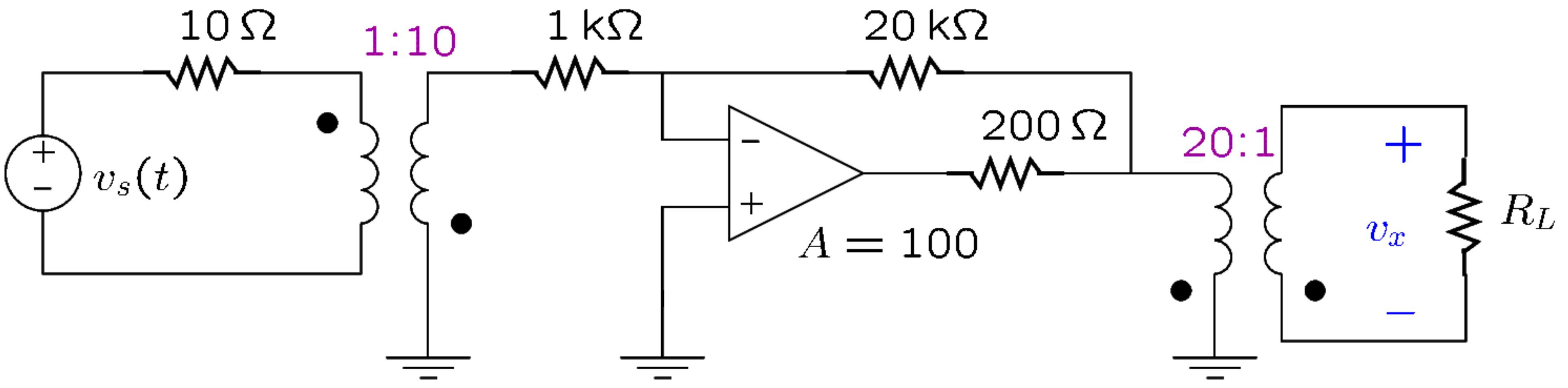
Review

Example 1

In the following example we will practice operational amplifiers, transformers, and the Thevenin equivalent.

Find the Thevenin equivalent at the terminals of the load resistor R_L. Assume $v_s(t) = 10\cos(\omega t)\ mV$.

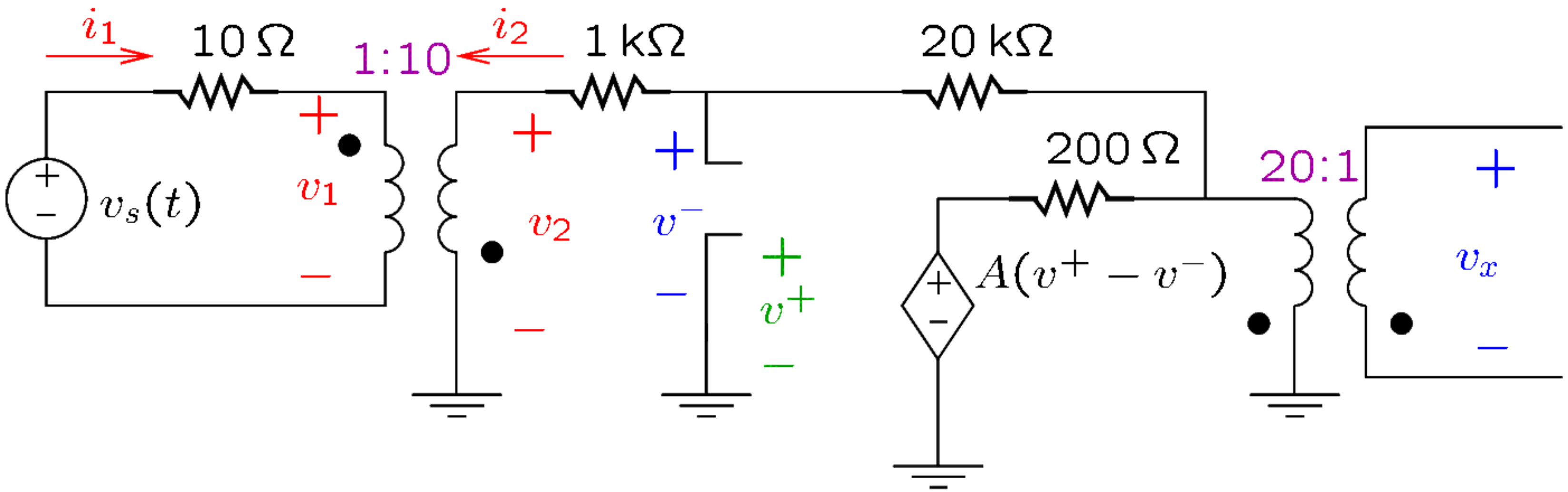
- *When a part of the circuit is considered to be the load, to find the Thevenin equivalent, remove first the load from the circuit.*



