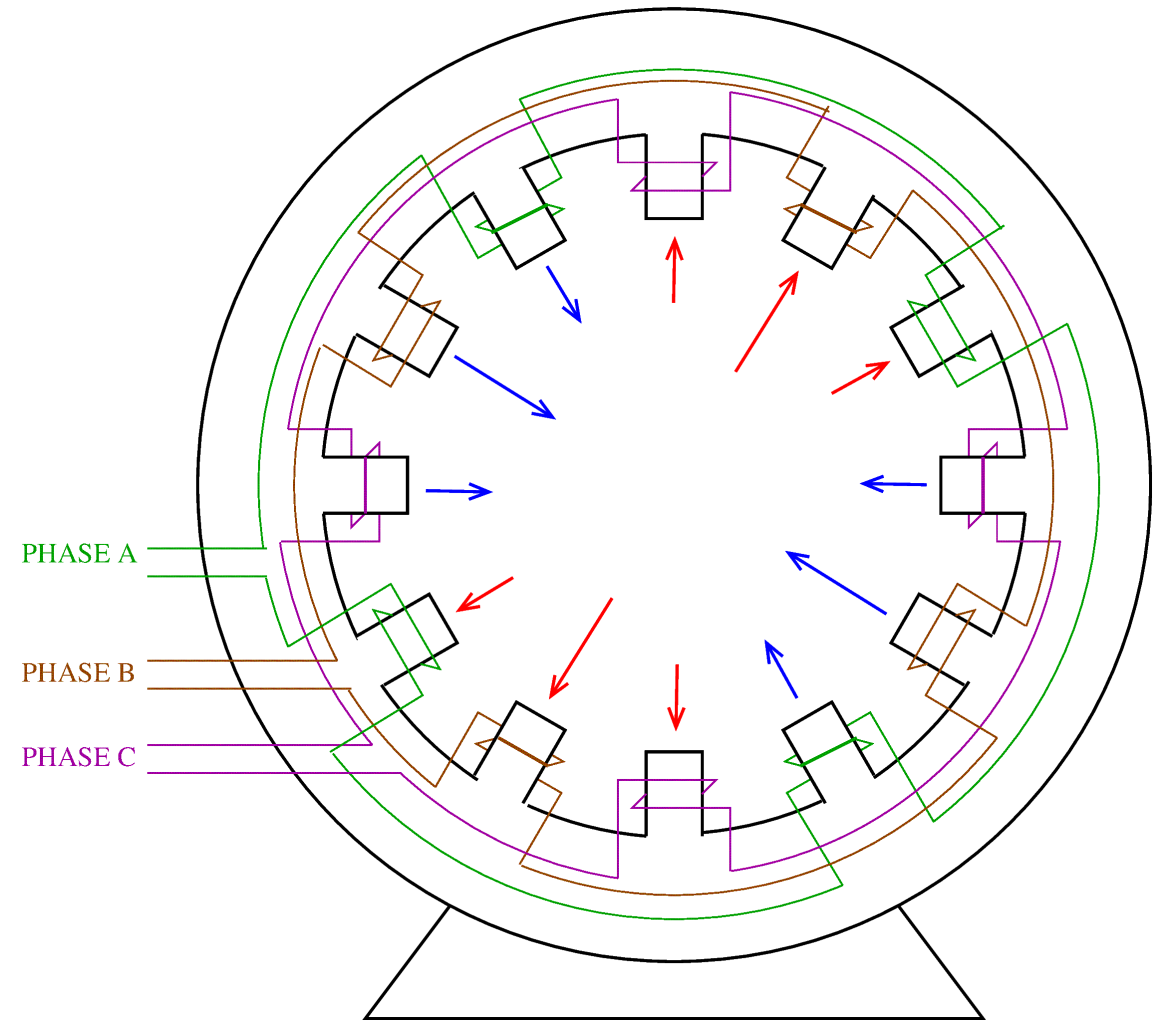
Principle of Operation

- Since phase C leads phase A by 120° , after all phases change by 60° , that is after $\Delta t = \frac{\pi}{3\omega}$, the phase C will have maximum current (but in the reverse direction).
- The figure shows the direction of the field at time $t_0 + \Delta t$.



Principle of Operation

- Since phase B leads phase C by 120° , after all phases change by 60° , that is after $\Delta t = \frac{\pi}{3\omega}$, the phase B will have maximum current (but in the reverse direction).
- The figure shows the direction of the field at time $t_0 + 2\Delta t$.
- Note that the magnetic field of the stator rotates clockwise!



Principle of Operation

AC motors use the rotating field of the stator in one of two ways.

- In *synchronous motors*, the rotor poles have a constant magnetic polarity. The rotor aligns itself with the rotating field and turns at the same speed.
- In *induction motors*, the rotating magnetic field induces currents in the rotor. The interaction of the induced currents with the rotating field creates torque.
- Note that the windings of the stator poles are usually overlapping (see Fig. 5-3 in the textbook).
 - This makes the stator magnetic field more uniform.

The Rotating Magnetic Field

- Let p be the number of stator poles per phase and f the frequency of the source.
- The pole pitch angle (the angle between two poles of the same phase) is

