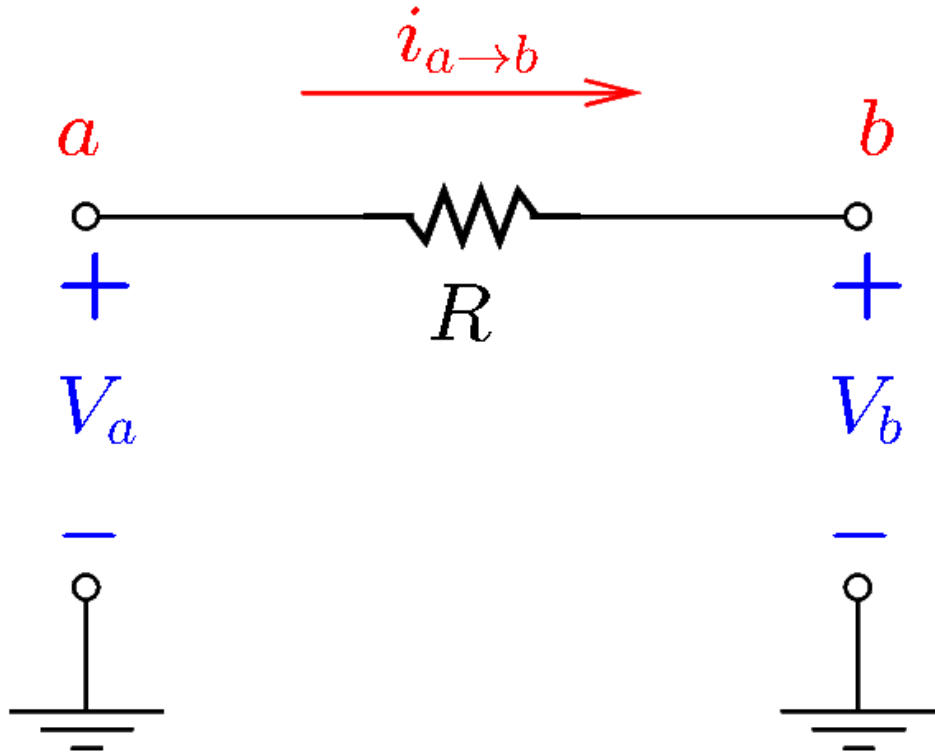


Nodal Analysis

A General Method for Solving Circuits

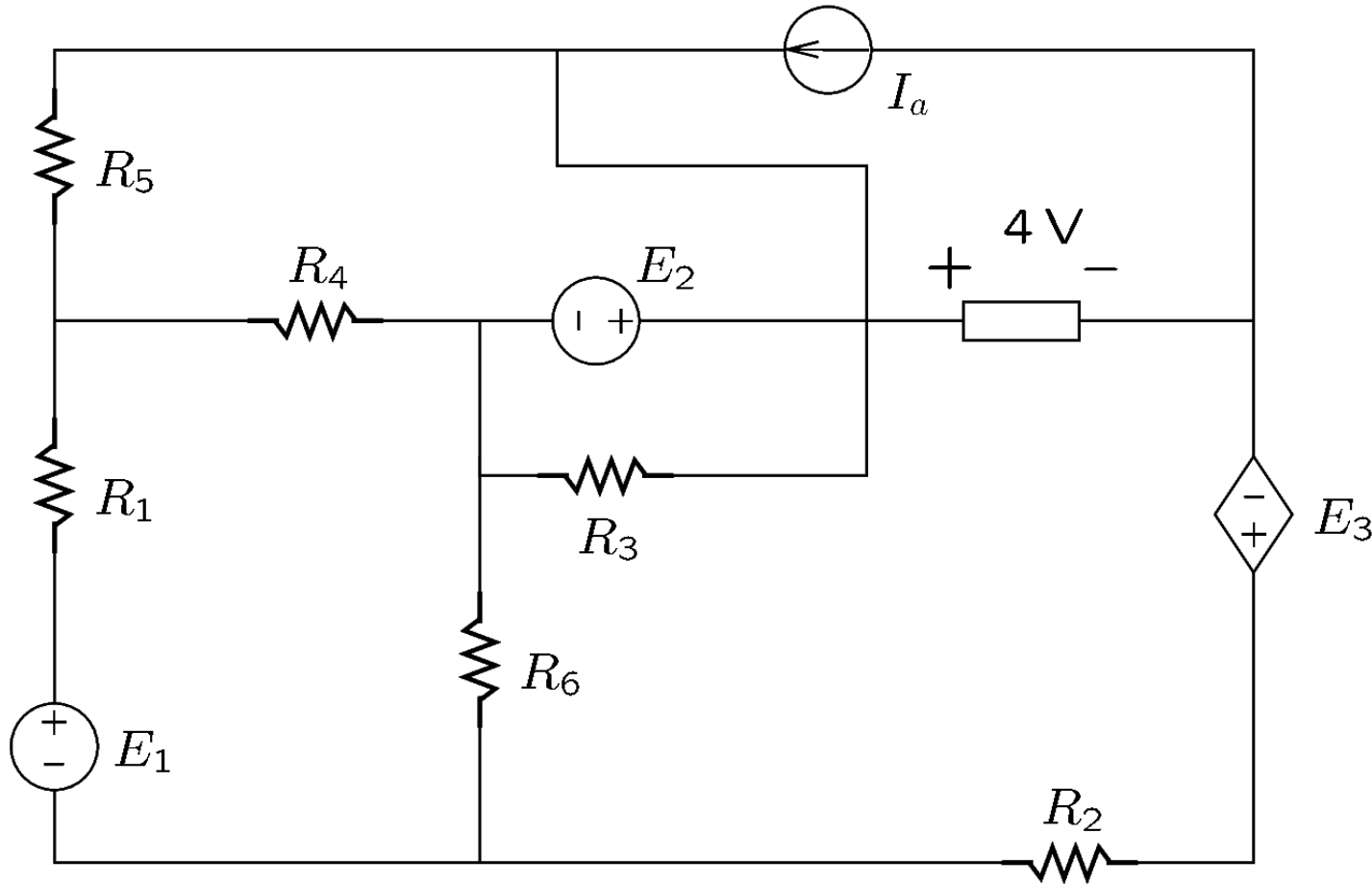
An important formula