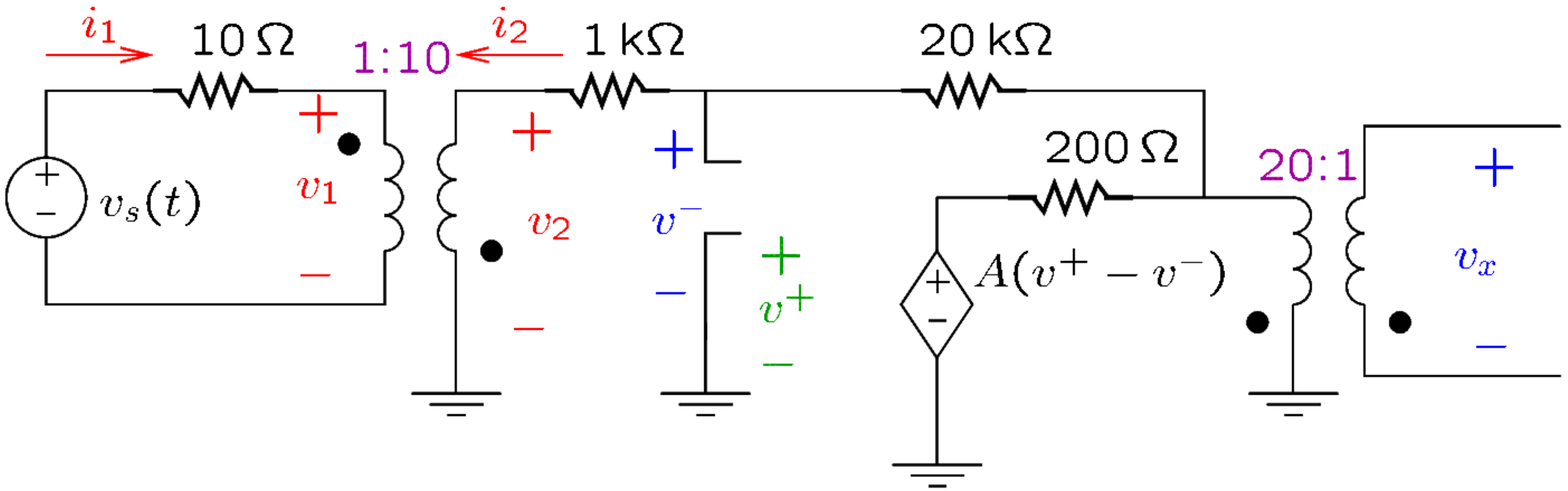
Example 1 (Continued)

- *By replacing the operational amplifier with its equivalent circuit we obtain the circuit shown in the figure.*
- *We will further simplify the circuit before finding the Thevenin equivalent.*
- *Note the transformer equations:*

$$\frac{-\boldsymbol{V_2}}{\boldsymbol{V_1}} = 10 \textit{ and } 1 \cdot \boldsymbol{I_1} + 10 \cdot (-\boldsymbol{I_2}) = 0$$



