Review—Part 2

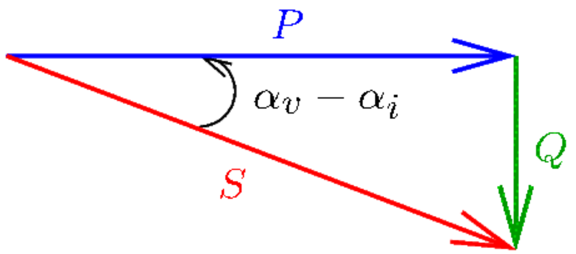
Example 1

A three-phase transformer steps down voltage to a Δ-connected load. The load operates at 30 kW and a power factor of 0.8 leading. The primary of the transformer is Δ-connected and the secondary is Y-connected. The turns ratio is a = 0.1. The line voltage of the primary is $2\ kV$ rms. Find the impedance of the load, per phase, and the line current drawn by the primary of the transformer.

- *Since the primary is Δ-connected, the phase voltage of the primary equals the line voltage of the primary:* $V_{p,phase} = V_{p,line} = 2\ kV\ rms$.
- *The phase voltage of the secondary is:* $V_{s,phase} = V_{p,phase} \cdot a = 200\ V\ rms$.
- *Since the secondary is Y-connected, the line voltage of the secondary is:* $V_{s,line} = \sqrt{3}V_{s,phase} = 200\sqrt{3}\ V\ rms$.
- *The load is connected to the secondary, so the secondary and the load have the same line voltage.*
- *The load is Δ-connected, so the phase voltage of the load equals the line voltage of the load:* $V_{l,phase} = V_{s,line} = 200\sqrt{3}\ V\ rms$.

Example 1 (Continued)

- *The total power of the load being* $30\ kW$*, the power per phase is* $10\ kW$.
- *The* kW *unit implies average power.*
- *The power factor is* 0.8*. Therefore,* $\alpha_v - \alpha_i = \pm\cos^{-1} 0.8 = \pm 36.87°$.
- *Since the power factor is 0.8 leading, current leads voltage (*$\alpha_v < \alpha_i$*).*
- *Therefore,* $\alpha_v - \alpha_i = -36.87°$.
- *The apparent power is* $S = \frac{P}{\cos(\alpha_v - \alpha_i)} = 12.5\ kVA$.



- *Assuming rms values,* $S = V_{l,phase} I_{l,phase}$.
- *Therefore, the phase current of the load is*

$$I_{l,phase} = \frac{S}{V_{l,phase}} = \frac{12.5\ kVA}{200\sqrt{3}\ V\ rms} = \frac{62.5}{\sqrt{3}}\ A\ rms.$$

- *The load impedance of each phase is:*

$$\boldsymbol{Z} = \frac{\boldsymbol{V}_{l,phase}}{\boldsymbol{I}_{l,phase}} = \frac{V_{l,phase}}{I_{l,phase}} \angle \alpha_v - \alpha_i = 9.6\angle - 36.87°\ \Omega$$

Example 1 (Continued)

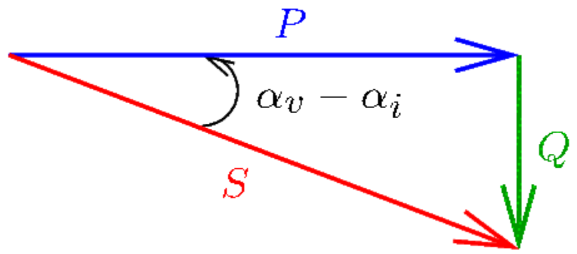
- *The phase current of the load was shown to be* $I_{l,phase} = \frac{62.5}{\sqrt{3}}\ A\ rms$.
- *Since the load is Δ-connected, the line current of the load is* $I_{l,line} = \sqrt{3} I_{l,phase} = 62.5\ A\ rms$.
- *Since the secondary of the transformer is* Y*-connected, it has the same line current:* $I_{s,line} = 62.5\ A\ rms$.
- *Since the secondary is* Y*-connected, the line and the phase currents are identical:* $I_{s,phase} = I_{s,line} = 62.5\ A\ rms$.
- *The phase current of the primary will be:* $I_{p,phase} = a \cdot I_{s,phase} = 6.25\ A\ rms$.
- *Since the primary is Δ-connected, the line current of the primary is*

$$I_{p,line} = \sqrt{3} I_{p,phase} = 10.825\ A\ rms.$$

Example 2

Indicate how the power factor can be corrected in the previous example.