Nodal equations can be written quickly if you remember this formula:



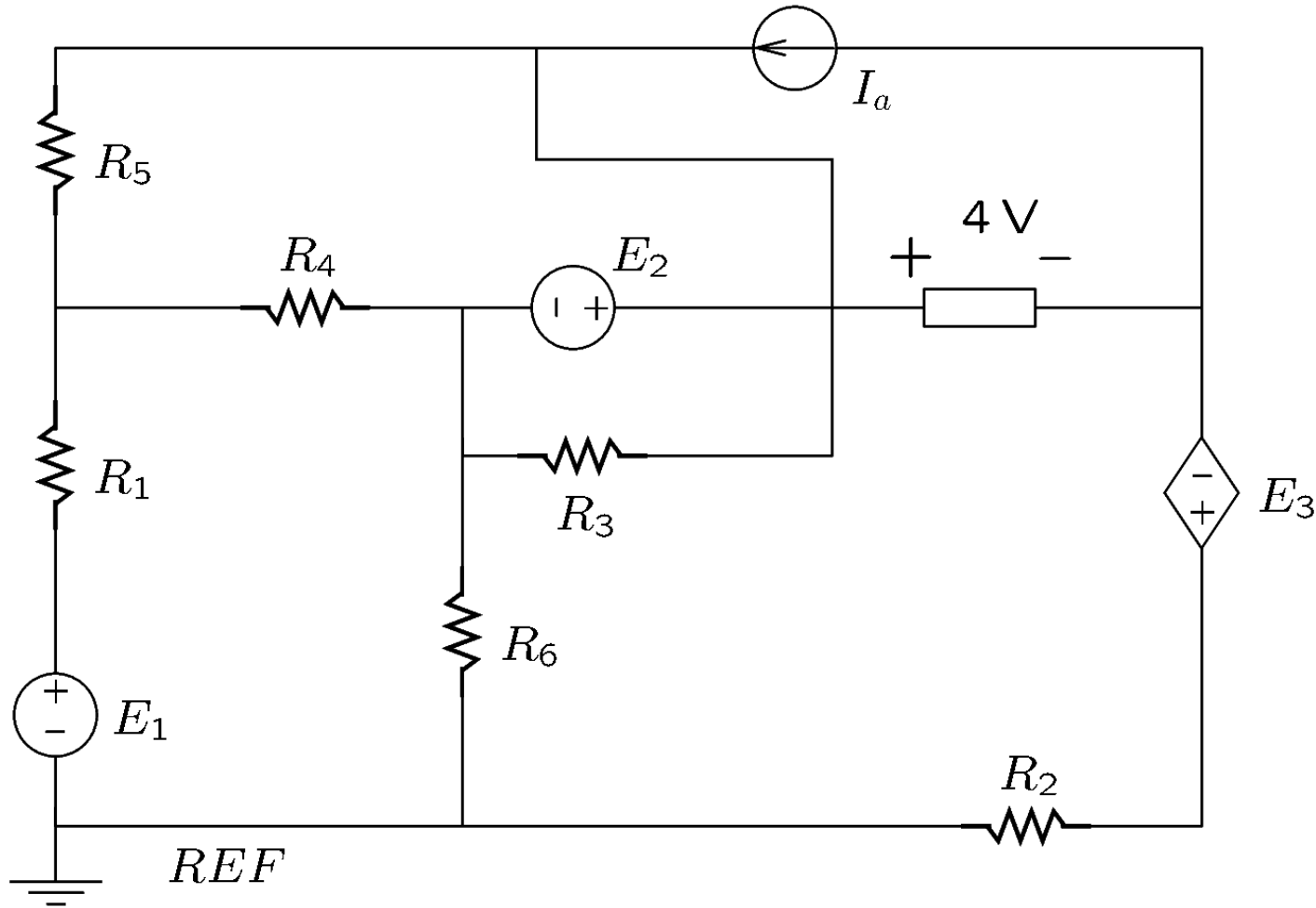
$$i_{a \rightarrow b} = \frac{V_a - V_b}{R}$$

