Laplace Transform Properties

Let a and b be constants, $F(s) = \mathcal{L}[f(t)]$, and $G(s) = \mathcal{L}[g(t)]$. The following table lists a number of basic properties of the Laplace transform.

$\mathcal{L}[t^{n-1}] = \frac{(n-1)!}{s^n}$, for $n \geq 1$	(1)	$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$
$\mathcal{L}[af(t) + bg(t)] = a\mathcal{L}[f] + b\mathcal{L}[g]$	(2)	$\mathcal{L}^{-1}[aF(s) + bG(s)] = a\mathcal{L}^{-1}[F] + b\mathcal{L}^{-1}[G]$
$\mathcal{L}\left[f(t/a)\right] = a\, \mathcal{L}[f]\vert_{s \to as}$	(3)	$\mathcal{L}^{-1}[F(as)] = \frac{1}{a}\mathcal{L}^{-1}\left[F\right]\vert_{t \to \frac{t}{a}}$
$\mathcal{L}[f(t)\mathcal{U}(t-a)] = \mathcal{L}\left[f(t+a)\right]e^{-as}$, for $a \geq 0$	(4)	$\mathcal{L}^{-1}[F(s)e^{-as}] = \mathcal{U}(t-a)\mathcal{L}^{-1}\left[F(s)\right]\vert_{t \to t-a}$, for $a \geq 0$
$\mathcal{L}[e^{-at}f(t)] = \mathcal{L}\left[f\right]\vert_{s \to s+a}$	(5)	$\mathcal{L}^{-1}[F(s)] = e^{-at}\mathcal{L}^{-1}\left[F(s-a)\right]$
$\mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0^-)$	(6)	

Based on the properties listed above, additional properties can be derived, such as the following.

	g(t)	G(s)
(7)	1	$\frac{1}{s}$
(8)	$\int_a^t f(\tau)d\tau$	$\frac{F(s)}{s} - \frac{1}{s}\int_0^a f(\tau)d\tau$
(9)	$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
(10)	e^{-at}	$\frac{1}{s+a}$
(11)	$\sin(at)$	$\frac{a}{s^2+a^2}$
(12)	$\cos(at)$	$\frac{s}{s^2+a^2}$

The formula of for the n'th order derivative can be derived from equation (6):

$$(13) \qquad \mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - f^{(n-1)}(0^-) - sf^{(n-2)}(0^-) - s^2 f^{(n-3)}(0^-) - \ldots - s^{n-1}f(0^-)$$

Assuming no discontinuities at $t = 0$, we can write $f(0)$ in the place of $f(0^-)$, $f'(0)$ in the place of $f'(0^-)$, and so on.