Introduction. System Modeling.

Electrical and Mechanical Components. Motion Mechanisms. Laplace Domain Equations.

M.V. Iordache, *EEGR3523 Mechatronics*, Spring 2021, LeTourneau University

Mechatronics—Introduction

What is Mechatronics?

At a first glance:

- Mechanics + Electronics → Mechatronics
- Mechatronics: *The introduction of electronic controls into mechanical components*. (Web definition)

Mechatronic designs involve:

- Mechanical systems
- Electrical systems

- Actuators (such as motors)
- Sensors

- Control systems
  - Implemented in software and/or hardware

The design approach of Mechatronics:

- Subsystems of different nature (such as electrical and mechanical) are designed together, rather than independently.
- Manufacturing considered also at design time.

Topics

- Modeling of mechanical and electrical systems
- Mathematical models
  - Provide a common representation of systems of different nature.
  - Transfer functions, state space models, block diagrams.
- System response
  - Describes how variables change in time.
  - Simulation.
- Control systems
  - Control algorithms that guarantee the system response to satisfy stability, time, and error specifications.
  - PID control.
- Actuators
  - Electric machines (motors and generators).

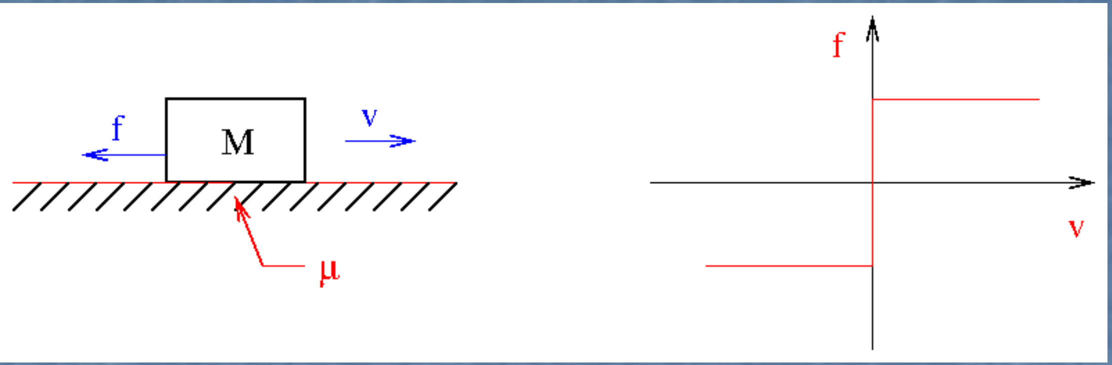
System Models

- A first step in the mathematical modeling of systems is obtaining a simplified model in terms of elementary mechanical and electrical components.
- Simplified mechanical models involve:
  - Blocks, springs, dampers, …
- Simplified electrical models involve:
  - Resistors, inductors, capacitors, dependent sources, …
- For a summary of mechanical and electrical components, see the models.pdf handout.

Mechanical Modeling—Friction

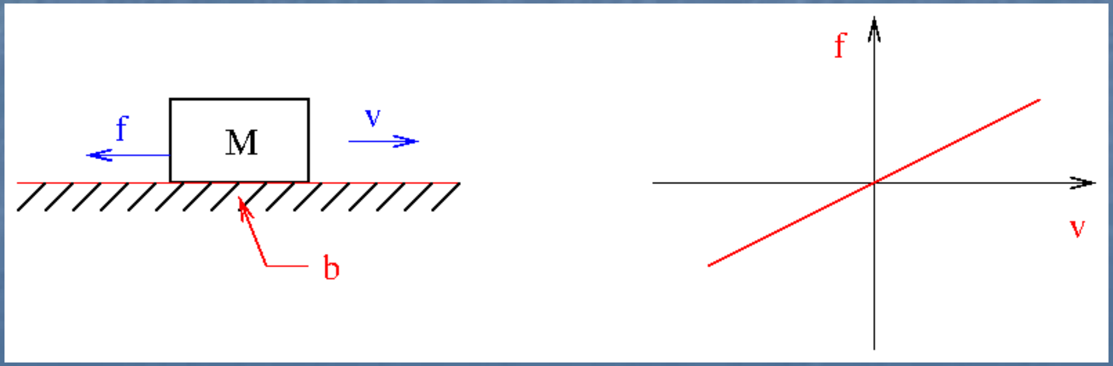
- Coulomb or dry friction.

$$f = \mu M g \cdot \text{sgn}(v)$$

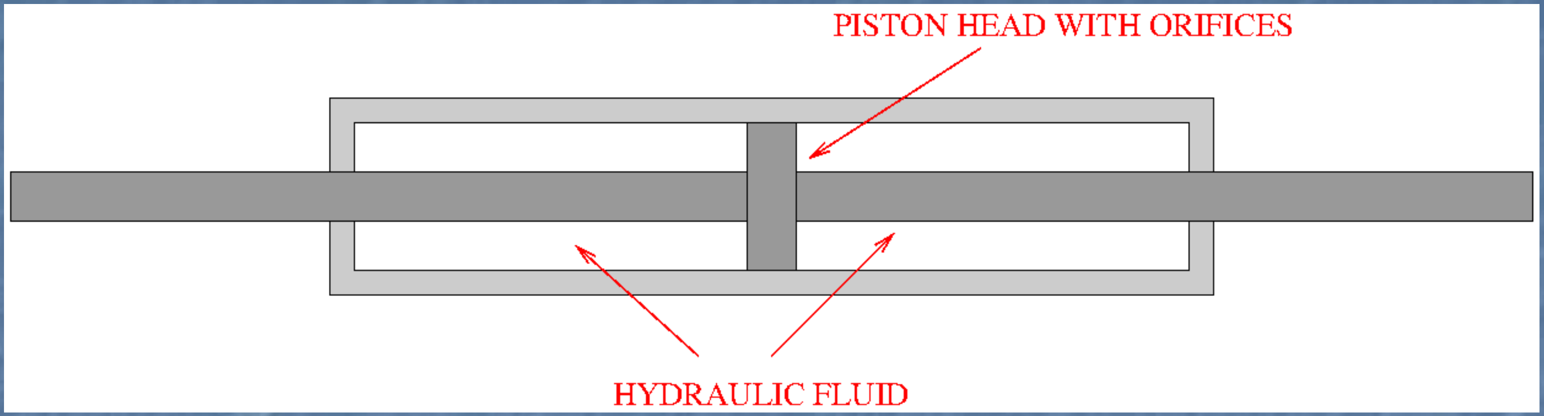


- Viscous friction (for lubricated surfaces).