Select a reference node



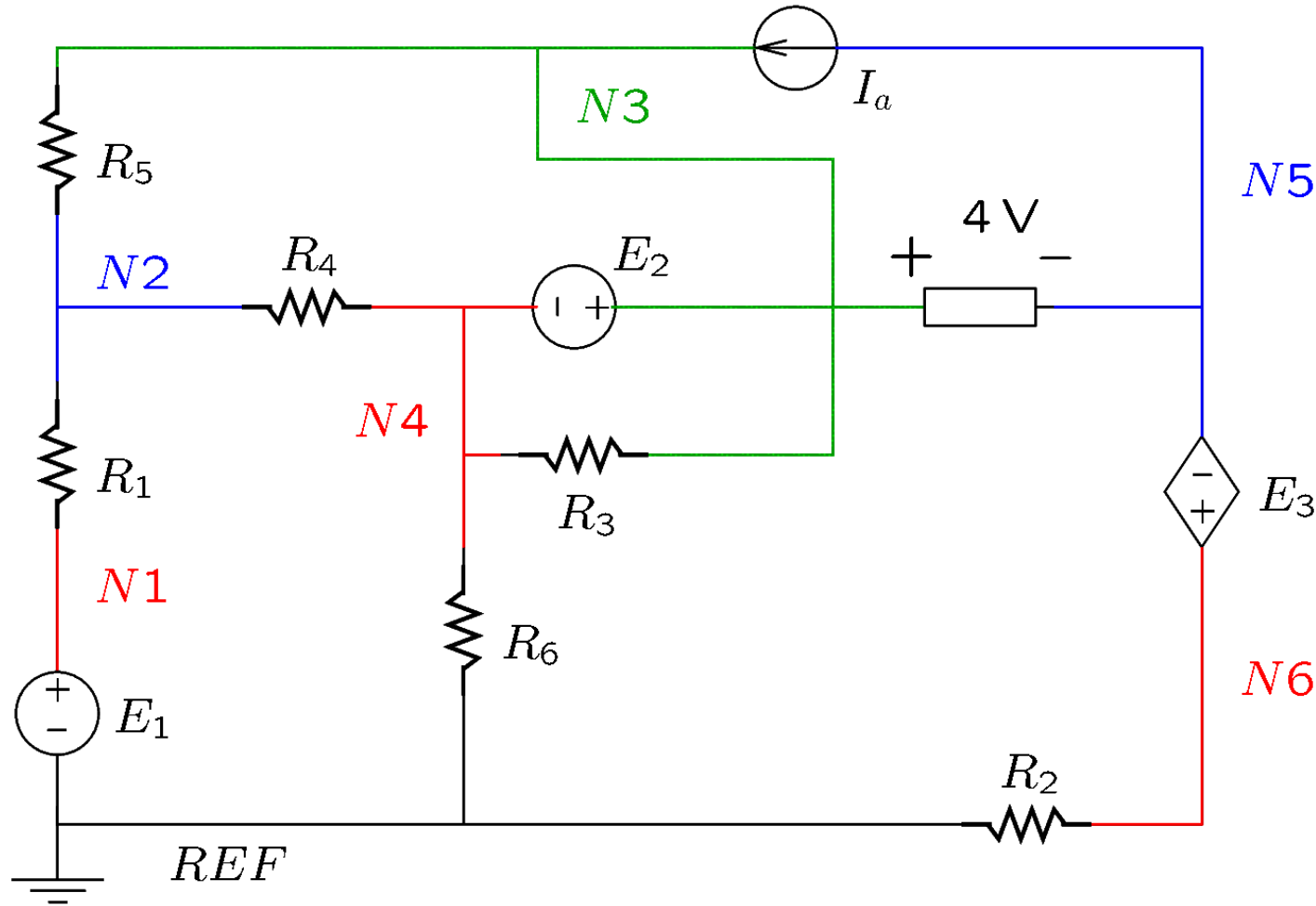
- Any node may serve as a reference node.
- The choice is arbitrary.
- Nodal analysis finds node voltages with respect to the reference node.
- A reference node will be denoted by REF, or by the electric-ground symbol.