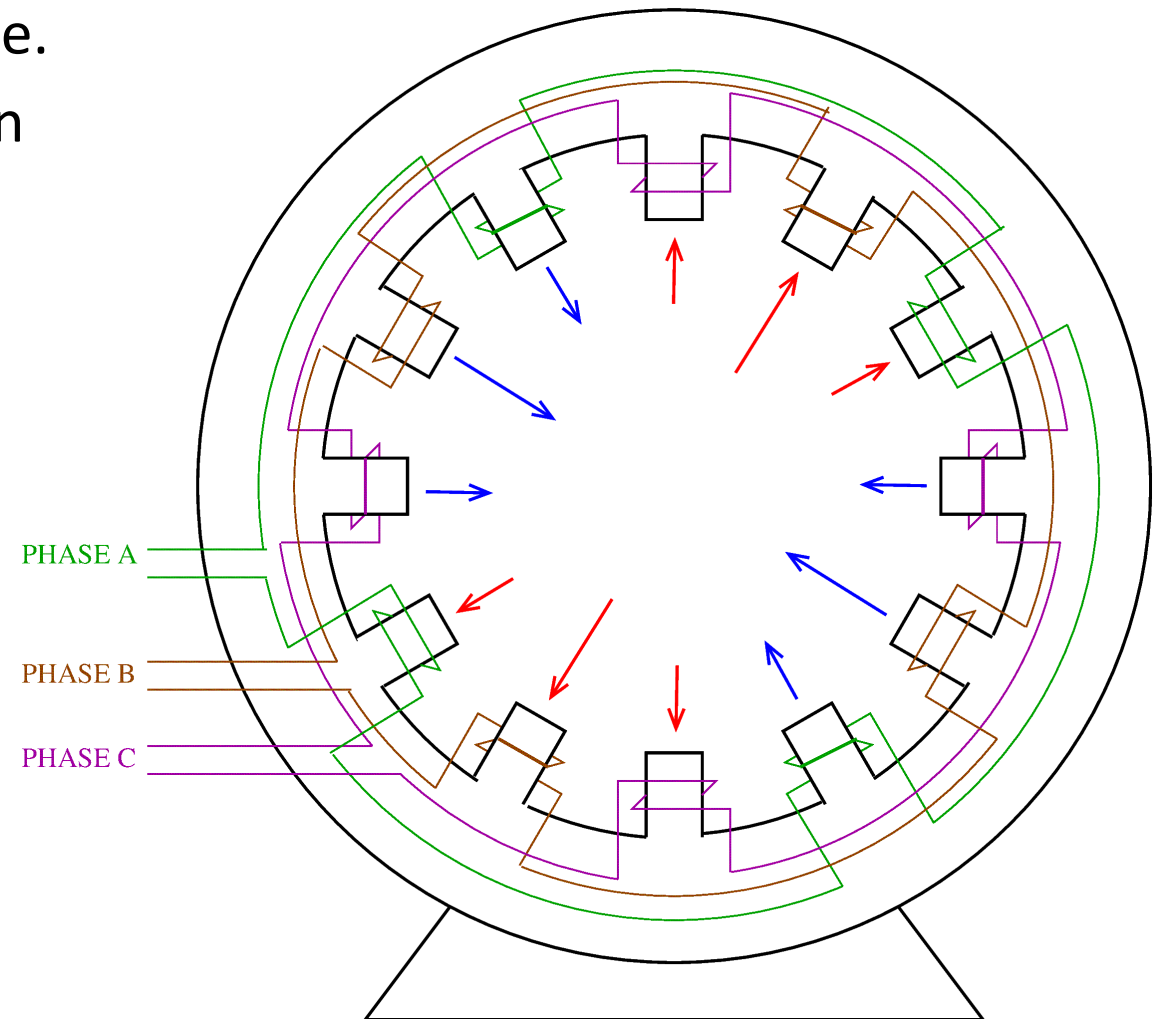
$$\theta = \frac{2\pi}{p}$$

- During half of a period, the rotating magnetic rotates by the angle θ .
- Therefore, the speed of the field is

$$\omega_s = \frac{\theta}{T/2} = \frac{4\pi}{Tp} = \frac{2\omega}{p} \text{ [rad/s]}$$

- In revolutions per minute:

$$n_s = \frac{120f}{p} \text{ [rpm]}$$



The Synchronous Speed

- The speed of the rotating field of the stator is called *synchronous speed*:

$$n_s = \frac{120f}{p} \text{ [rpm]}$$

- The speed of a synchronous motor equals the synchronous speed, since the rotor has constant magnetic polarity and aligns itself to the rotating field.
- The speed of an induction motor is less than the synchronous speed.
- If the induction motor would run at the synchronous speed, the rotating field would not induce any currents in the rotor, and therefore the motor would have no torque.
- Let n be the speed of an induction motor. The *slip* is defined as

$$s = \frac{n_s - n}{n_s}$$

Examples

A synchronous motor has 4 poles per phase and is supplied at 230 V rms and 60 Hz. Find the speed of the motor.

$$n = \frac{120f}{p} = 1800 \text{ rpm.}$$

An induction motor has 6 poles per phase and is supplied at 230 V rms and 60 Hz. What is the motor speed at a slip of 3%?

$$n = \frac{120f}{p} (1 - s) = 1164 \text{ rpm.}$$

Induction Motors

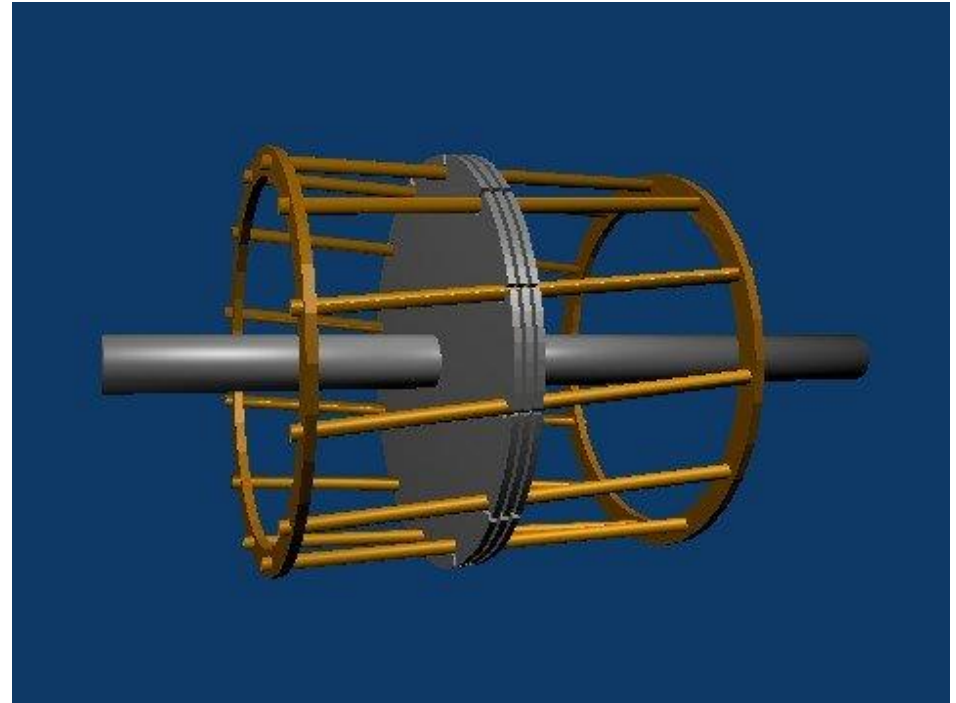
- In an induction motor, the rotor coils are shorted.
- Thus, there are closed current paths through the rotor and the rotating magnetic field of the stator will induce currents in the rotor.
- The interaction of the rotor currents and the stator field creates torque.
- The rotor may be:
 - A *squirrel cage rotor*.
 - A *wound rotor*.

Induction Motors—Wound Rotor

- The wound-rotor motor, also known as *slip-ring motor*, connects the rotor windings to external variable resistors.
- The resistors are connected to the rotor by means of brushes touching slip rings attached to the rotor.
- The resistors can be adjusted to control the induced currents, and in this way torque and speed.

Induction Motors—Squirrel Cage

- Squirrel cage motors are brushless.
- They are very common.
- A copper structure resembling a cage is enclosed in the rotor core.
- Since the copper bars are shorted by the end rings, currents can flow freely through the bars.
- The stator rotating field induces currents in the bars, and the interaction of the currents with the field creates torque.

