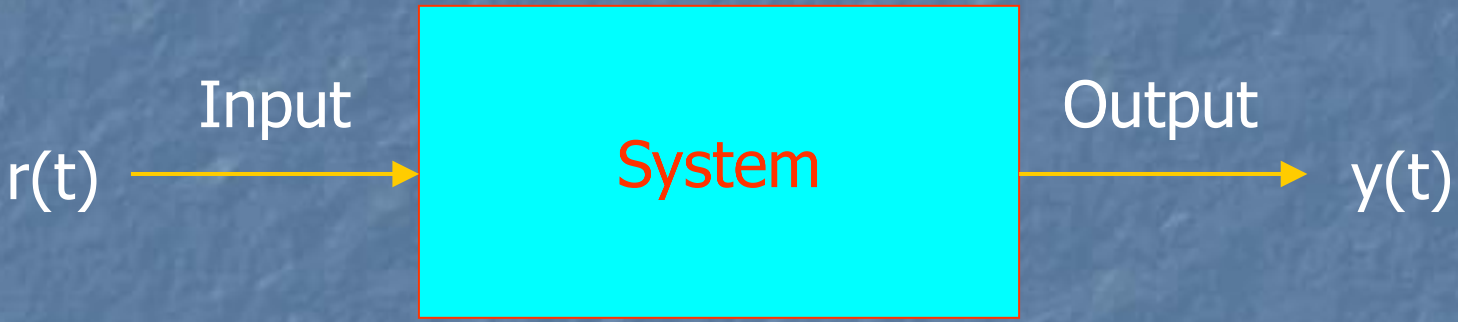
# Transfer Functions

## Obtaining TF Models. Transient and Steady-State Responses.

M.V. Iordache, *EEGR3523 Mechatronics*, Spring 2019, LeTourneau University

# System Response

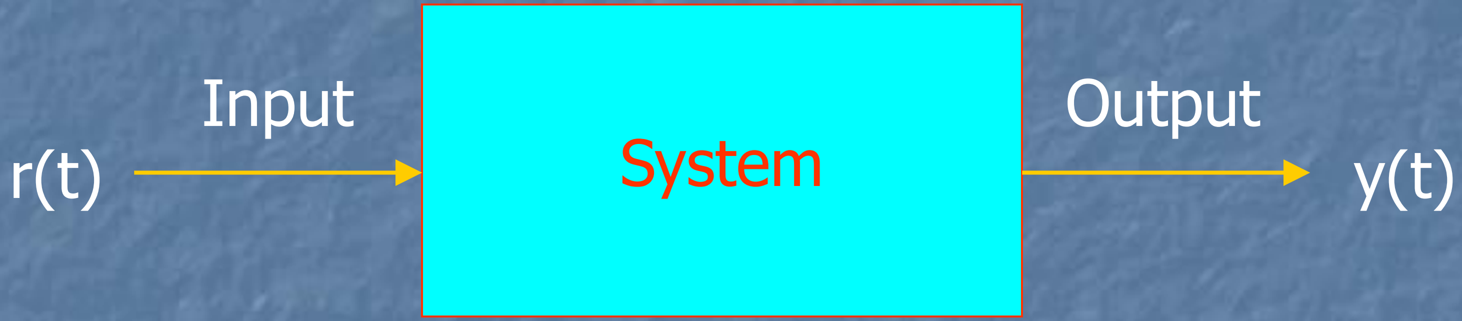
- Consider the response (output) of a system to a given an excitation (input).



- Note that $y(t) = y_r(t) + y_i(t)$, where
  - $y_r(t)$ is determined by the input $r(t)$
  - $y_i(t)$ is determined by the initial conditions
  - Typically, $y_i(t)$ goes quickly to zero, so $y(t) \simeq y_r(t)$.

# Transfer Functions

- Are system models allowing to determine the response (output) of the system based on the excitation (input) of the system.