Mark the reference node



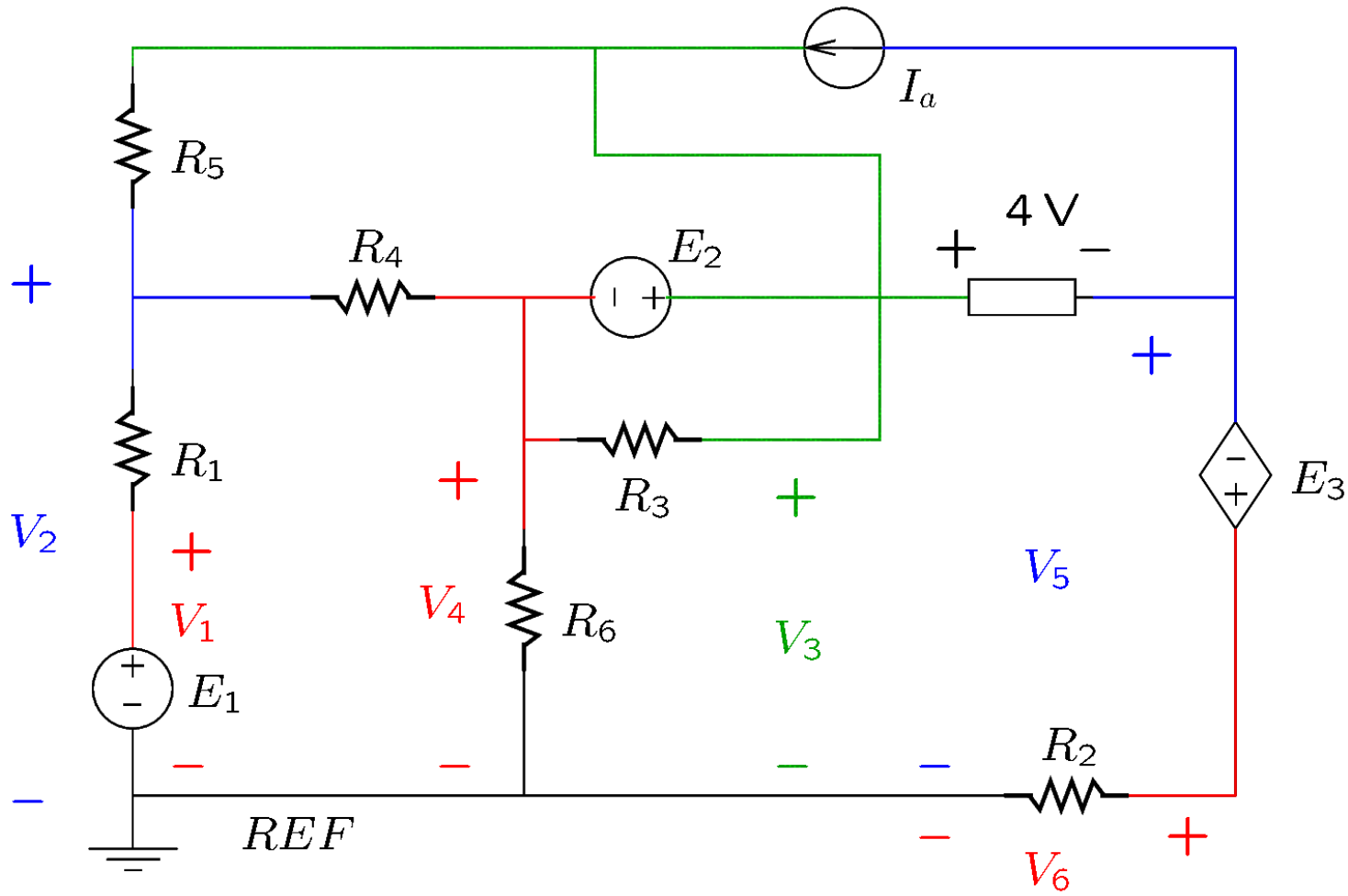
The node at the bottom left corner was chosen as the reference.

Identify the nodes of the circuit



The nodes are $N1 \dots N6$ and REF .