$$\boxed{f = b \cdot v}$$

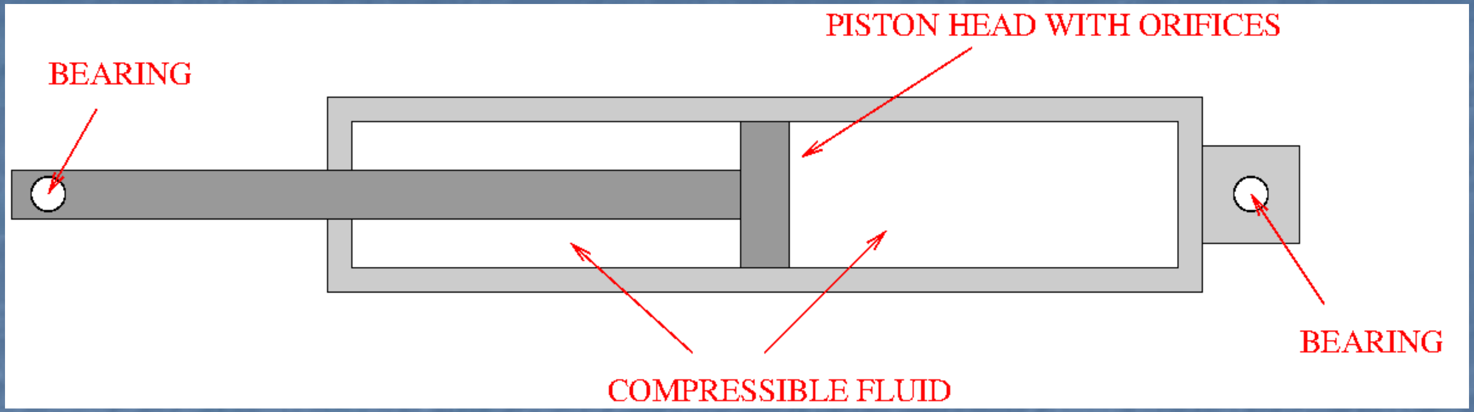


How to make a damper …



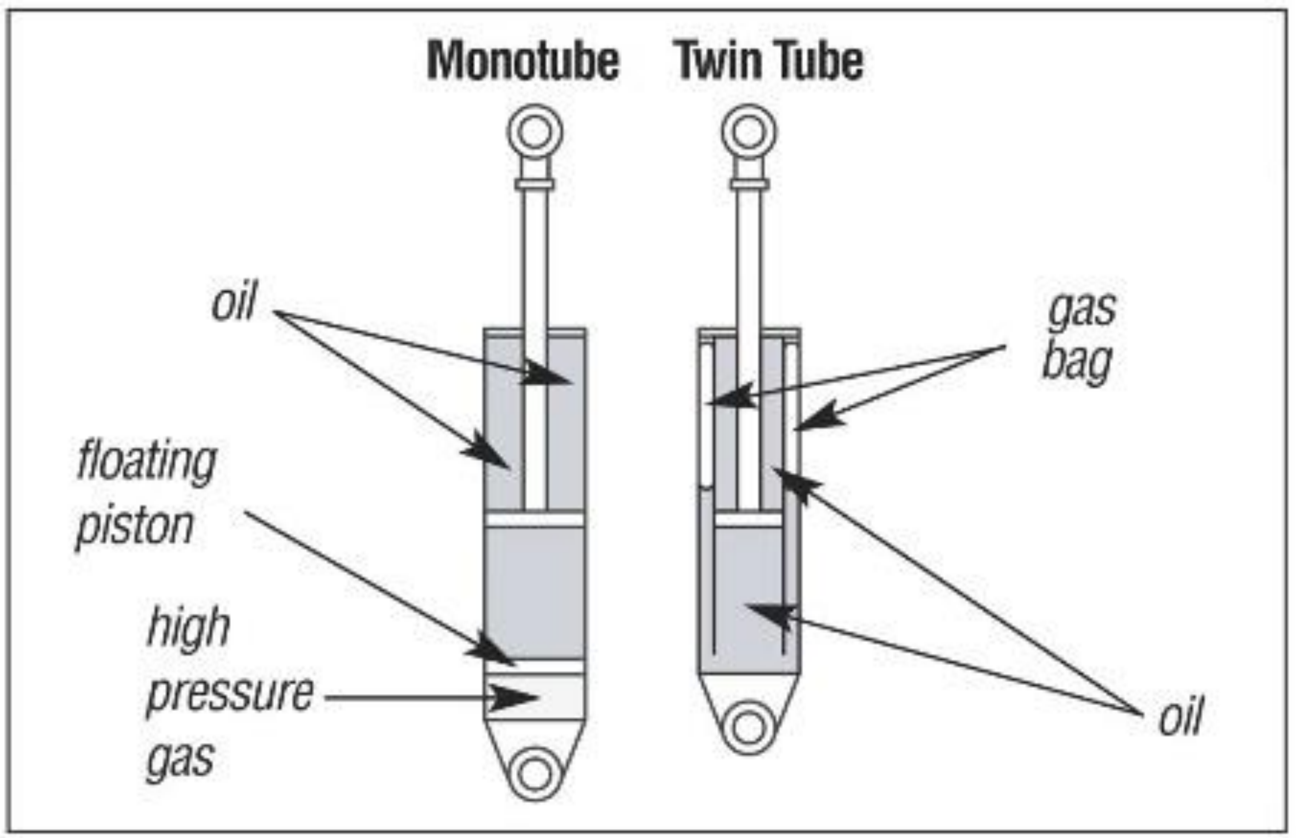
- A damper resembles a shock absorber.
- A shock absorber might not operate according to the same linear equation.

How to make a damper ...



- The figure illustrates the principle of a damper or shock absorber.
- Not only friction, but also compression/extension due to stem volume.
- Various enhancements possible.

Shock absorbers



From https://en.wikipedia.org/wiki/Shock_absorber#/media/File:Shock_Absorbers_Detail.jpg
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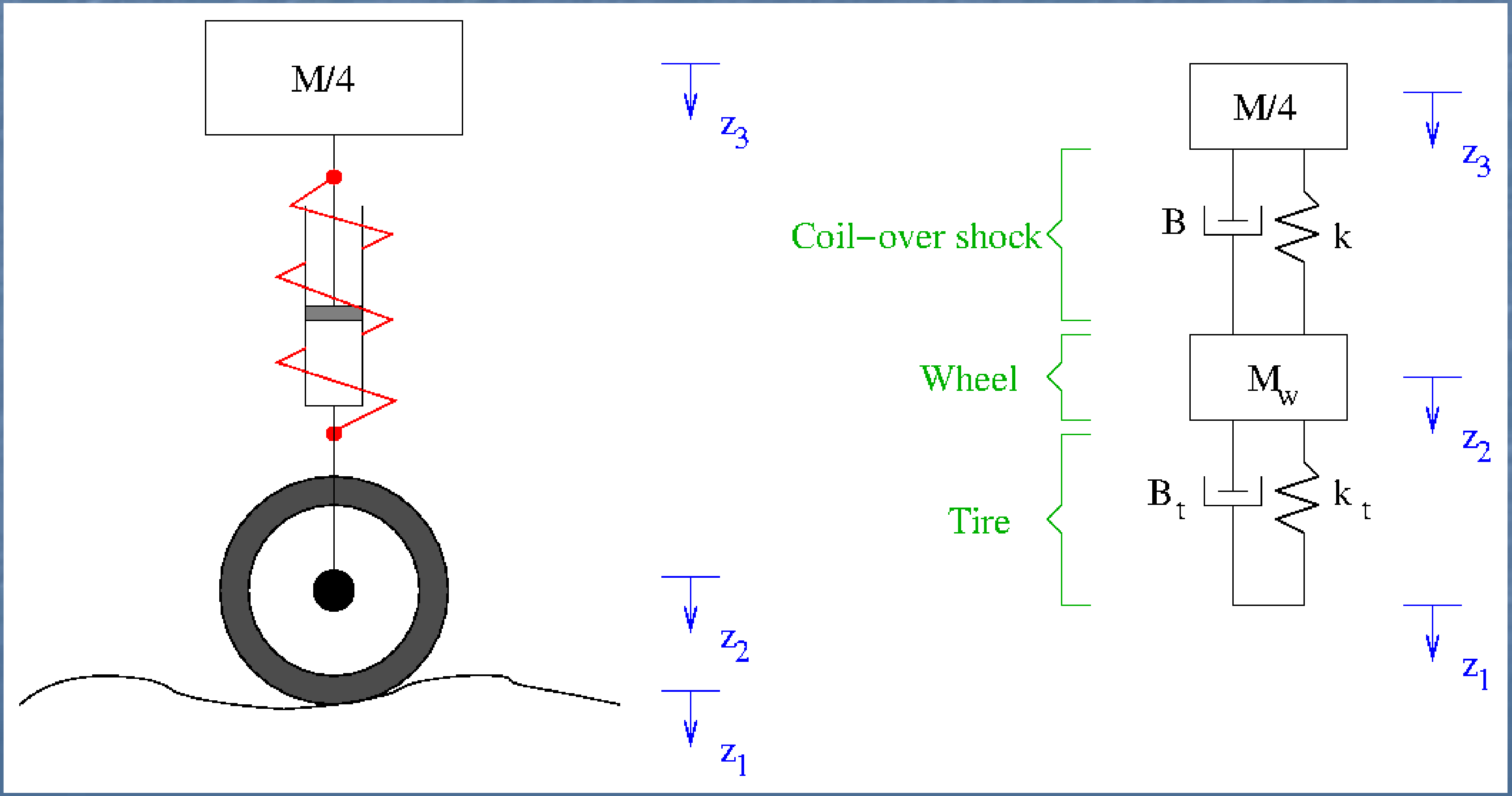
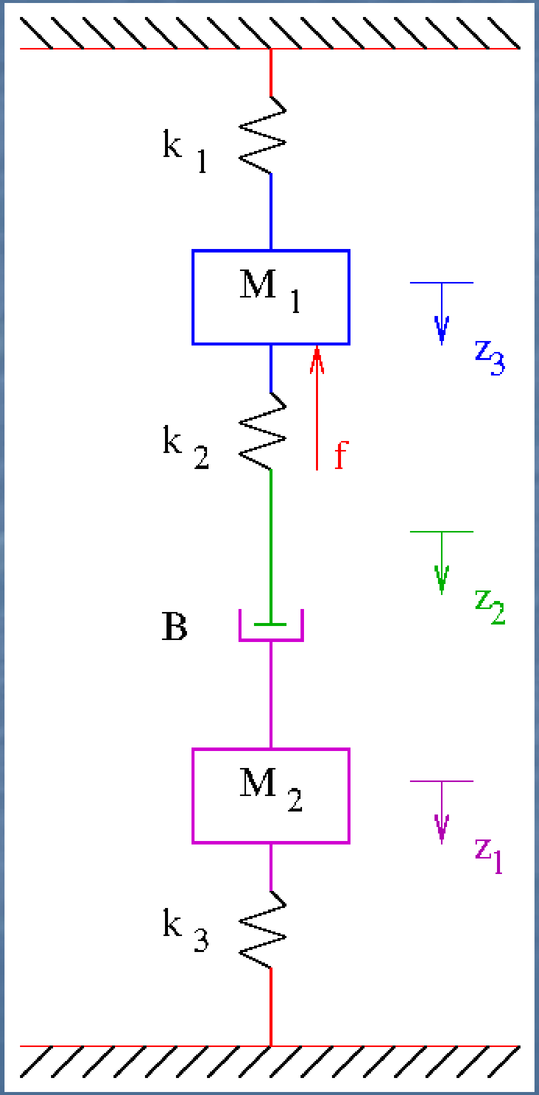
System Models -- Example