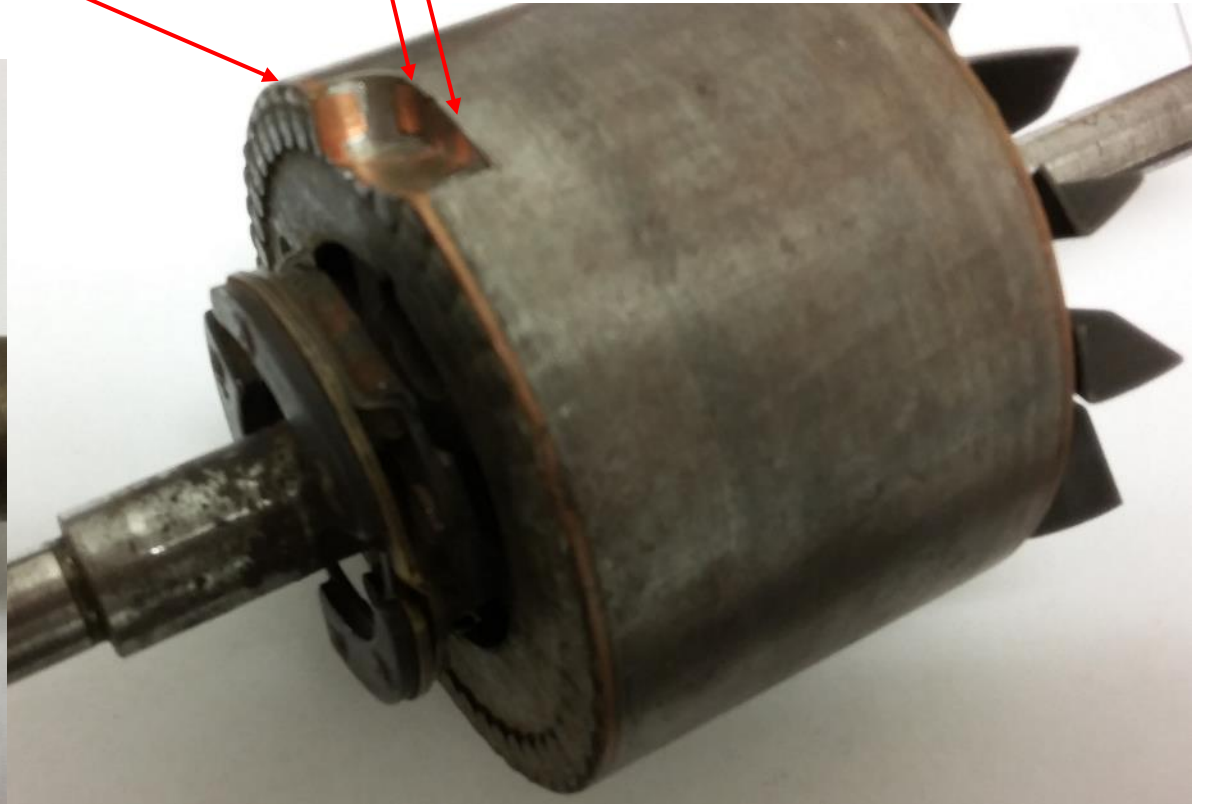


The figure shows the cage-like structure and three core laminations in the middle. The cage is enclosed in the core of the rotor.

Figure from https://en.wikipedia.org/wiki/Induction_motor, CC BY-SA 3.0, downloaded on Apr. 25, 2020.

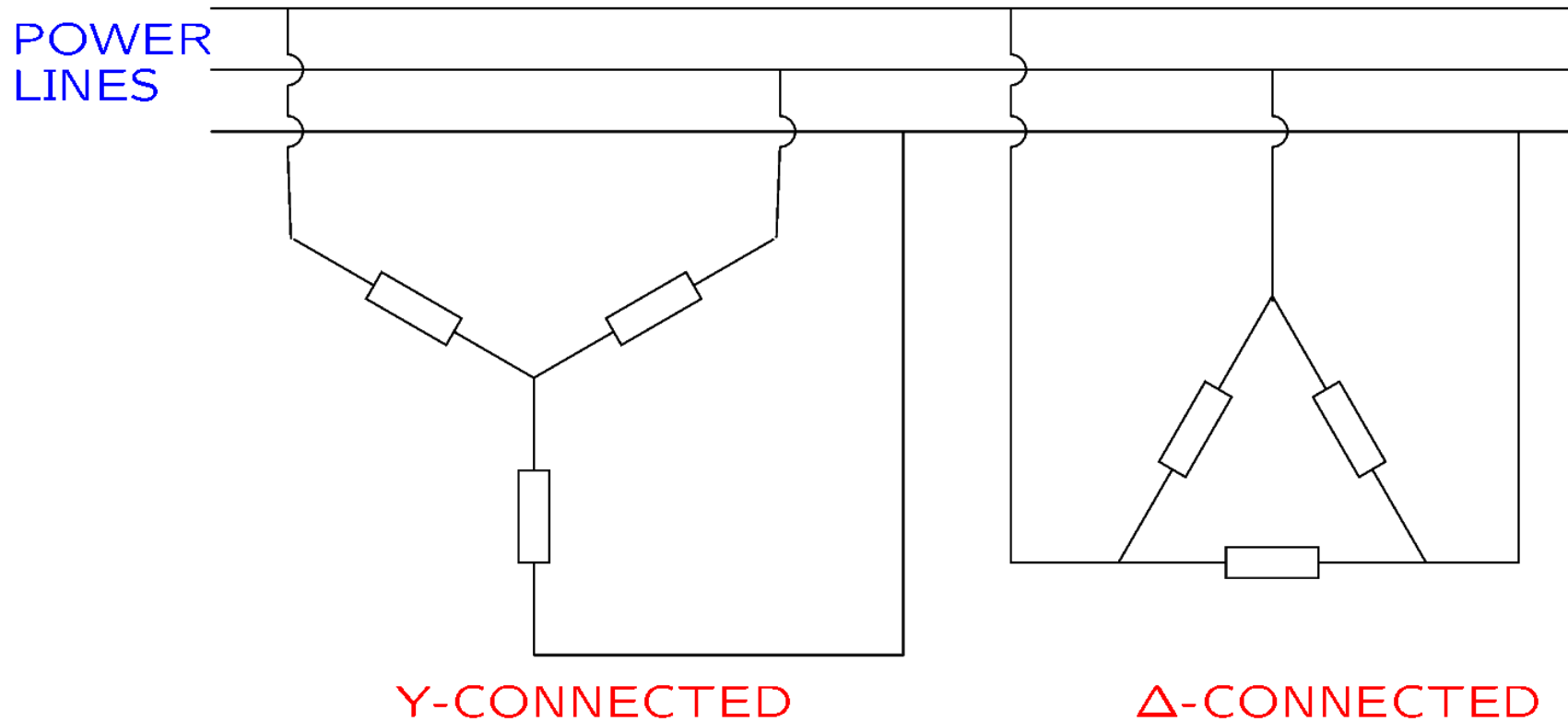
Induction Motors—Squirrel Cage

- Squirrel cage rotors.
- A cut in the rotor reveals an end ring and copper bars inside the rotor core.