Identify nodal voltages



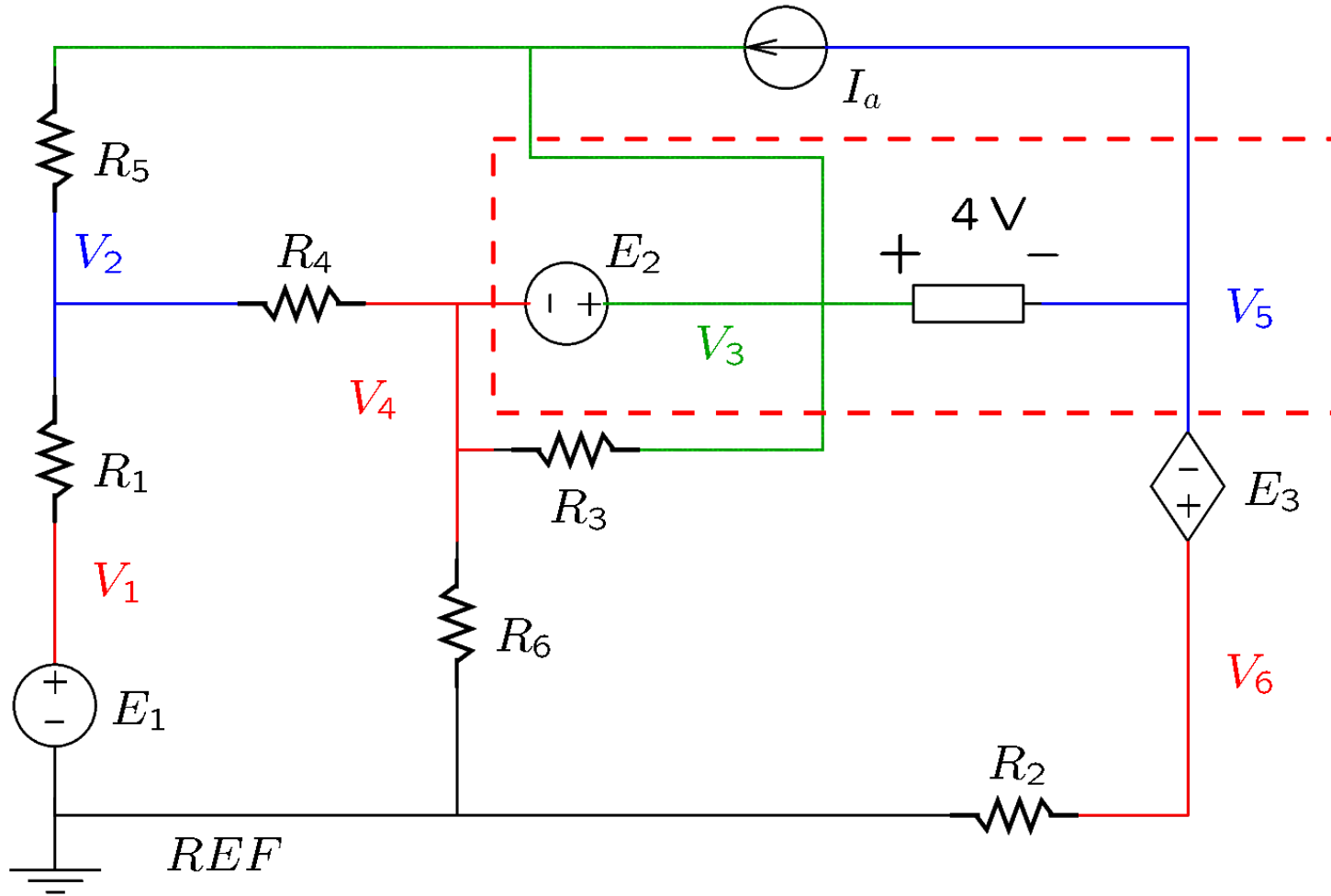
Nodal analysis solves for node voltages with respect to REF.

Identify the supernodes

A **supernode** is a network in which every node is connected to some other node by a source of voltage or by a component of known voltage.

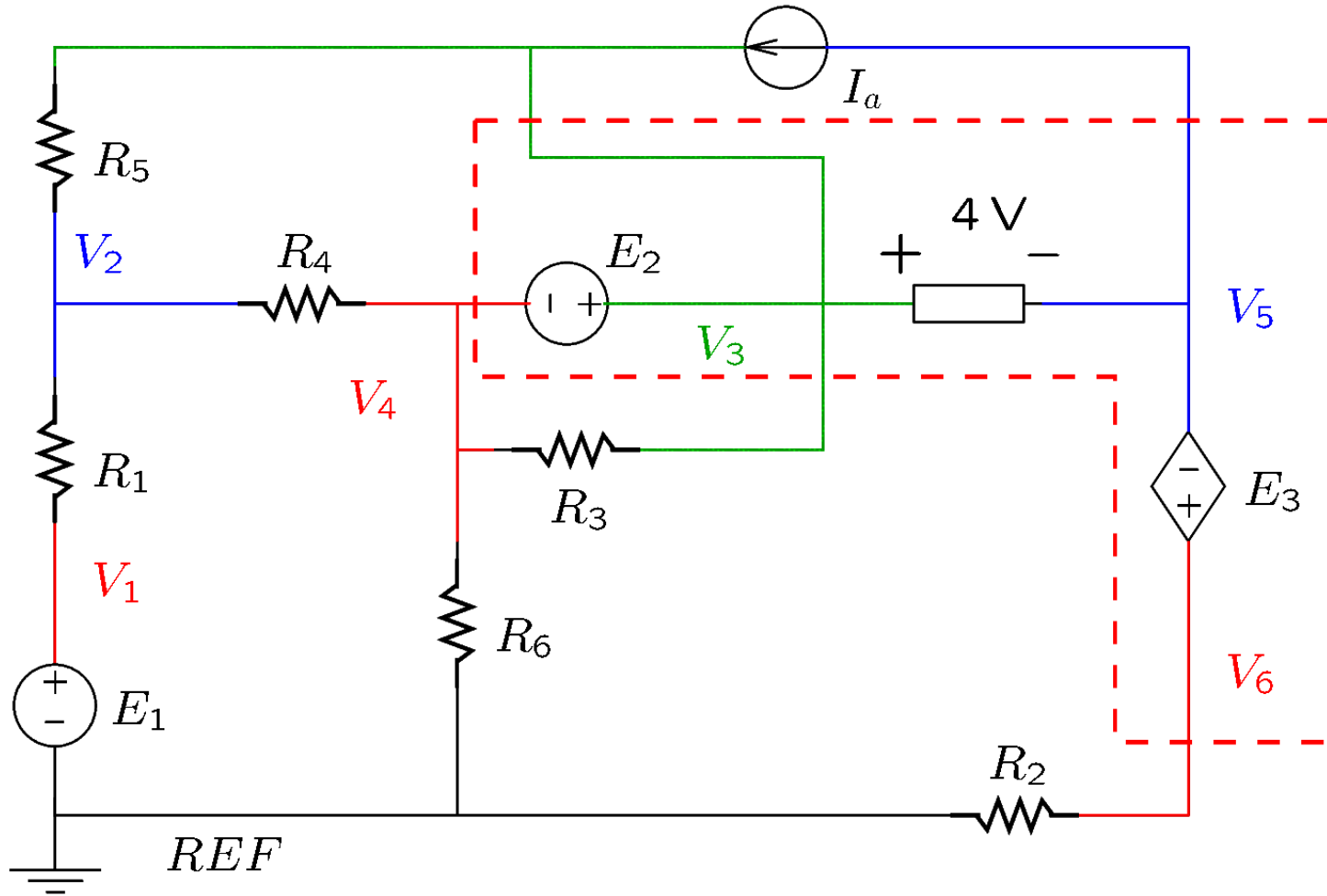
- A path must exist between any two nodes that goes only through branches containing sources of voltage or components of known voltage.
- The network must be **maximal**; do not leave outside of the supernode a node that is connected to it by a source of voltage or a component of known voltage.
- Include into the network all components connected between any two nodes of the supernode.