Input

r(t)

System

Output

y(t)

- If H(s) is the TF:

$$y_r(t) = \mathcal{L}^{-1}\{H(s)R(s))\}$$

# Transfer Functions

An I/O relationship

$$y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1 y' + a_0 y = b_m r^{(m)} + b_{m-1} r^{(m-1)} + \ldots b_0 r$$

is expressed by a TF

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots b_0}{s^n + a_{n-1} s^{n-1} + \ldots a_1 s + a_0}$$

# Recall DEQ terminology …

- Ordinary, when no partial derivatives.

- Linear, if all factors depend linearly on $y^{(0)} \ldots y^{(n)}$.
  - $y'' + t^2y = r$ is linear
  - $yy''+5y' = r$ is nonlinear

- The LT used for ordinary linear diff eqs with constant coefficients.
  - $y'' + t^2y = r$ (NO LT)
  - $y'' + 6y = r$ (USE LT)

# Transfer Functions

- Describe any linear time-invariant (LTI) system.
  - Linear: $ar_1+br_2 \rightarrow ay_1+by_2$
  - Time-invariant: system equations independent of time.
- Multiple inputs and/or outputs → H(s) is a matrix.

# Transfer Functions

Note that

$$H(s) = \frac{Y(s)}{R(s)}$$

assuming zero initial conditions.

$y(t) = \mathcal{L}^{-1}\{H(s)R(s))\}$, but

$y(t) \neq H(s)r(t)$, and $y(t) \neq h(t)r(t)$

# More on Transfer Functions G(s)

- g(t) is called the *impulse response.*
- G(jw) describes the magnitude and phase shift of the steady state response to a sinewave of angular frequency w.

$$a \cdot \sin(\omega t + \phi) \longrightarrow \boxed{G(s)} \longrightarrow a \cdot |G(j\omega)| \cdot \sin(\omega t + \phi + \angle G(j\omega))$$

- This property of G(jw) applies when all singularities of G(s) are in the LHP.

# Measuring Transfer Functions

- Using LabVIEW.
- Bode analyzers

# What can we tell about y(t) by looking at Y(s)?

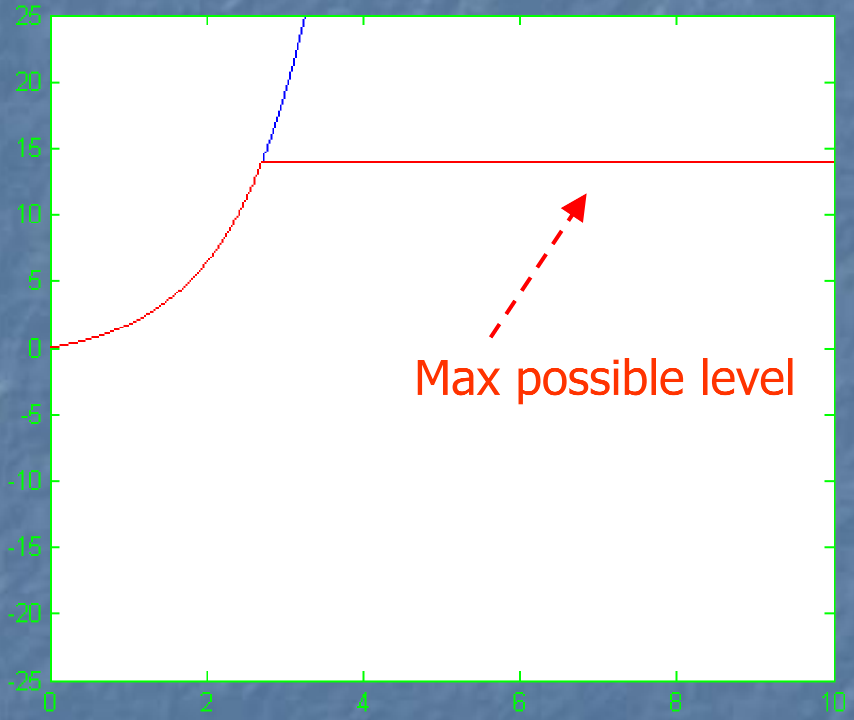
- Singularities (poles)
  - Stable response: y(t) converges to a constant value.