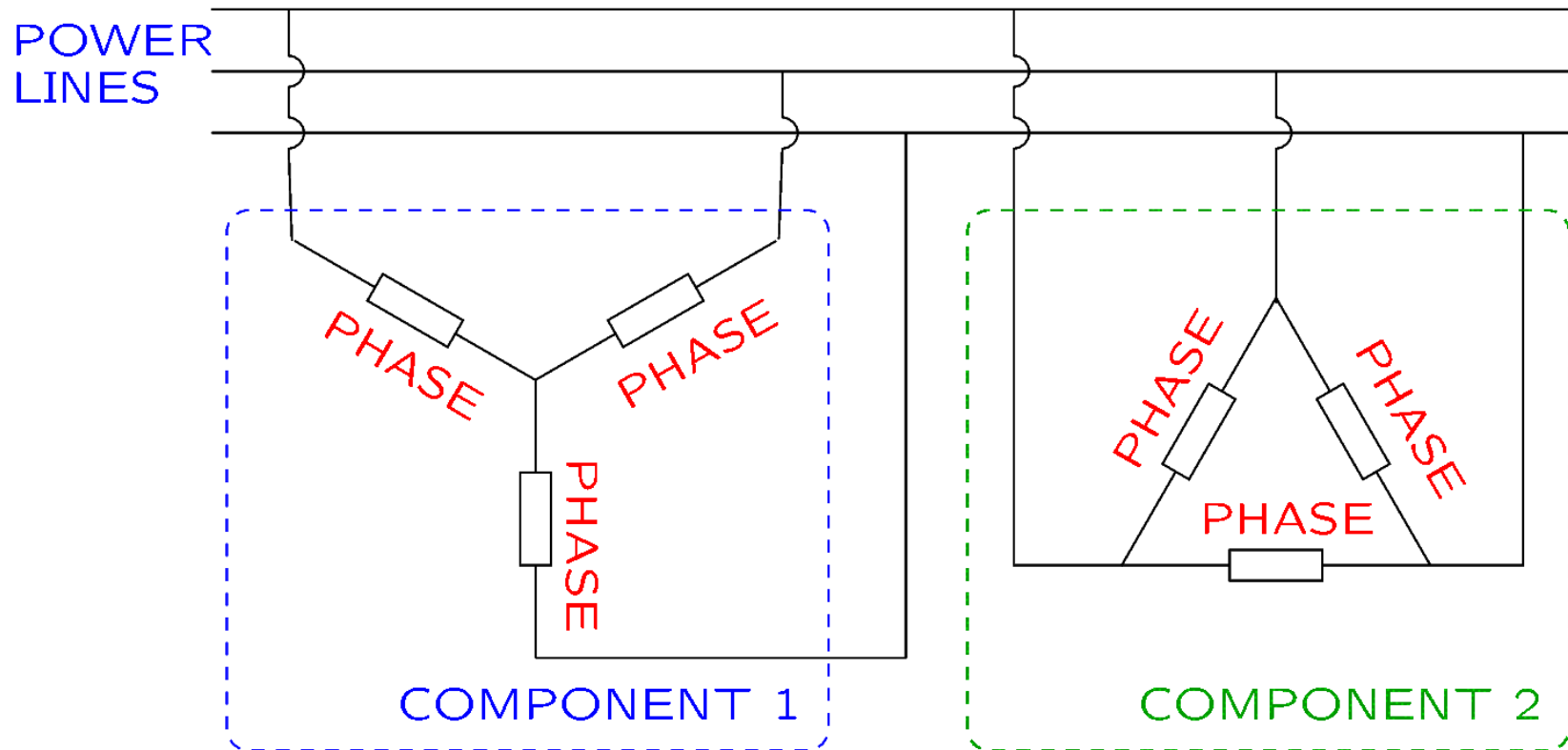
Review of Three Phase Systems

- In a three phase system, sources and loads are connected in the *wye (Y) configuration* or the *delta (Δ) configuration*.



Review of Three Phase Systems

- A three phase component can be represented with three subcomponents, called *phases*, connected in Y or Δ .



Review of Three Phase Systems

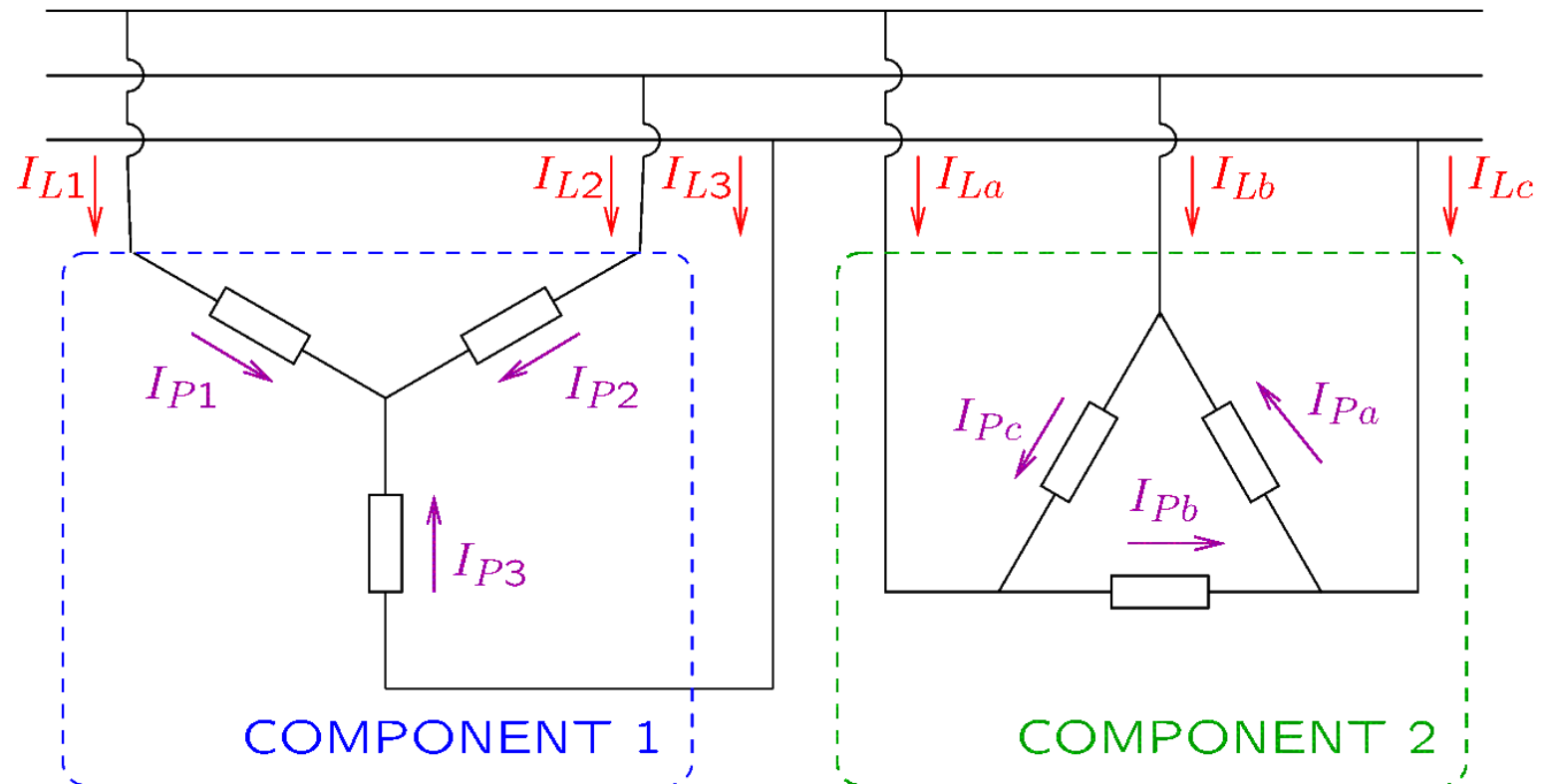
- The current flowing through a phase is called *phase current*.
- The current flowing through a conductor that powers a component is called *line current*.

- In the figure, the phase currents are:

- I_{P1}, I_{P2}, I_{P3}
- I_{Pa}, I_{Pb}, I_{Pc}

- The line currents are:

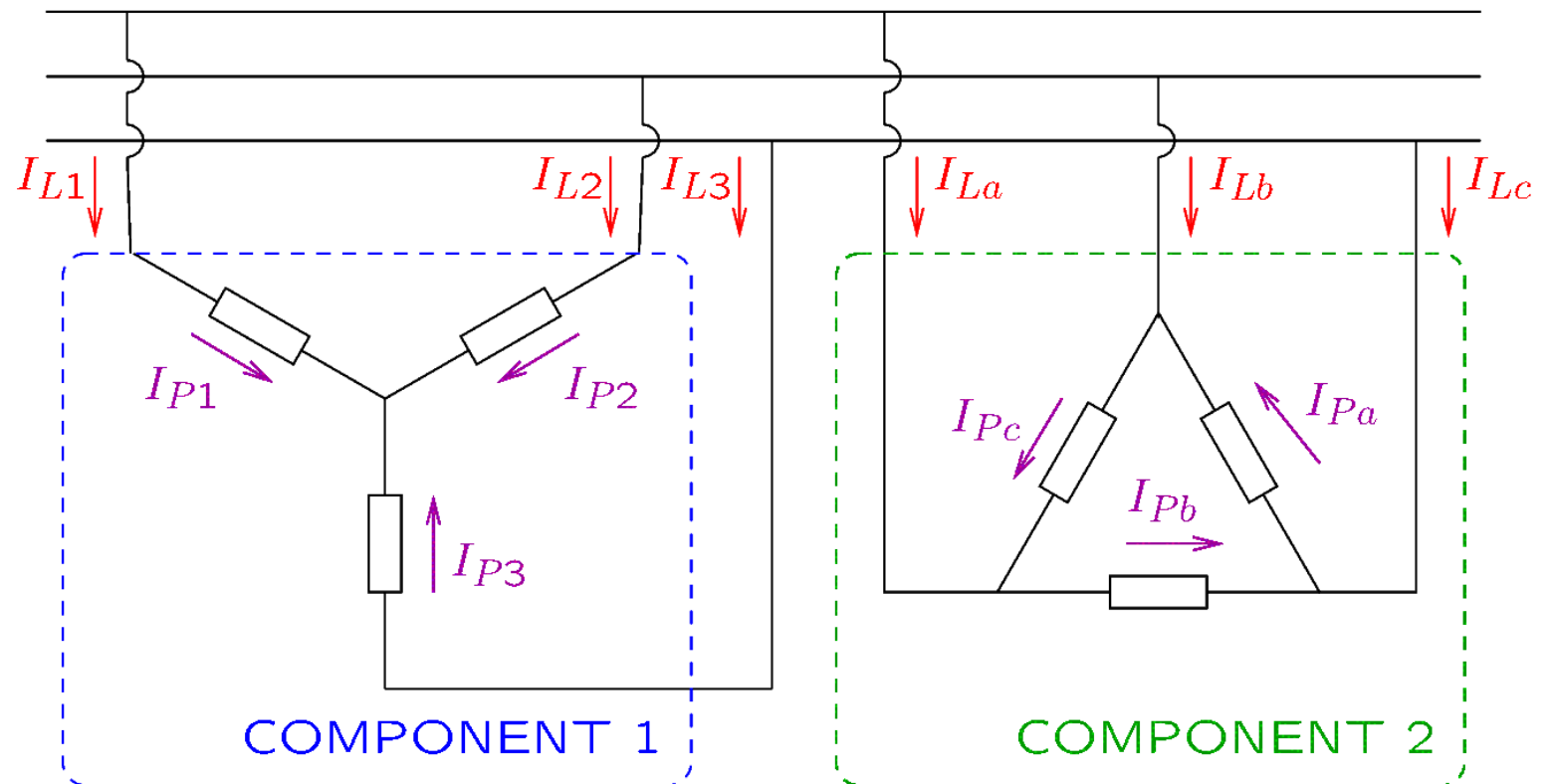
- I_{L1}, I_{L2}, I_{L3}
- I_{La}, I_{Lb}, I_{Lc}



Review of Three Phase Systems

- For a Y -connected component, phase and line currents are identical.
- For a Δ -connected component, the rms values are related by $I_L = \sqrt{3}I_P$.

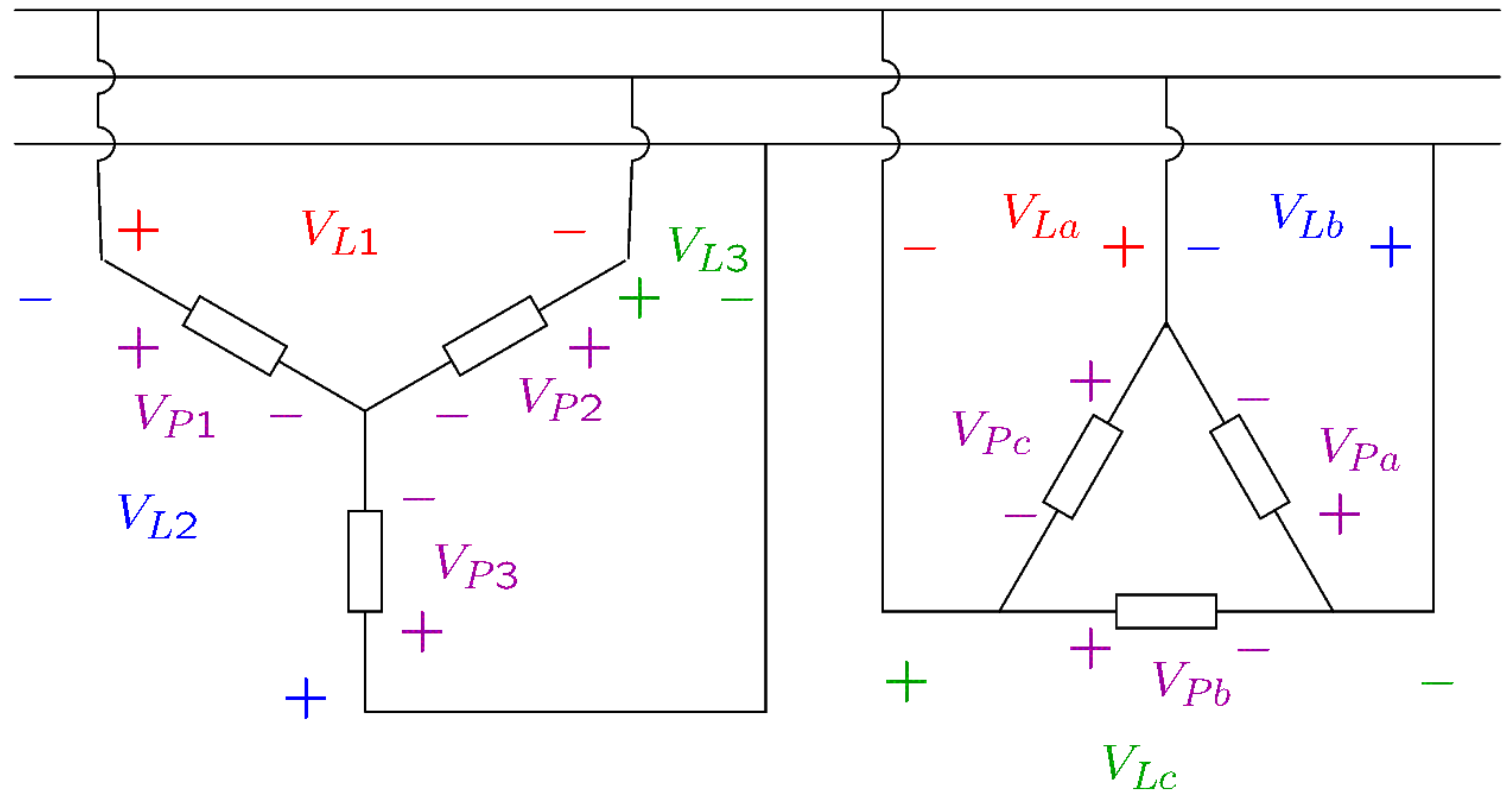
- In the figure, the phase currents are:
 - I_{P1}, I_{P2}, I_{P3}
 - I_{Pa}, I_{Pb}, I_{Pc}
- The line currents are:
 - I_{L1}, I_{L2}, I_{L3}
 - I_{La}, I_{Lb}, I_{Lc}



Review of Three Phase Systems

- The voltage on a phase is called *phase voltage*.
- The voltage between two power lines is called *line-to-line voltage*, or shortly, *line voltage*.