Supernodes



This network is not a supernode: it does not include node 6, so it is not maximal.

Supernodes

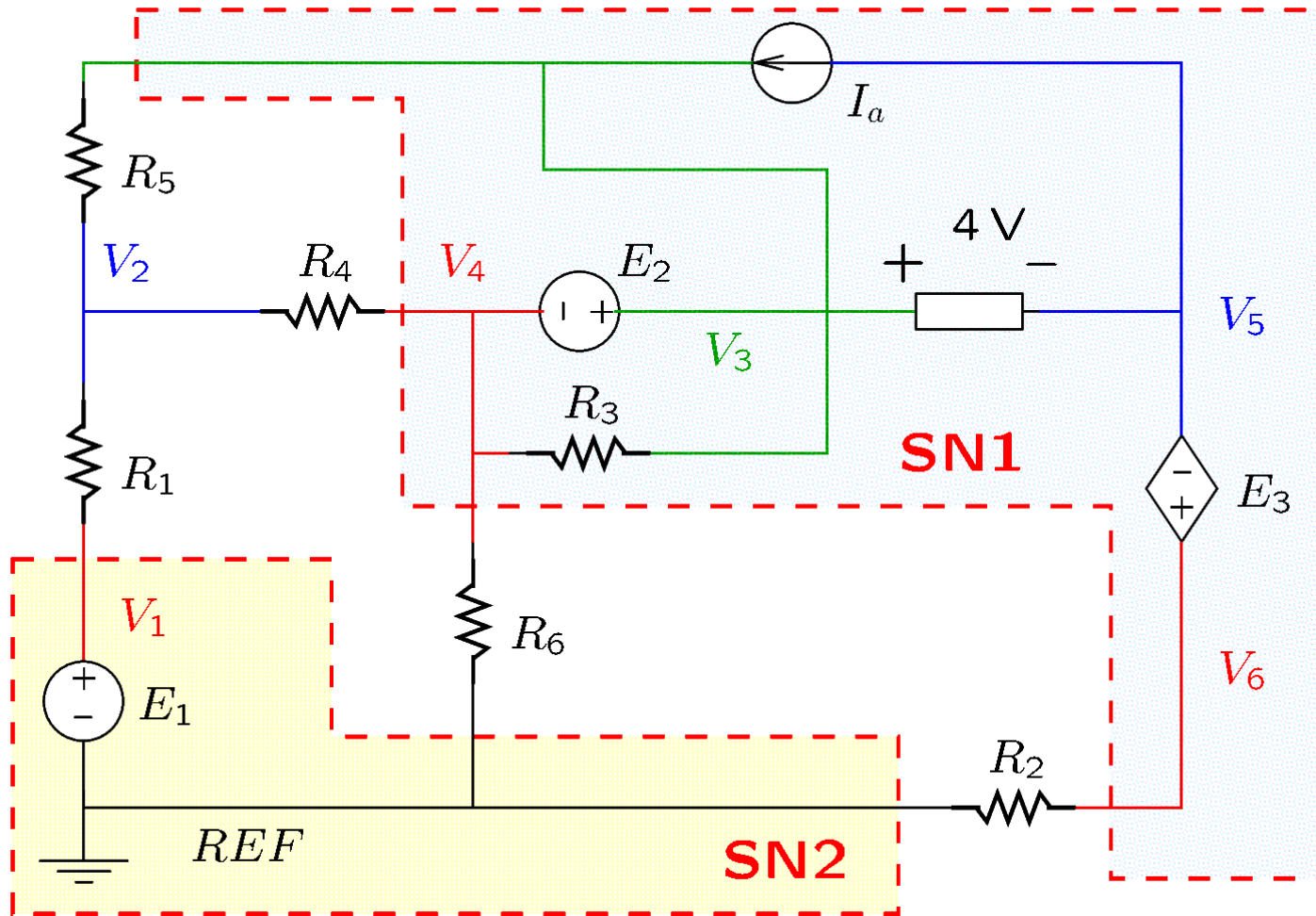


This is not the best way to define the supernode: it does not include R_3 and I_a .

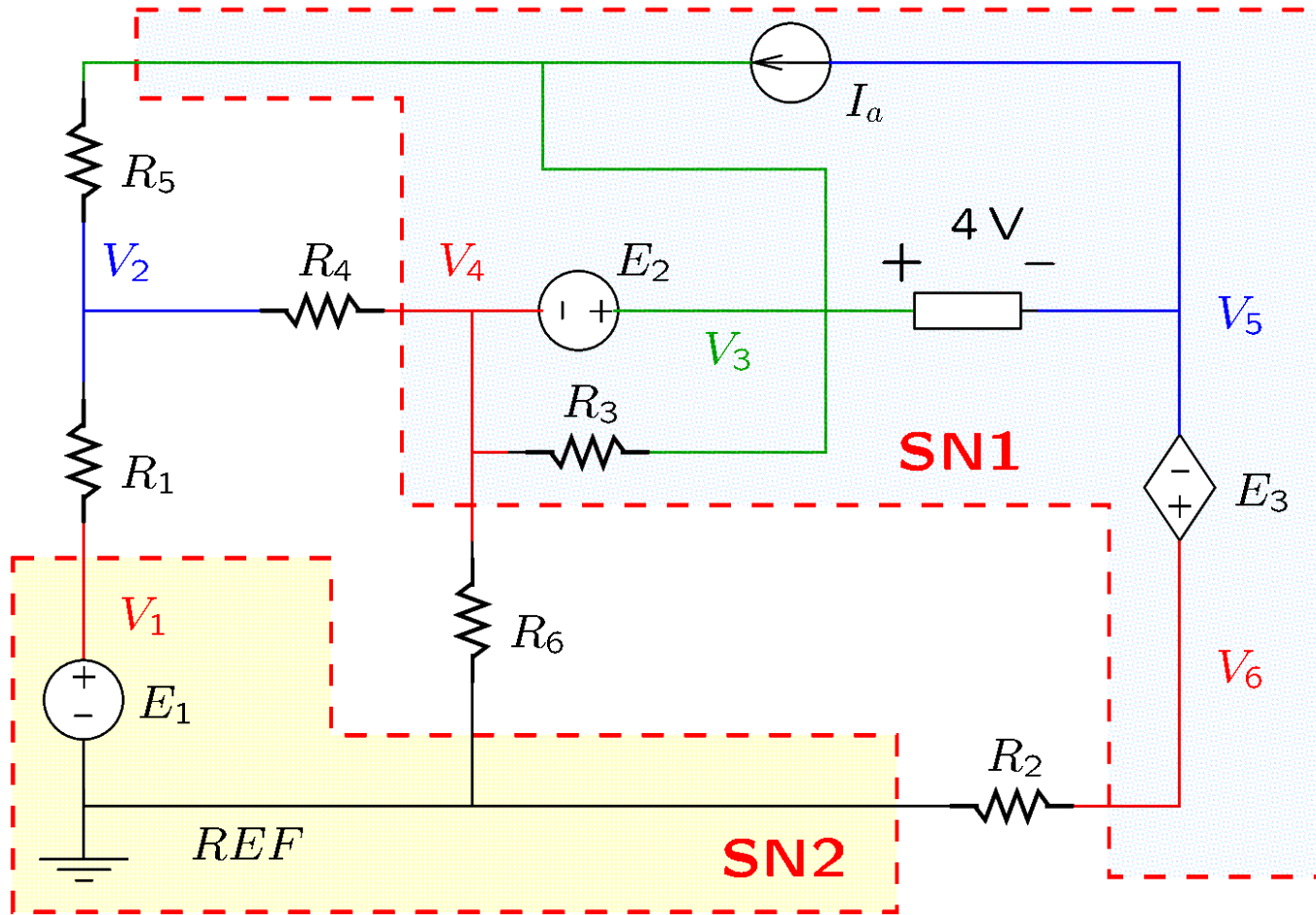
It is best to include the components connected between the nodes of the network.

Supernodes

The circuit has two supernodes:
SN1 and SN2.