- A first step in the mathematical modeling of systems is obtaining a simplified model in terms of elementary mechanical and electrical components.



Writing the Equations …

- Identify the rigid parts of the system.
- Associate a displacement variable to each moving part.
  - For simplicity, use the same direction for all variables.
- Each displacement variable will have one equation.



Writing the Equations …

- Method 1: Use free body diagrams.
- Method 2:
  - Write weights and external forces on one side of the equation and the remaining forces on the other side.

$$M_2 g = \quad M_2 \ddot{z}_1 \quad k_3 z_1 \quad B(\dot{z}_2 - \dot{z}_1)$$

  - Select arbitrarily the sign of one of the terms.

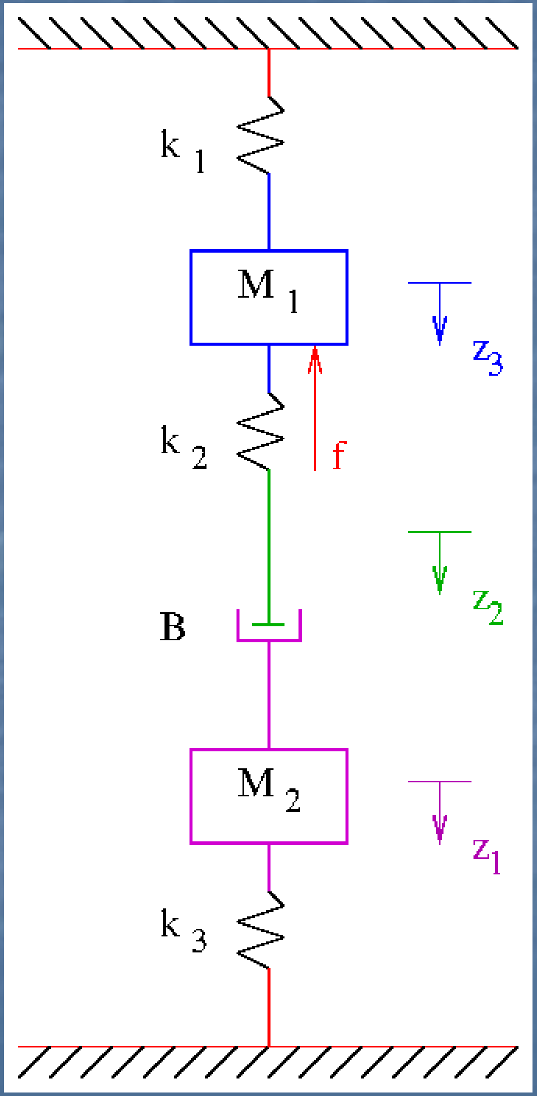
$$+M_2 g = \quad M_2 \ddot{z}_1 \quad k_3 z_1 \quad B(\dot{z}_2 - \dot{z}_1)$$

  - The other signs must be consistent with the force and displacement directions. *$M_2 g$ and z_1 in the same direction imply:*

$$+M_2 g = +M_2 \ddot{z}_1 \quad k_3 z_1 \quad B(\dot{z}_2 - \dot{z}_1)$$

  - In the equation of z, all terms in z must have same sign (stability). *Here, in the equation of z_1, all terms in z_1 (including its derivatives) must have the same sign.*

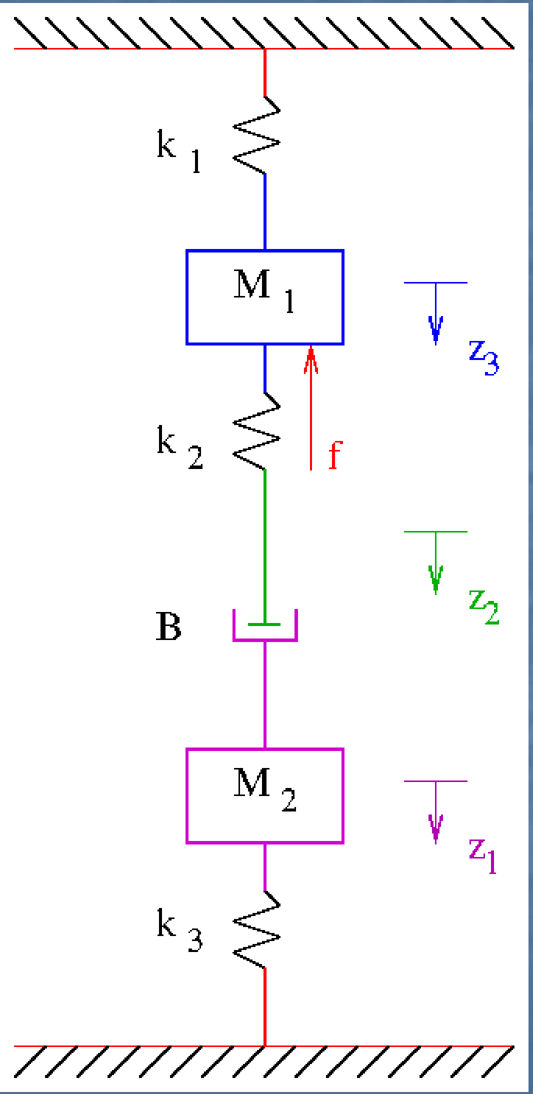
$$+M_2 g = +M_2 \ddot{z}_1 + k_3 z_1 - B(\dot{z}_2 - \dot{z}_1)$$