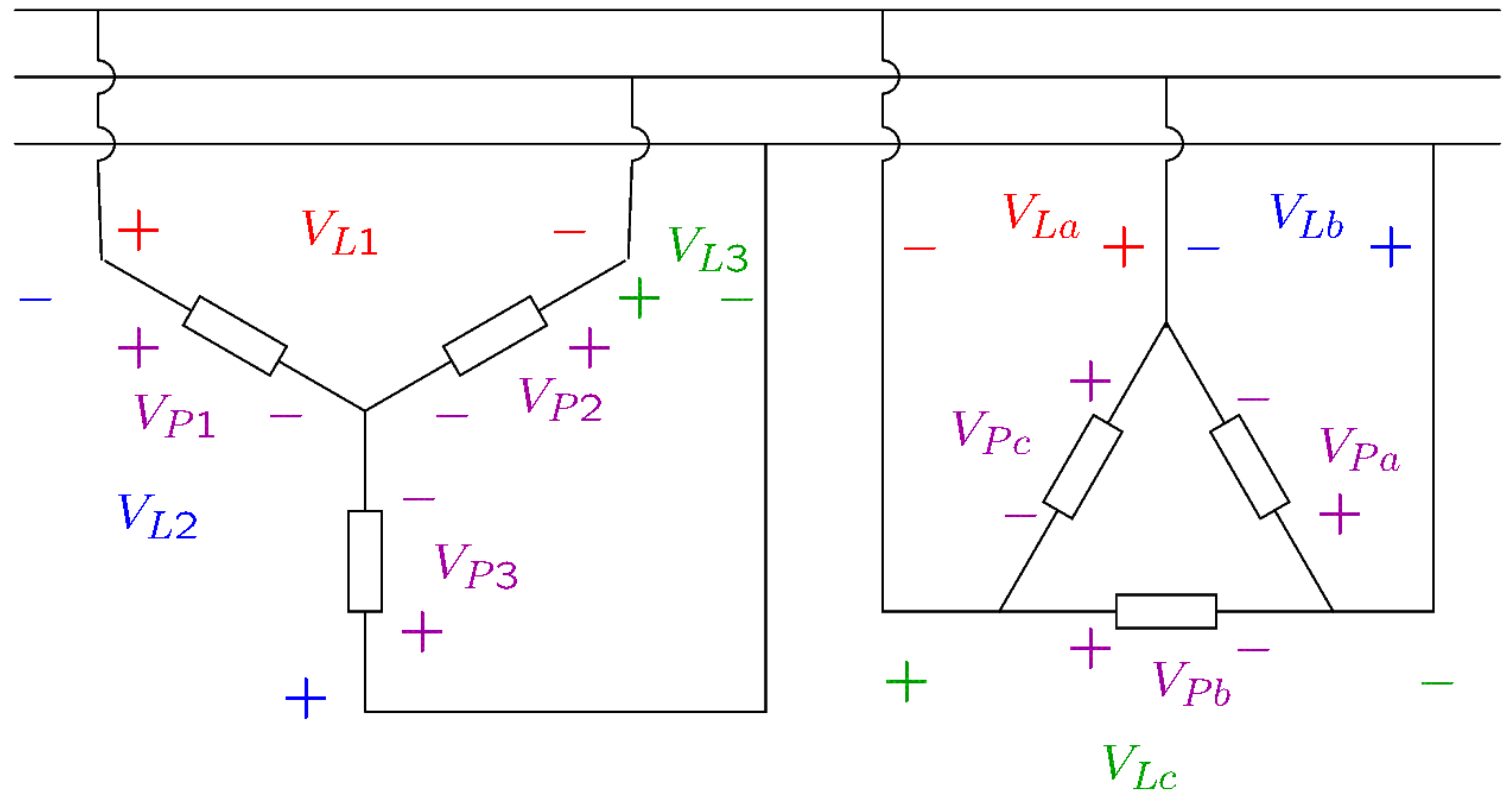
- In the figure, the phase voltages are:
 - V_{P1}, V_{P2}, V_{P3}
 - V_{Pa}, V_{Pb}, V_{Pc}
- The line voltages are:
 - V_{L1}, V_{L2}, V_{L3}
 - V_{La}, V_{Lb}, V_{Lc}



Review of Three Phase Systems

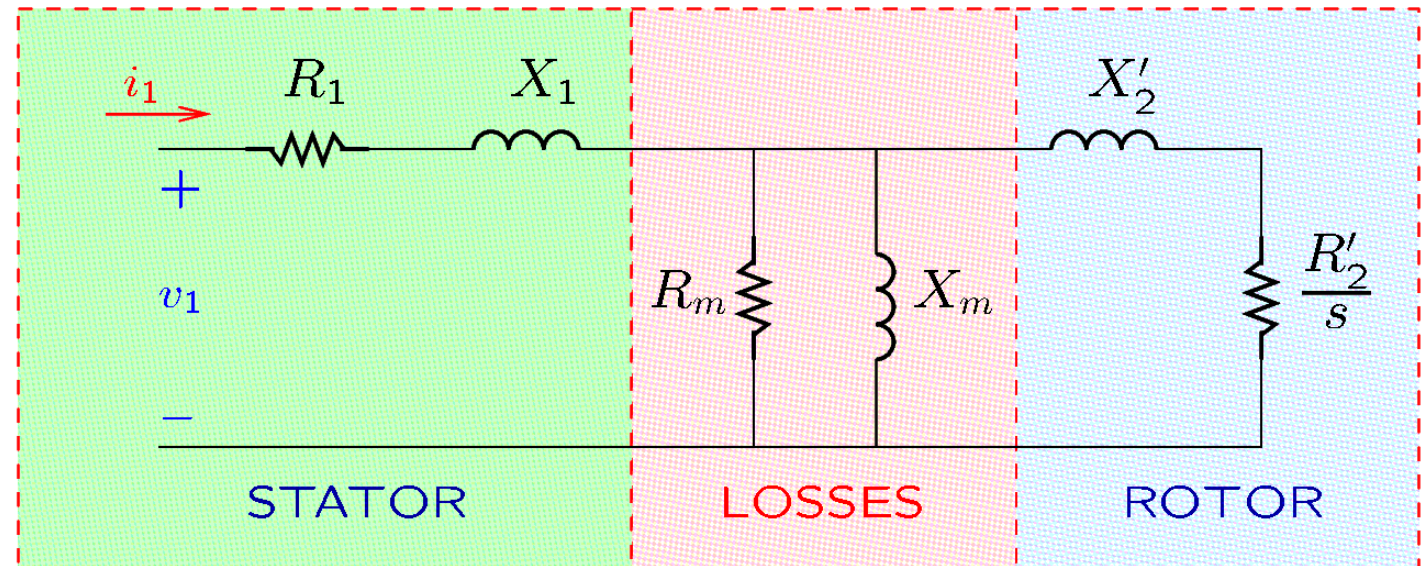
- For a Δ -connected component, phase and line voltages are identical.
- For a Y -connected component, the rms values are related by $V_L = \sqrt{3}V_P$.