    When sY(s) has no poles in the RHP and on the imaginary axis.
  - Marginally stable response: y(t) is a bounded oscillation.

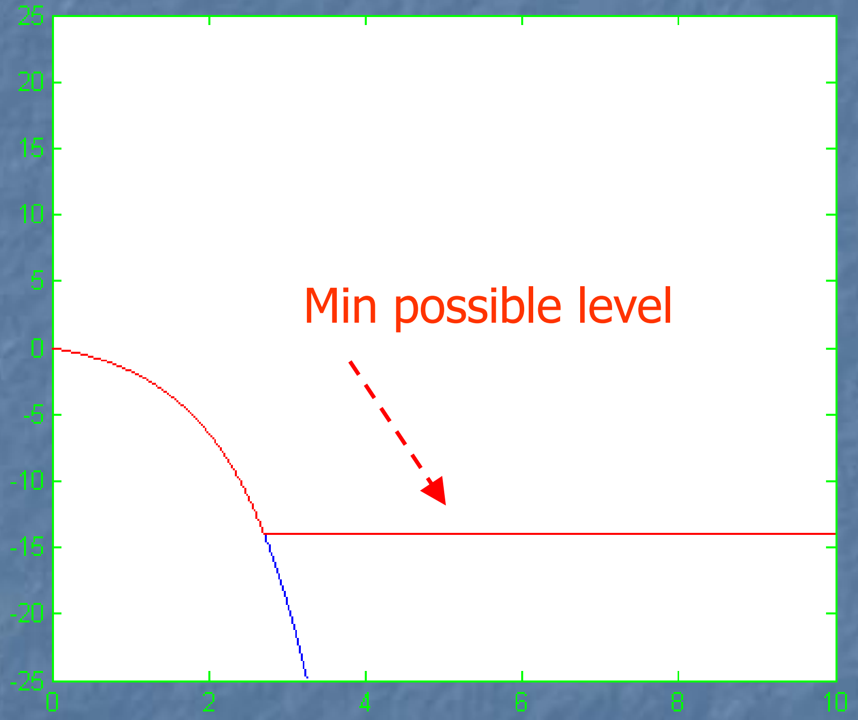
    When sY(s) has no poles in the RHP and at the origin and has simple poles on the imaginary axis.
  - Unstable response: y(t) is unbounded.

    When Y(s) has poles in the RHP or multiple poles at the same location on the imaginary axis.
- The Final Value Theorem.
  - Assumes sY(s) has no poles in the RHP and on the imaginary axis.
- The Initial Value Theorem.