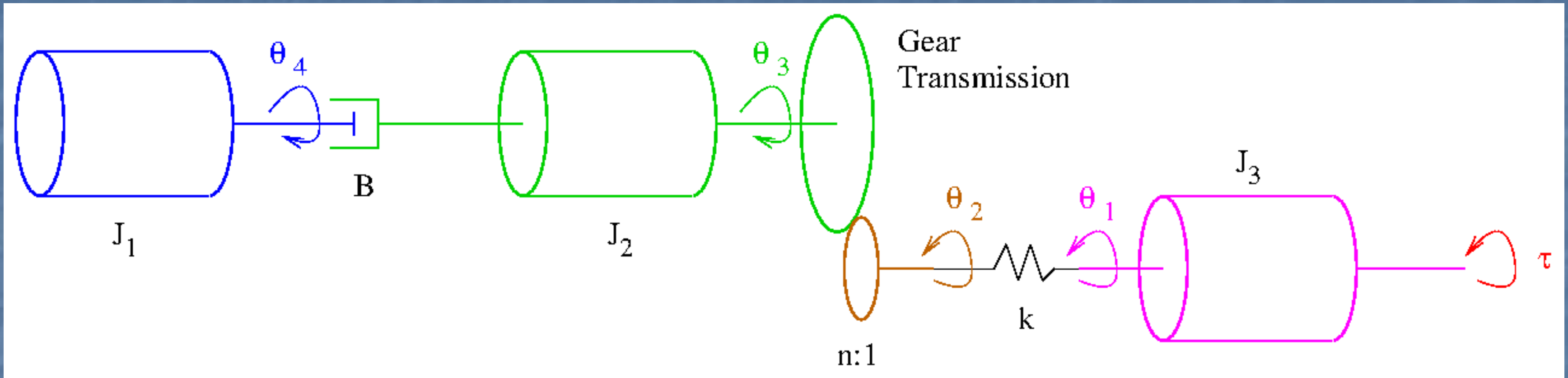
Writing the Equations …

- There will be one equation for each displacement variable:

$$
\begin{aligned}
+M_1 g - f &= M_1 \ddot{z}_3 + k_1 z_3 + k_2(z_3 - z_2) \\
0 &= +B(\dot{z}_2 - \dot{z}_1) - k_2(z_3 - z_2) \\
+M_2 g &= +M_2 \ddot{z}_1 + k_3 z_1 - B(\dot{z}_2 - \dot{z}_1)
\end{aligned}
$$

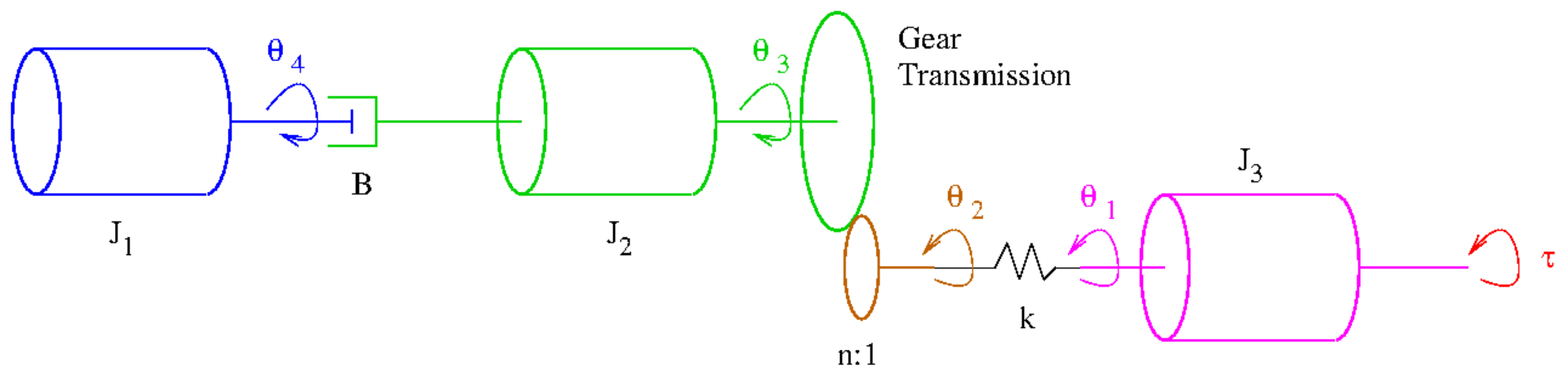


Rotatory Systems



- The same methods apply:
  - $z \leftrightarrow \theta,\ M \leftrightarrow J,\ f \leftrightarrow \tau$
- Identify the rigid parts of the system.
- Associate a displacement variable to each moving part.
  - Preferably, all displacements should have the same sign.
- Each displacement variable will have one equation.

Writing the Equations …



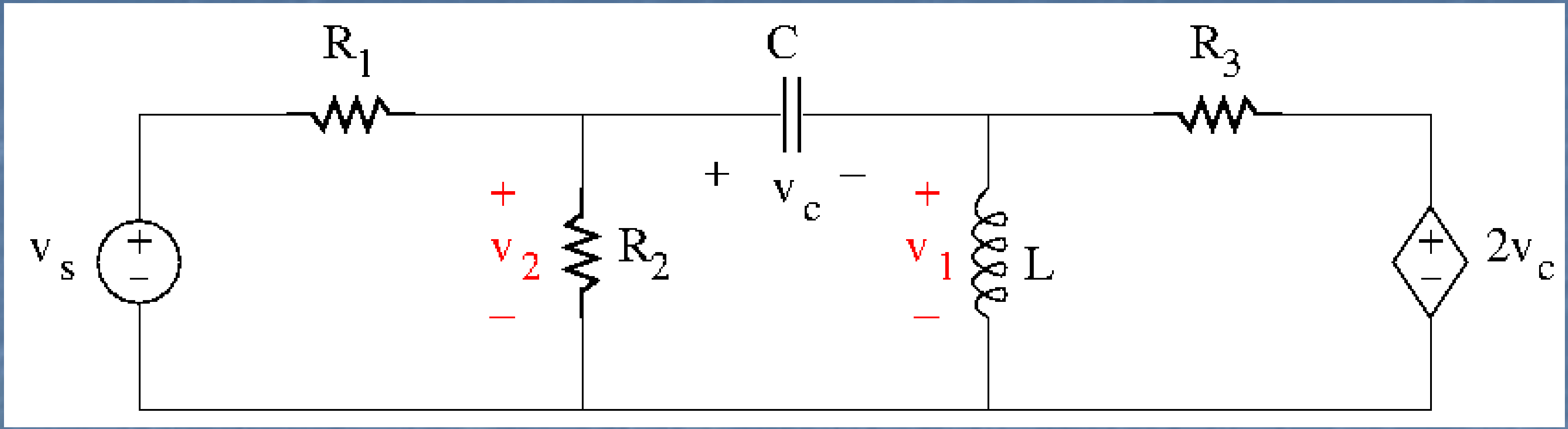
- There will be one equation for each displacement variable:

$$0 = J_1\ddot{\theta}_4 - B(\dot{\theta}_3 - \dot{\theta}_4)$$
$$0 = J_2\ddot{\theta}_3 + B(\dot{\theta}_3 - \dot{\theta}_4) + nk(\theta_2 - \theta_1)$$
$$\theta_2 = n\theta_3$$
$$\tau = J_3\ddot{\theta}_1 + k(\theta_1 - \theta_2)$$

The gear ratio n modifies both torque and displacement.