- In the figure, the phase voltages are:
 - V_{P1}, V_{P2}, V_{P3}
 - V_{Pa}, V_{Pb}, V_{Pc}
- The line voltages are:
 - V_{L1}, V_{L2}, V_{L3}
 - V_{La}, V_{Lb}, V_{Lc}



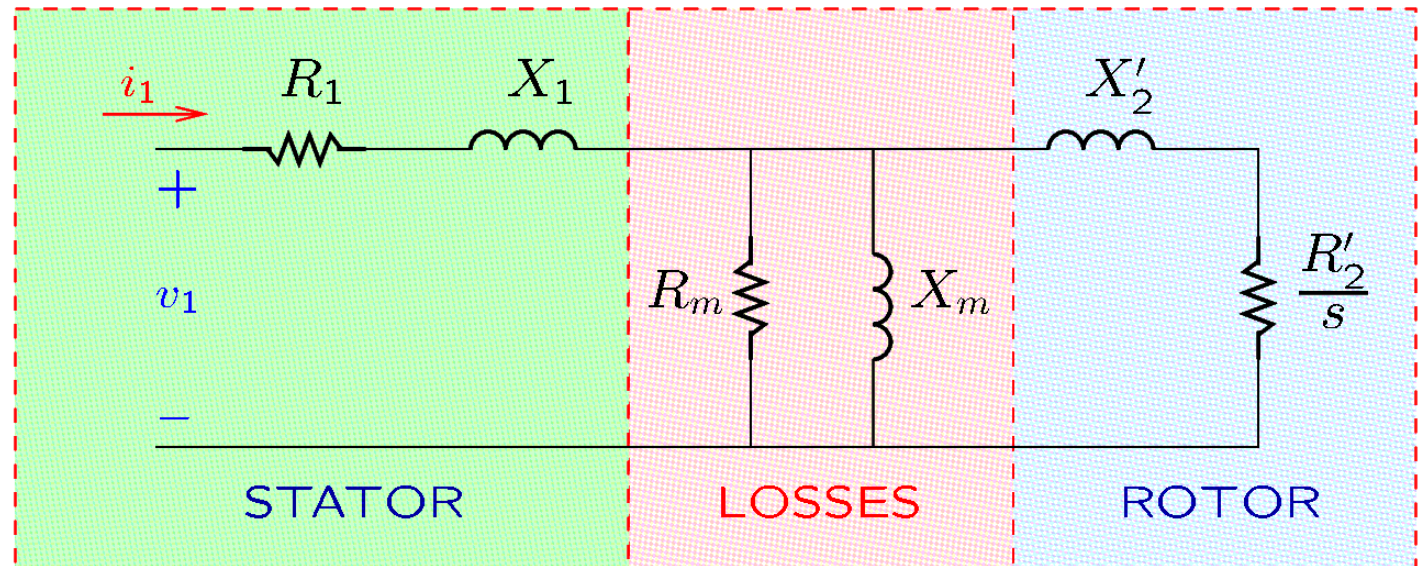
Induction Motors—Electrical Model

- Each phase of a three phase induction motor is equivalent to the circuit shown in the figure.
- R_1 and X_1 are the resistance and reactance, respectively, of a stator phase.
- R'_2 and X'_2 correspond to the resistance and reactance, respectively, of the rotor.
- The difference $R'_2/s - R'_2$ is a resistance modeling the effect of the mechanical work of the motor.
- Note that s is the slip.



Induction Motors—Electrical Model

- R_m models core losses.
- X_m models that the magnetic coupling of the rotor and stator is imperfect.
- *The per phase equivalent circuit is obtained from the equivalent circuit of a practical transformer.*



Induction Motors—Electrical Model

- The circuit can be analyzed in the frequency domain.
- We will use **rms values** for all voltages and currents.
- The input average power per phase is $P_1 = V_1 I_1 \cos(\alpha_v - \alpha_i)$, where α_v and α_i denote the angles of the voltage and of the current, respectively.

- Recalling that reactances do not dissipate power,

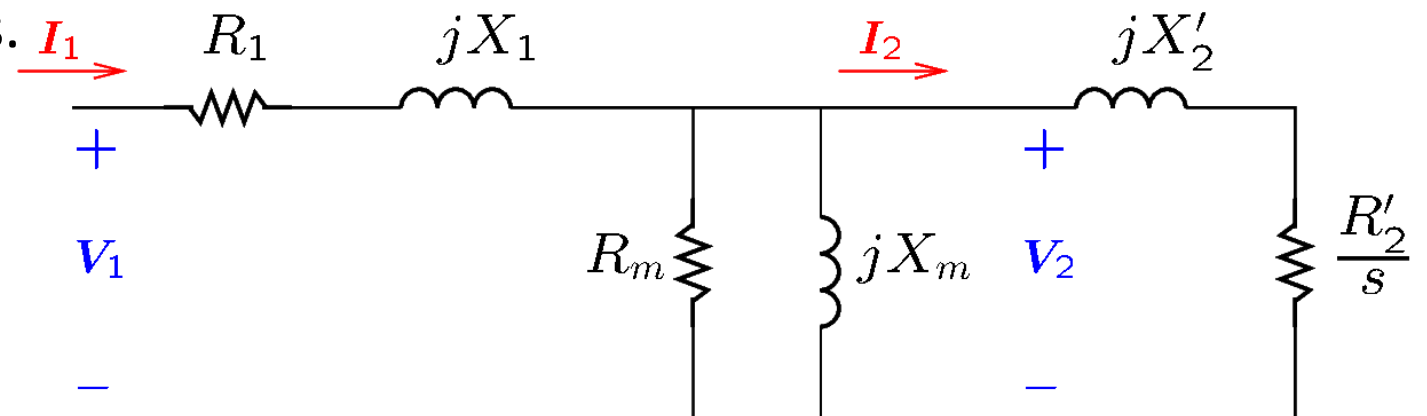
$$P_1 = I_1^2 R_1 + I_2^2 R'_2 / s + V_2^2 / R_m$$

- Since there are three phases, the total power is $P_{in} = 3P_1$.

- $3I_1^2 R_1$ is the stator copper loss.

- $3V_2^2 / R_m$ is the core loss.

- $3I_2^2 R'_2 / s$ is the rotor power.



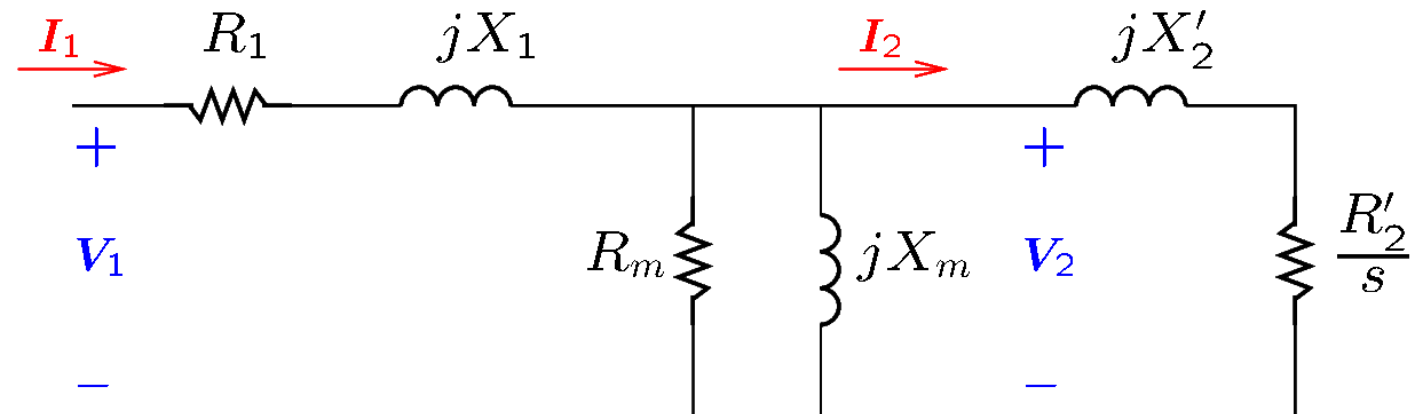
Induction Motors—Electrical Model

- The rotor power is $3P_g = 3I_2^2 R'_2 / s$, where P_g is the power crossing the air gap between the stator and the rotor, per phase.
- P_g has two components:
 - $I_2^2 R'_2$ is the power dissipated on the rotor resistance, per phase.
 - $P_d = P_g - I_2^2 R'_2 = P_g(1 - s)$ is the electromagnetic developed power, per phase.
- Thus, the electromagnetic torque of the motor satisfies:

$$T\omega = 3P_d$$

- Since $\omega = \omega_s(1 - s)$, (where ω_s is the synchronous speed):

$$T = \frac{3I_2^2 R'_2}{s\omega_s}$$



Induction Motors—Example

Assuming a Y -connected motor with 4 poles per phase, a line voltage of 200 V rms and 60 Hz, $R_1 = 1 \Omega$, $R'_2 = 0.5 \Omega$, $X_1 = X'_2 = 0.5 \Omega$, and $X_m = 100 \Omega$, find the starting torque of the motor. Neglect R_m .