# Unstable Responses

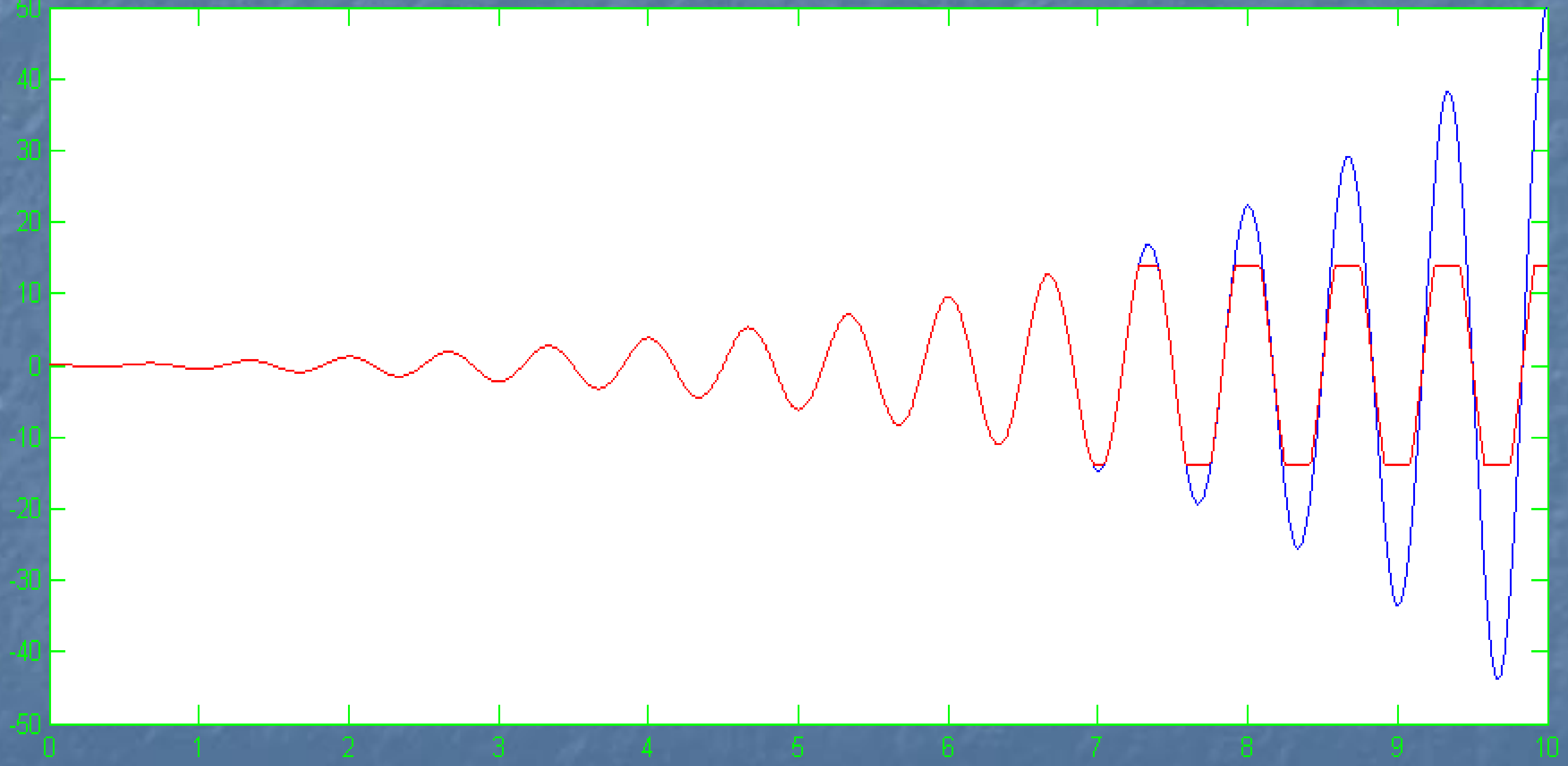


Max possible level



Min possible level

# Unstable Response



# The Steady State Response

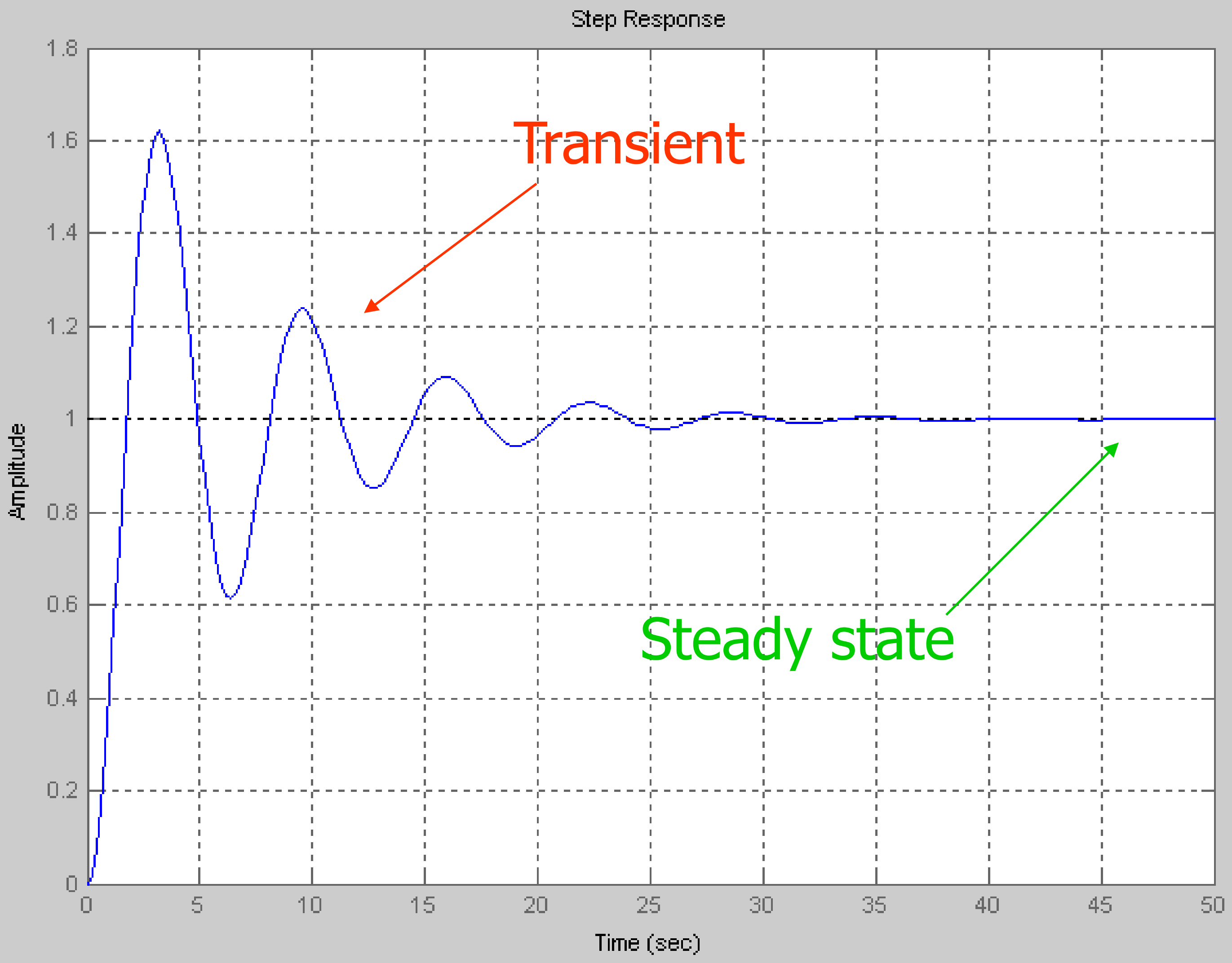
- If the Final Value Theorem Applies:

$$r(t) \longrightarrow \boxed{G(s)} \longrightarrow \lim_{s\to 0} sG(s)R(s)$$

- For harmonic excitation, use the Fourier transform approach:

$$a\cdot\sin(\omega t+\phi) \longrightarrow \boxed{G(s)} \longrightarrow a\cdot|G(j\omega)|\cdot\sin(\omega t+\phi+\angle G(j\omega))$$