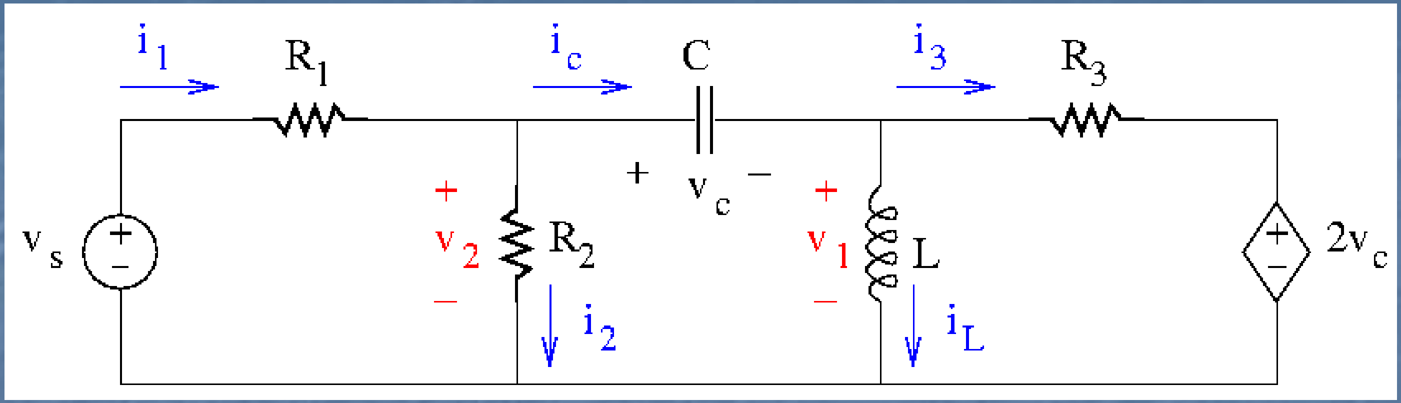
- In the equation of θ all terms in θ must have the same sign.
  - In the equation of θ_3 all terms in θ_3 have the same sign. This remains true after substituting $\theta_2 = n\theta_3$.

Electrical Systems



- Nodal analysis recommended.
  - Select reference node.
  - Mark unknown nodal voltages.
  - Write KCL for each node of unknown voltage.
  - Substitute the current of each circuit element using the element equation.

Writing the Equations …



- Write KCL for each node of unknown voltage.

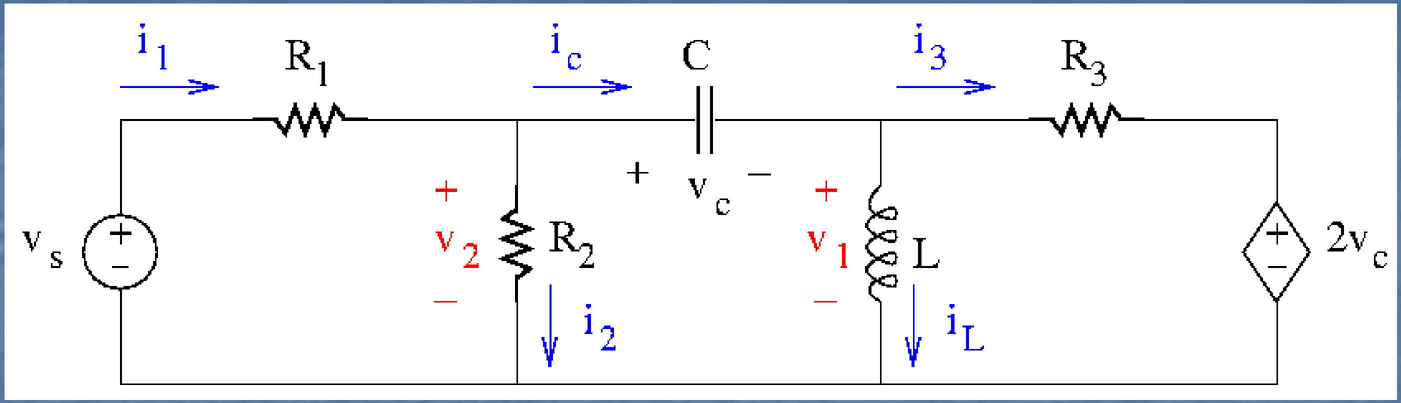
$$i_1 = i_2 + i_c \qquad i_c = i_L + i_3$$

- Substitute the current of each circuit element using the element equation.

$$\overbrace{\frac{v_s - v_2}{R_1}}^{i_1} = \overbrace{\frac{v_s}{R_2}}^{i_2} + \overbrace{C(\dot{v}_2 - \dot{v}_1)}^{i_c}$$

$$\underbrace{C(\dot{v}_2 - \dot{v}_1)}_{i_c} = \underbrace{i_L(0) + \frac{1}{L}\int_0^t v_1(x)dx}_{i_L} + \underbrace{\frac{v_1 - 2\overbrace{(v_2 - v_1)}^{v_c}}{R_3}}_{i_3}$$

Signs ...



- In the equation of the node of voltage v, before substituting dependent source expressions, if all terms in v are written on the same side of the equation, they have the same sign.

$$0 = \overbrace{\frac{v_s}{R_2}}^{i_2} + \overbrace{C(\dot{v}_2 - \dot{v}_1)}^{i_c} - \overbrace{\frac{v_s - v_2}{R_1}}^{i_1}$$

$$0 = \underbrace{i_L(0) + \frac{1}{L}\int_0^t v_1(x)dx}_{i_L} + \underbrace{\frac{v_1 - 2v_c}{R_3}}_{i_3} - \underbrace{C(\dot{v}_2 - \dot{v}_1)}_{i_c}$$

Laplace Domain Equations

- Models → systems of differential equations.
- Differential equations can be studied in the Laplace domain.
- The Laplace transform substitutes functions of time f(t) with functions F(s) that depend on the Laplace variable s.
- With zero initial conditions, the following substitutions apply.

TIME DOMAIN	LAPLACE DOMAIN
$\int_0^t z(x)dx$	$s^{-1}Z(s)$
$z(t)$	$Z(s)$
$\frac{dz}{dt}$	$sZ(s)$
$\frac{d^2z}{dt^2}$	$s^2Z(s)$
$\frac{d^nz}{dt^n}$	$s^nZ(s)$

Laplace Domain Equations

- Example:

$$f = M\ddot{z} + b\dot{z} + kz$$

Write the equation in the Laplace domain (a) in terms of z; (b) in terms of the velocity v. Assume zero initial conditions.

- Solution:

(a)

$$F(s) = Ms^2Z(s) + bsZ(s) + kZ(s)$$

(b) Since $v = \frac{dz}{dt}$ it must be that $V(s) = sZ(s)$. Therefore:

$$F(s) = MsV(s) + bV(s) + ks^{-1}V(s)$$

Laplace Domain Equations

- With nonzero initial conditions, the following substitutions apply.

TIME DOMAIN	LAPLACE DOMAIN
$\int_0^t z(x)dx$	$s^{-1}Z(s)$
$z(t)$	$Z(s)$
$\frac{dz}{dt}$	$sZ(s) - z(0^-)$
$\frac{d^2z}{dt^2}$	$s^2Z(s) - \frac{dz}{dt}(0^-) - s \cdot z(0^-)$
$\frac{d^nz}{dt^n}$	$s^nZ(s) - \frac{d^{n-1}z}{dt^{n-1}}(0^-) - s \cdot \frac{d^{n-2}z}{dt^{n-2}}(0^-) - \cdots - s^{n-1} \cdot z(0^-)$

Laplace Domain Equations

- Example:

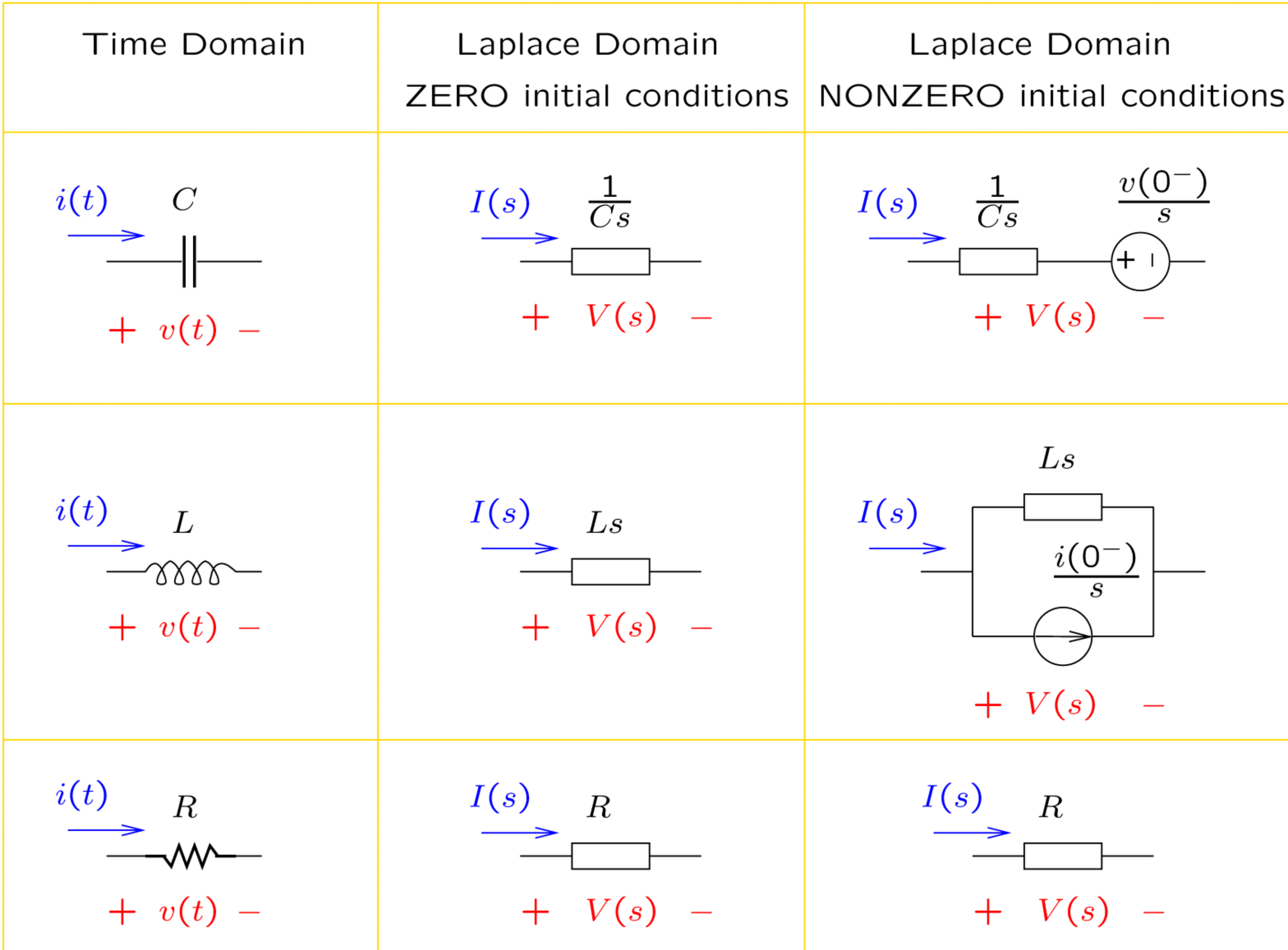
$$f = M\ddot{z} + b\dot{z} + kz$$

Write the equation in the Laplace. Assume <u>nonzero</u> initial conditions.

- Solution:

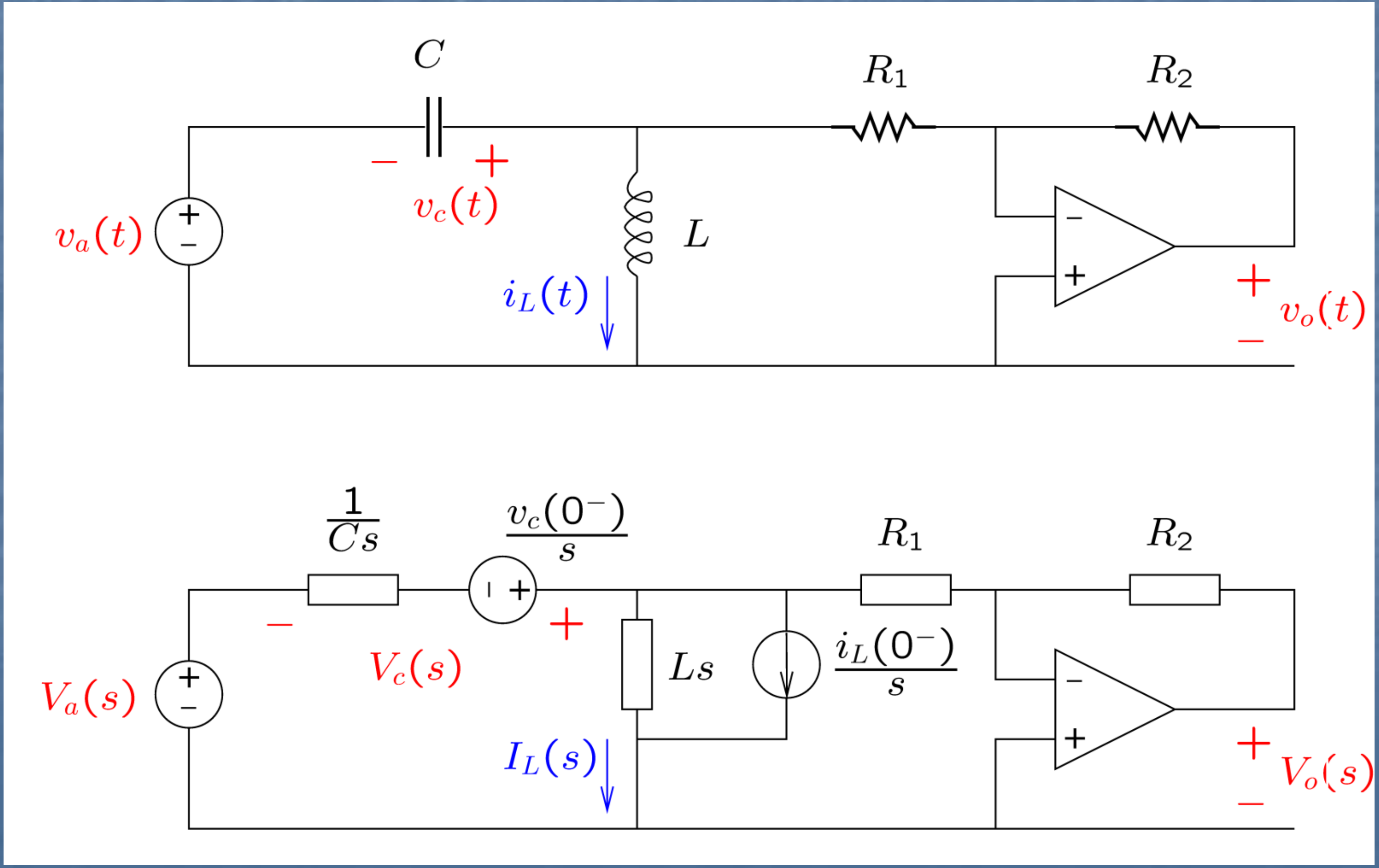
$$F(s) = Ms^2Z(s) - M\dot{z}(0^-) - Msz(0^-) + bsZ(s) - bz(0^-) + kZ(s)$$