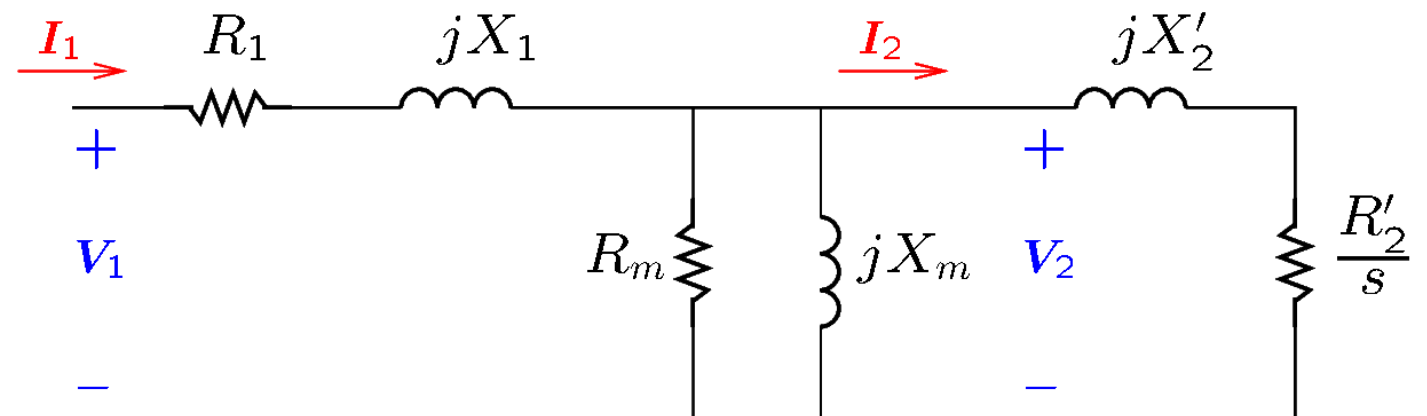
- Since the motor is Y -connected, the voltage V_1 is the phase voltage: $V_1 = \frac{200}{\sqrt{3}} \text{ V}$.
- When the motor starts, the speed is $\omega = 0$, therefore the slip is $s = 1$.
- The total impedance of the phase is $\mathbf{Z}_1 = R_1 + jX_1 + (jX_m) || (jX'_2 + R'_2/s)$.
- By current division, $I_2 = I_1 \frac{jX_m}{jX_m + jX'_2 + R'_2/s}$, where $I_1 = V_1/\mathbf{Z}_1$.
- Numerically,

$$\omega_s = \frac{4\pi f}{p} = 60\pi \text{ rad/s.}$$

$$\mathbf{Z}_1 = 1.80 \angle 33.78^\circ \Omega$$

$$I_2 = 63.88 \text{ A rms}$$

$$T = \frac{3I_2^2 R'_2}{s\omega_s} = 32.472 \text{ Nm}$$

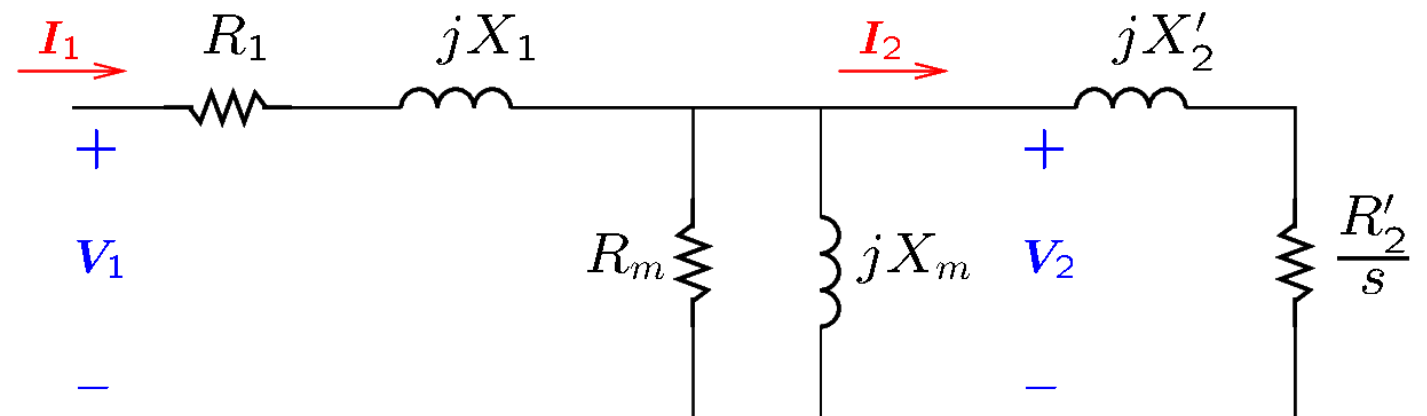


Induction Motors—Example 2

In the previous example, how will the torque change if the motor is connected in the Δ -configuration?

- The circuit per phase will not change.
- V_1 will change to $V_1\sqrt{3} = 200 \text{ V rms}$.
- Thus, all currents and voltages will increase by a factor of $\sqrt{3}$.
- Thus I_2 , will change to $I_2\sqrt{3}$ and I_2^2 to $3I_2^2$.
- Therefore, the torque will triple.

$$T = \frac{3I_2^2 R'_2}{s\omega_s} = 97.416 \text{ Nm}$$



Synchronous Machines

- Are used as motors or generators of three-phase voltage.
- The motors operate at the synchronous speed n_s .
- Purely synchronous motors cannot start.
- They can be started using:
 - electronic control or
 - damper windings (bars)
- Damper bars: make up an induction motor that operates at speeds $n < n_s$.
- Damper bars have no effect if $n = n_s$.
- Thus, the rotor has a *field winding* and a *damper winding*.
- The field winding is powered with DC current, to ensure that the polarity of the rotor poles does not change.
- The rotor is powered by means of slip rings and brushes.

Synchronous Machines and BLDC motors

- Synchronous motors in which the rotor consists of permanent magnets are similar to brushless DC motors.
- While synchronous motors use three-phase voltage, brushless DC (BLDC) motors use an electronic controller that converts DC voltage to pulses.
- By applying pulses sequentially to each phase, a rotating magnetic field is obtained.
- To control accurately the speed, and so that the BLDC motors are self starting, the controllers need feedback.
- Feedback is provided by
 - Hall sensors (which sense a magnetic field) or
 - Shaft encoders (which determine the angular position of the motor shaft).