- *The reactive power of the load, per phase, is* $Q = S\sin(\alpha_v - \alpha_i) = -7.5\ kVAR$.
- *To correct the power factor, we need to cancel the reactive power of the load.*



- *Only reactive elements have reactive power:*
  - *Capacitors generate reactive power.*
  - *Inductors absorb reactive power.*
- *Therefore, we need an additional inductive load of reactive power* $Q_i = -Q = +7.5\ kVAR$*, per phase.*
- *Let* L *be the inductance of each phase and* $V_{i,phase}$ *the rms phase voltage.*
- *Let* $X = L\omega$ *be the reactance of* L.

$$Q_i = \frac{V_{i,phase}^2}{X} \Rightarrow L = \frac{V_{i,phase}^2}{Q_i\omega}$$

Example 2 (Continued)

- *Note that if the inductors are Y-connected, their value will be three times smaller than if they are Δ-connected.*
- *Assuming Y-connected inductors and a $60\ Hz$ frequency:*

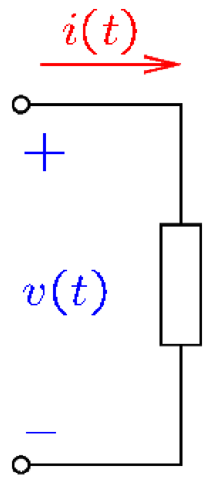
$$L = \frac{V_{i,phase}^2}{Q_i \omega} = \frac{\left(200\sqrt{3}\ V/\sqrt{3}\right)^2}{7.5\ kVAR \cdot 2\pi \cdot 60\ Hz} = 14.147\ mH$$

Remarks

- Note that the admittance $\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = G + jB$.
    - $G = Real(\boldsymbol{Y})$, the conductance, is the real part of $\boldsymbol{Y}$.
    - $B = Imag(\boldsymbol{Y})$, the susceptance, is the imaginary part of $\boldsymbol{Y}$.
- Any impedance $\boldsymbol{Z}$ can be represented as $\boldsymbol{Z} = R_p||(jX_p)$, where $R_p = \frac{1}{G}$ and $X_p = -\frac{1}{B}$.
- Any impedance $\boldsymbol{Z}$ can be represented also as $\boldsymbol{Z} = R + jX$, where
    - $R = Real(\boldsymbol{Z})$, the resistance, is the real part of $\boldsymbol{Z}$.
    - $X = Imag(\boldsymbol{Z})$, the reactance, is the imaginary part of $\boldsymbol{Z}$.

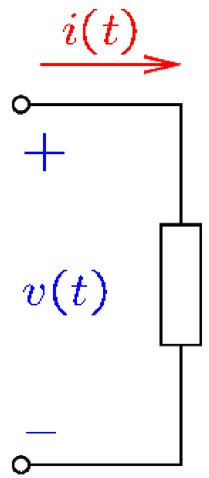
Remarks

- Let V_{rms} and I_{rms} be the rms voltage and current on an impedance $\boldsymbol{Z}$.
- If $\boldsymbol{Z} = R + jX$, note that:
  - The average power dissipated on $\boldsymbol{Z}$ is $P = RI_{rms}^2$.
  - The reactive power absorbed by $\boldsymbol{Z}$ is $Q = XI_{rms}^2$.
  - Let us note that X may be positive or negative.
- If we represent $\boldsymbol{Z}$ instead as $\boldsymbol{Z} = R_p || (jX_p)$, then:
  - The average power dissipated on $\boldsymbol{Z}$ is $P = V_{rms}^2 / R_p$.
  - The reactive power absorbed by $\boldsymbol{Z}$ is $Q = V_{rms}^2 / X_p$.
- These equations can be derived immediately from $\boldsymbol{S} = P + jQ$, $\boldsymbol{S} = \boldsymbol{V}_{rms} \boldsymbol{I}_{rms}^*$, and $\boldsymbol{V}_{rms} = \boldsymbol{Z} \boldsymbol{I}_{rms}$.