Circuits in the Laplace Domain



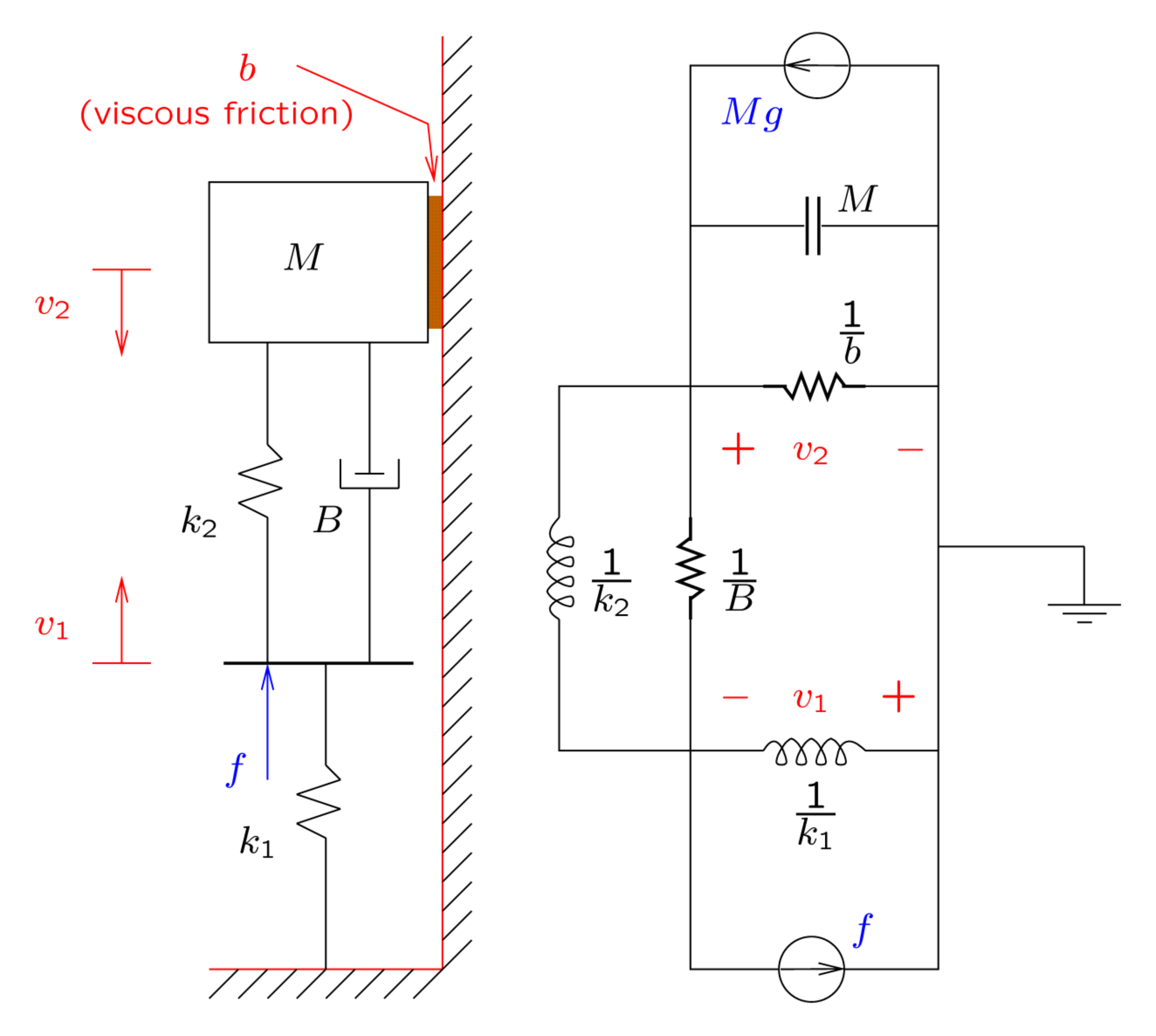
Impedance Diagrams

- Impedance diagrams represent electric circuits in the Laplace domain.



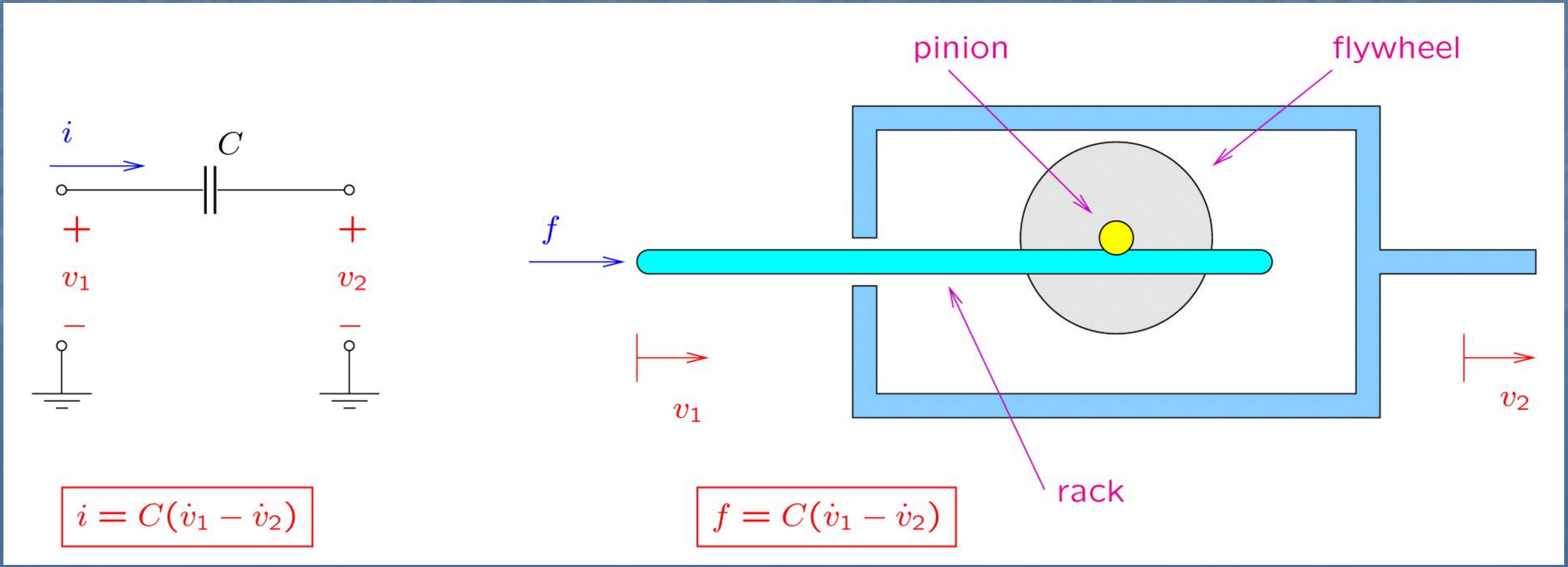
Analogies

- Electric analogy of mechanical systems:
  - M → C
  - k → 1/L
  - B → 1/R
  - f → i
  - v → v.



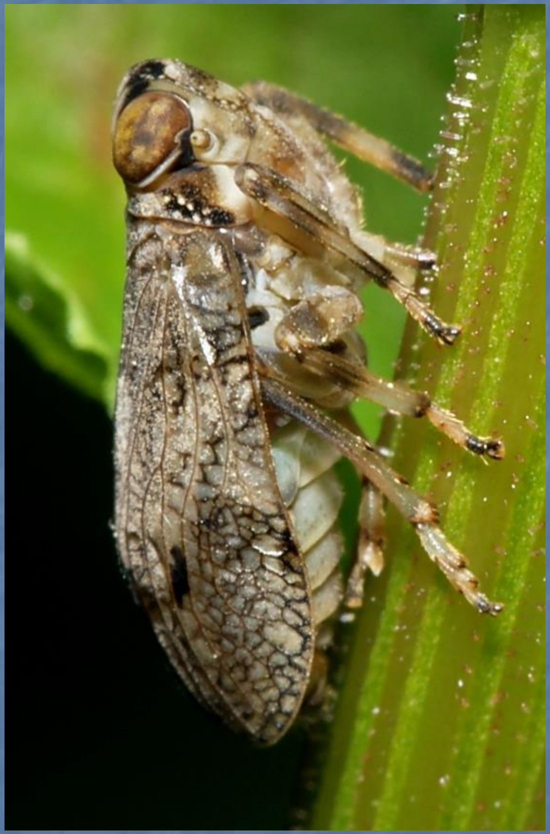
Impedance Diagrams

- Impedance diagrams applied also to mechanical systems, using the electric system analogy:
  M → C, k → 1/L, B → 1/R, f → i, and v → v.
- A block is represented by a ground connected capacitor.
- Mechanical equivalent of an ungrounded capacitor: the inerter (invented by Prof. Malcolm C. Smith).