Supernode voltages



- For each supernode, select a node n .
- Let V_n be the voltage of node n .
- Express the voltage of every other node of the supernode in terms of V_n (by KVL).

Example: Select node 5 for SN1.

$$V_3 = V_5 + 4,$$

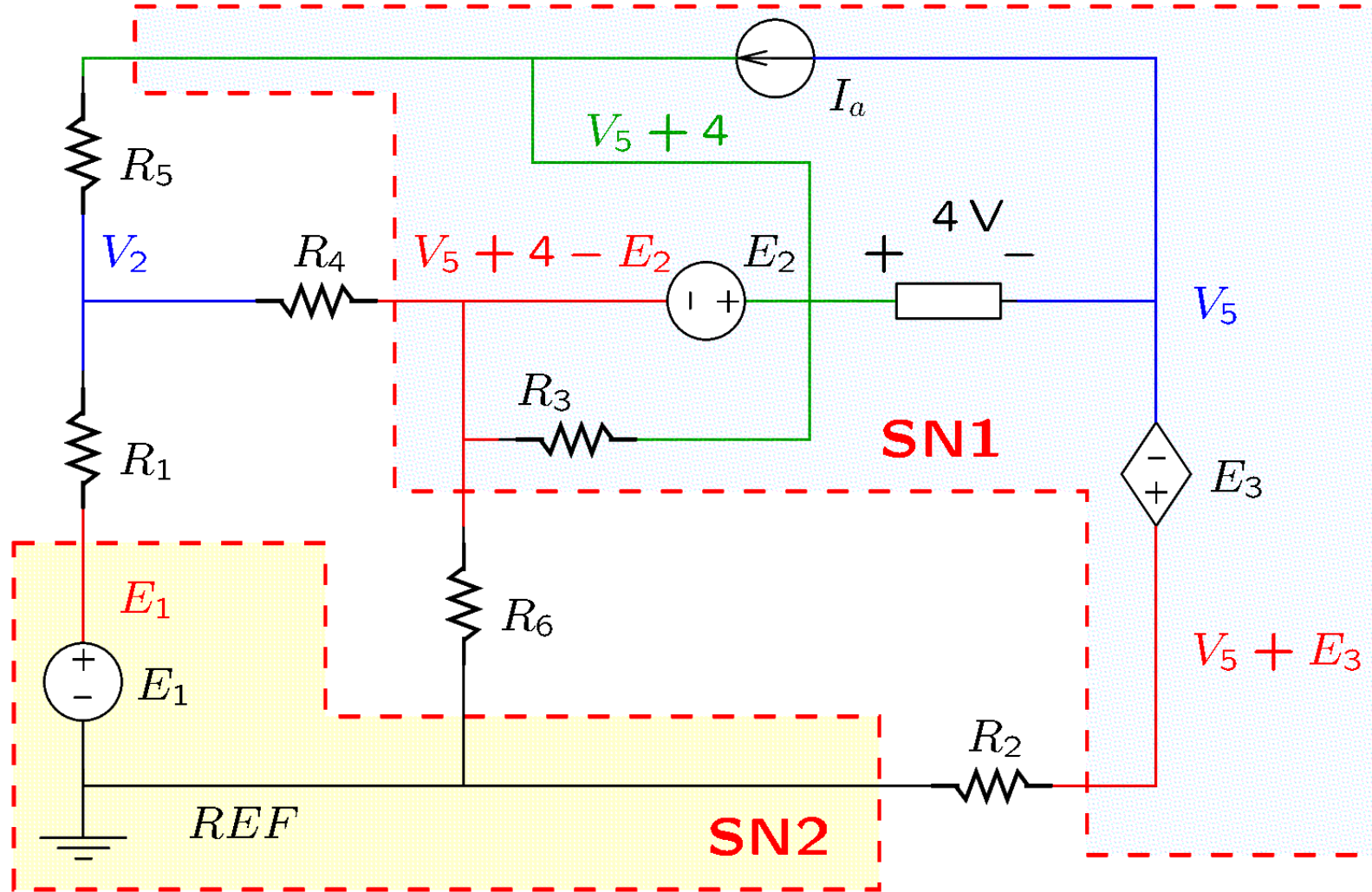
$$V_4 = V_5 + 4 - E_2,$$

$$V_6 = V_5 + E_3.$$

Example: Select REF for SN2

$$V_1 = E_1.$$

Supernode voltages



Write KCL

- Write KCL for every supernode that does not contain the REF node.
- Write KCL for every node that is not part of a supernode and that is not the REF node.

KCL for SN1:

$$\frac{V_5 + 4 - V_2}{R_5} + \frac{V_5 + 4 - E_2 - V_2}{R_4} + \frac{V_5 + 4 - E_2}{R_6} + \frac{V_5 + E_3}{R_2} = 0$$

KCL for node 2:

$$\frac{V_2 - E_1}{R_1} + \frac{V_2 - (V_5 + 4)}{R_5} + \frac{V_2 - (V_5 + 4 - E_2)}{R_4} = 0$$