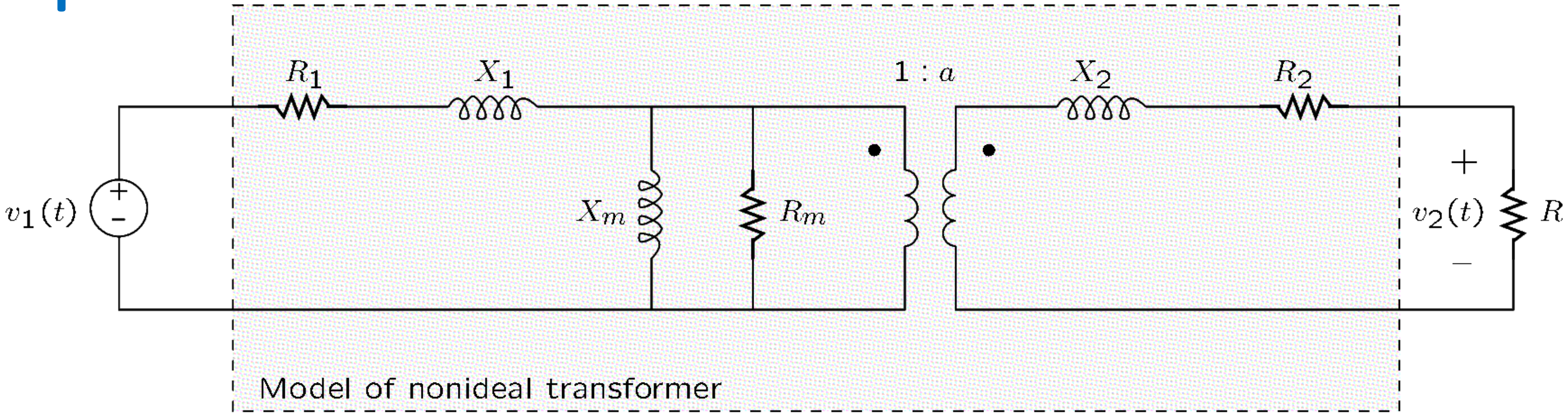


Remarks

- Thus, for a resistor: $\boldsymbol{Z} = R$, $P = RI_{rms}^2 = V_{rms}^2/R$ and $Q = 0$.
- For a capacitor: $\boldsymbol{Z} = jX$, $X = -1/\omega C$, $P = 0$ and $Q = XI_{rms}^2 = V_{rms}^2/X$.
- For an inductor: $\boldsymbol{Z} = jX$, $X = \omega L$, $P = 0$ and $Q = XI_{rms}^2 = V_{rms}^2/X$.



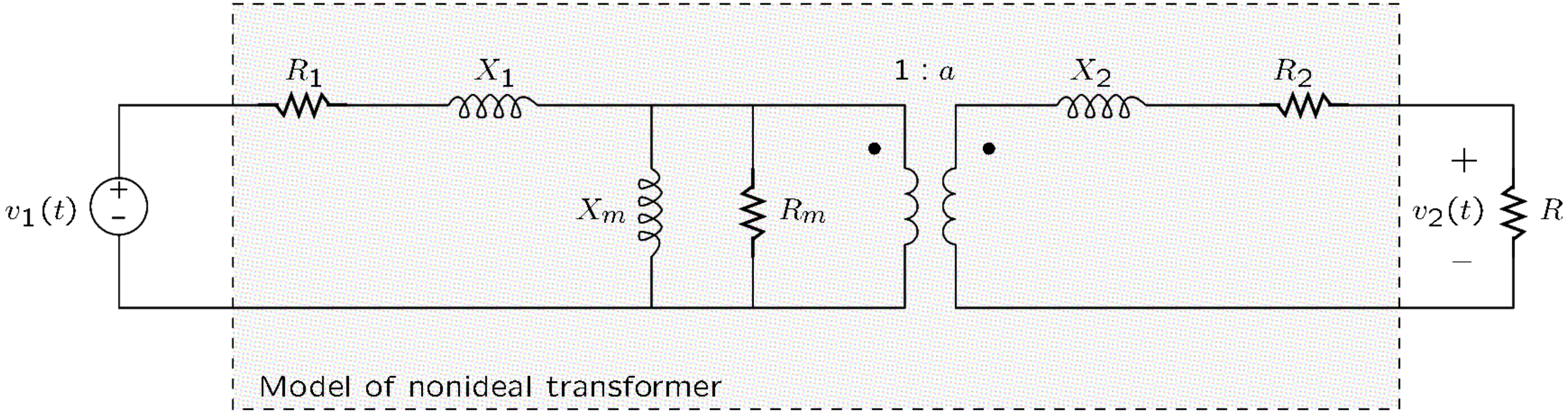
Example



- One of the homework problems involves the circuit above.
- The average power of the resistor R is $P = V_2^2/R$, where V_2 is the rms value of $v_2(t)$.
- A possible way to find the average power of the source is by finding the total power dissipated by the circuit.
- Let $\boldsymbol{Z}$ be the total impedance seen by the source:

$$\boldsymbol{Z} = R_1 + jX_1 + (jX_m)||R_m||\left(\frac{jX_2}{a^2} + \frac{R_2}{a^2} + \frac{R}{a^2}\right)$$

Example (Continued)



- Let $G = Imag\left(\frac{1}{Z}\right)$ be the imaginary part of the admittance.
- Let $R_p = \frac{1}{G}$.
- The average power dissipated by the circuit will be V_1^2/R_p, where V_1 is the rms value of $v_1(t)$.