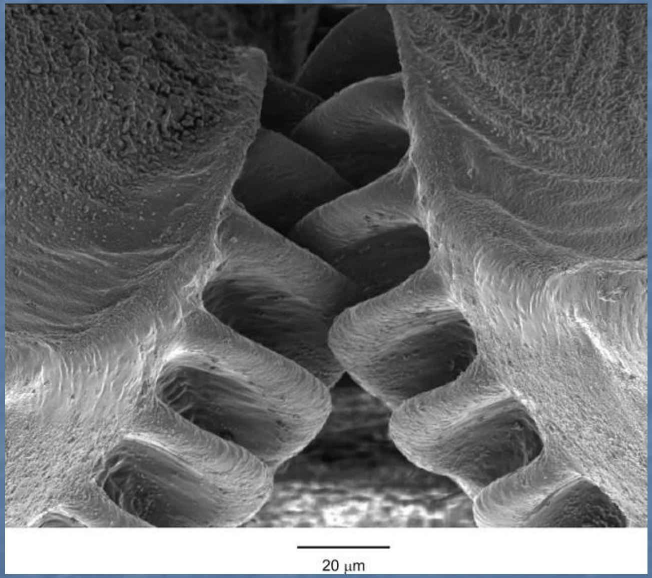


Motion Mechanisms

- Gear transmission in creation.



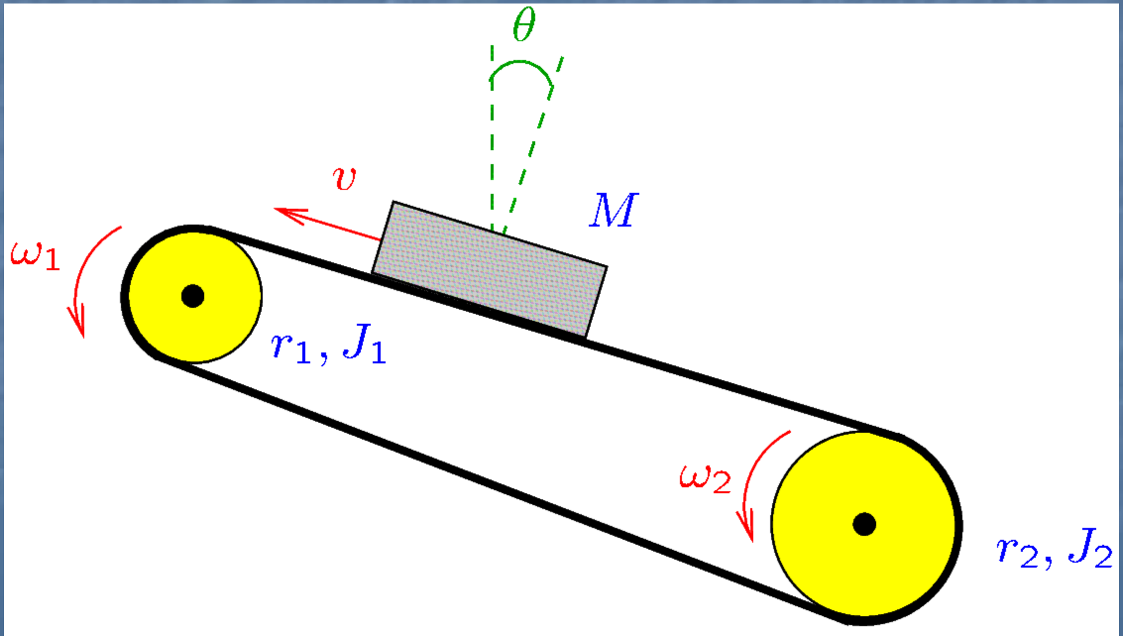
Sarefo, Issus.coleoptratus.1, CC BY-SA 4.0



University of Cambridge (Profs. Malcolm Burrows & Gregory Sutton), Interactive gears in the hind legs of Issus coleoptratus from Cambridge gears-3, CC BY-SA 3.0

Motion Mechanisms

- Can be solved by considering the power balance.
- Example: Consider the following conveyor mechanism.
  - The motor is connected to the first pulley by means of a reducer (gearbox) of ratio n.
  - The combined inertia of the motor and gearbox is J_m.
  - The mass of the belt is m.



Motion Mechanisms

Let τ_m and ω_m be the motor torque and velocity. Let $G = Mg$.

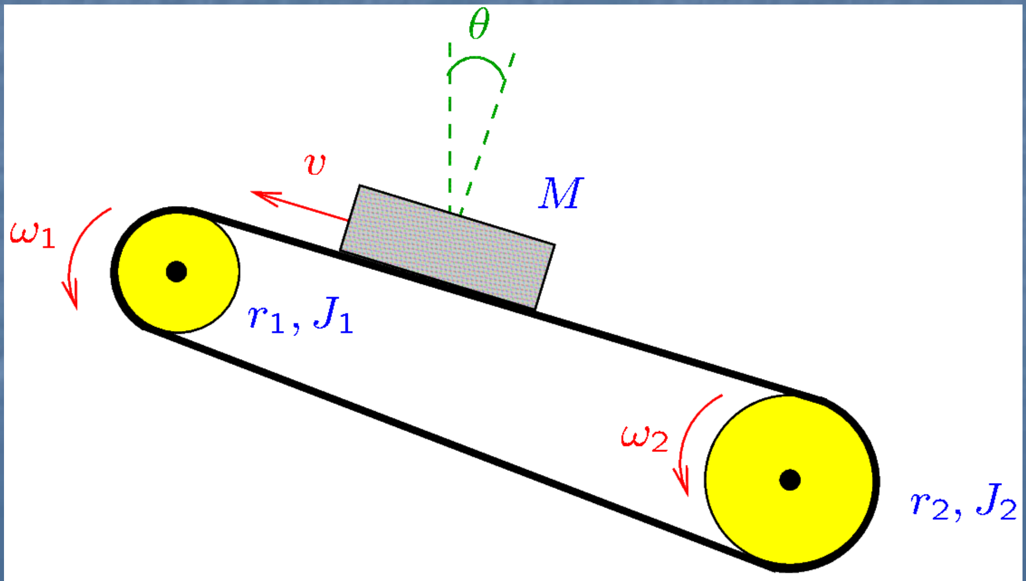
$$\tau_m \omega_m = J_m \dot{\omega}_m \omega_m + (M+m)\dot{v}v + J_1 \dot{\omega}_1 \omega_1 + J_2 \dot{\omega}_2 \omega_2 + vG \sin\theta$$

Substituting $\omega_m = n\omega_1,\ \ \omega_2 = \frac{r_1}{r_2}\omega_1$, and $v = r_1\omega_1$:

$$\tau_m = J_t \dot{\omega}_m + Mgr_1 \frac{1}{n} \sin\theta$$

where

$$J_t = J_m + \frac{(M+m)r_1^2 + J_1}{n^2} + \frac{J_2 r_1^2}{r_2^2 n^2}$$



Motion Mechanisms

Suppose the mechanism has an efficiency η.

The power balance equation becomes:

$$\tau_m \omega_m = J_m \dot{\omega}_m \omega_m + \frac{1}{\eta} \cdot [(M+m)\dot{v}v + J_1 \dot{\omega}_1 \omega_1 + J_2 \dot{\omega}_2 \omega_2 + vG \sin\theta]$$