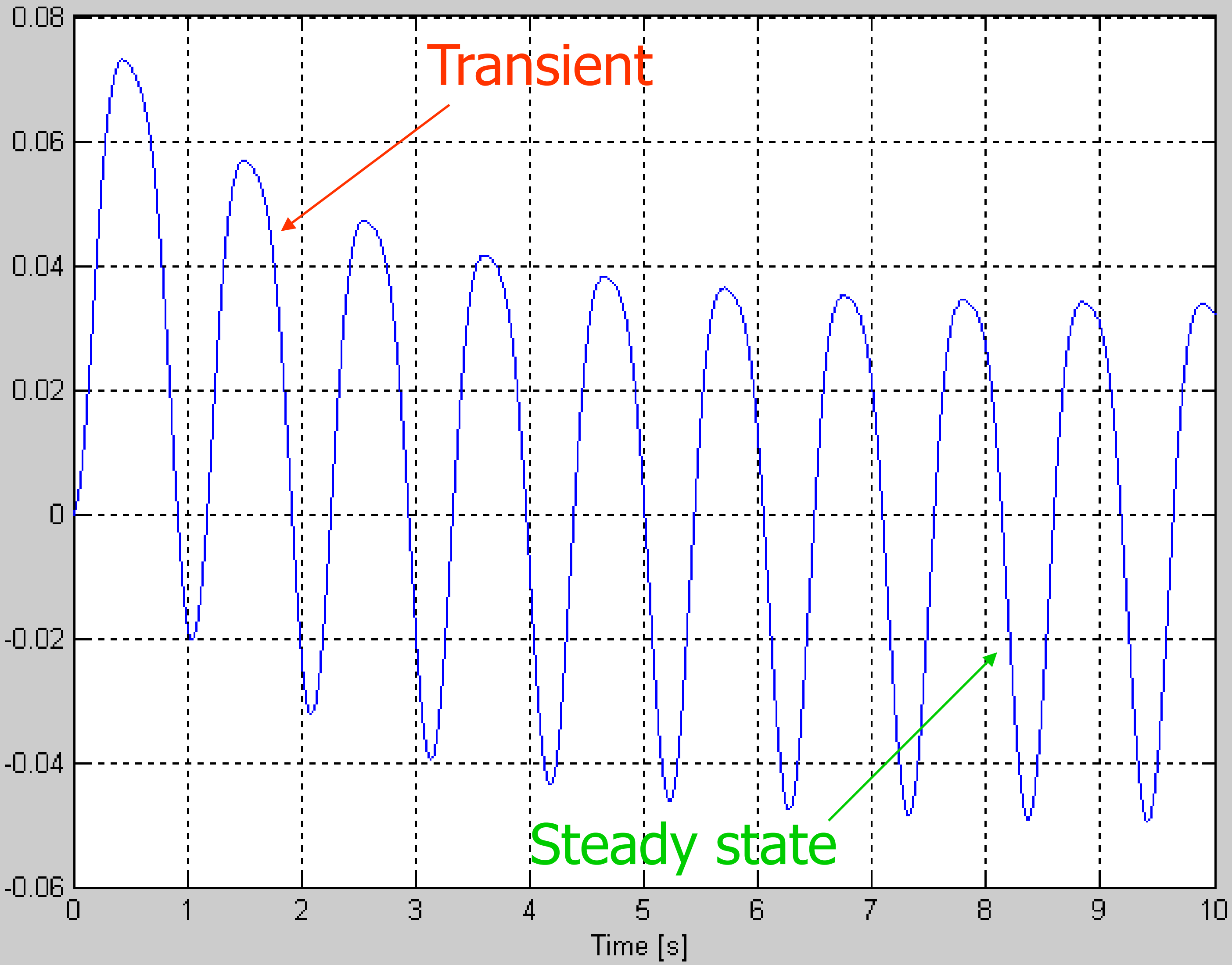
# The Steady State Response



Transient

Steady state

# The Steady State Response



# The Steady State Response

- The transient response $y(t)$ can be found with the inverse Laplace transform.
  - *It depends on initial conditions.*
- The steady state response $y_{ss}$:
  - Is the transient response $y(t)$ as $t \rightarrow \infty$.
  - When $y_{ss}$ is a constant, it is easily found with the Final Value Theorem.
  - When the input is sinusoidal, $y_{ss}(t)$ may be found with the Fourier Transform approach.
  - *Typically, independent of initial conditions.*