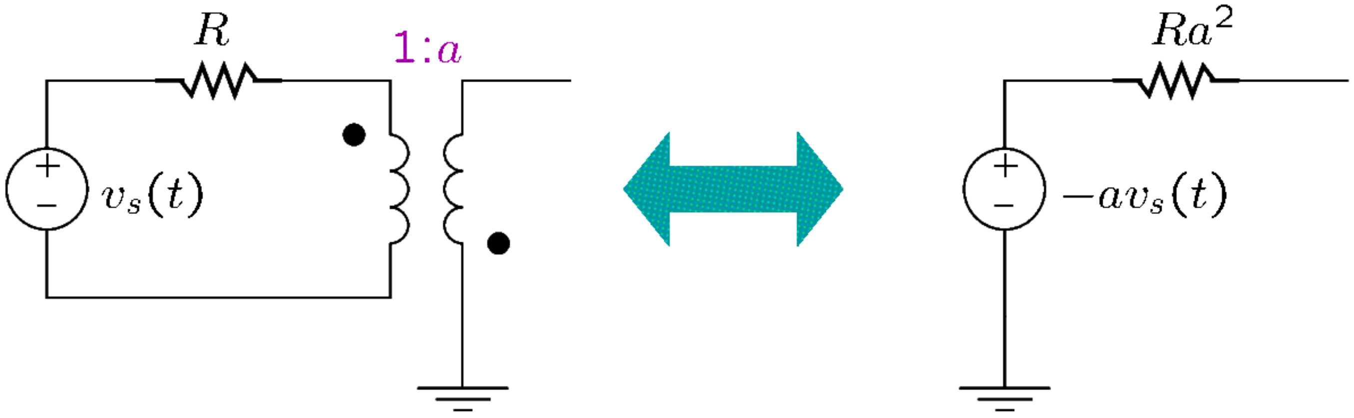
Example 1 (Continued)



- *Substituting* $\boldsymbol{V_1} = -\boldsymbol{V_2}/10$ *and* $\boldsymbol{I_1} = 10\boldsymbol{I_2}$ *in* $\boldsymbol{V_1} = \boldsymbol{V_s} - \boldsymbol{I_1} \cdot 10\ \Omega$*, we find that*

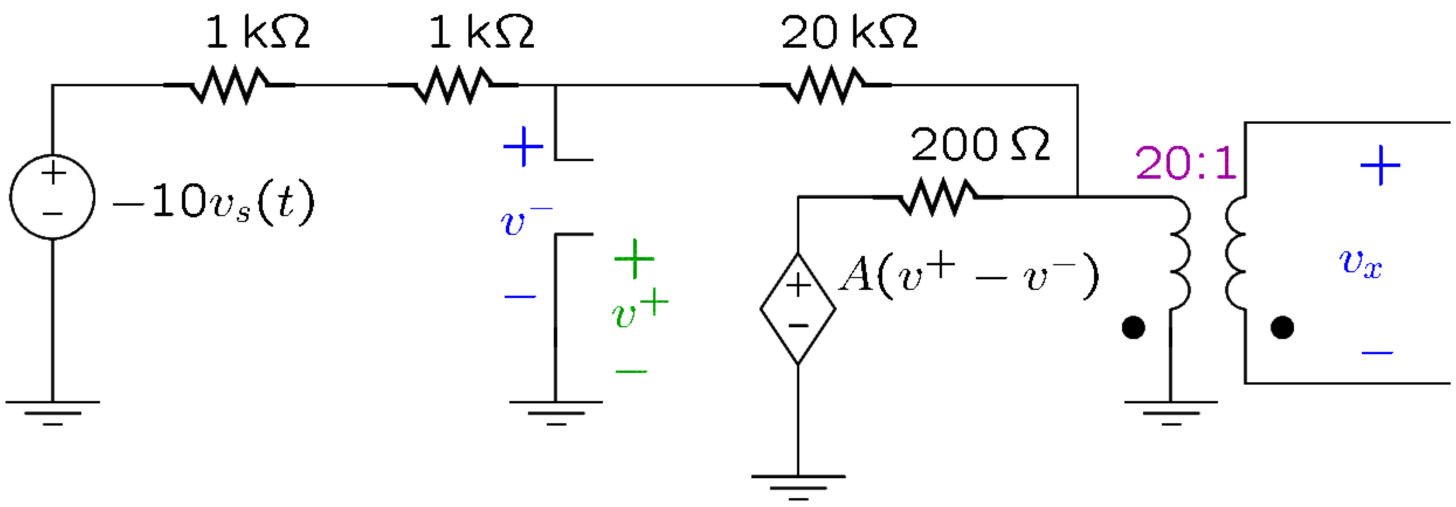
$$\boldsymbol{V_2} = -10\boldsymbol{V_s} + \boldsymbol{I_2} \cdot 10^2 \cdot 10\ \Omega$$

- *Thus, we have proved the following equivalence:*



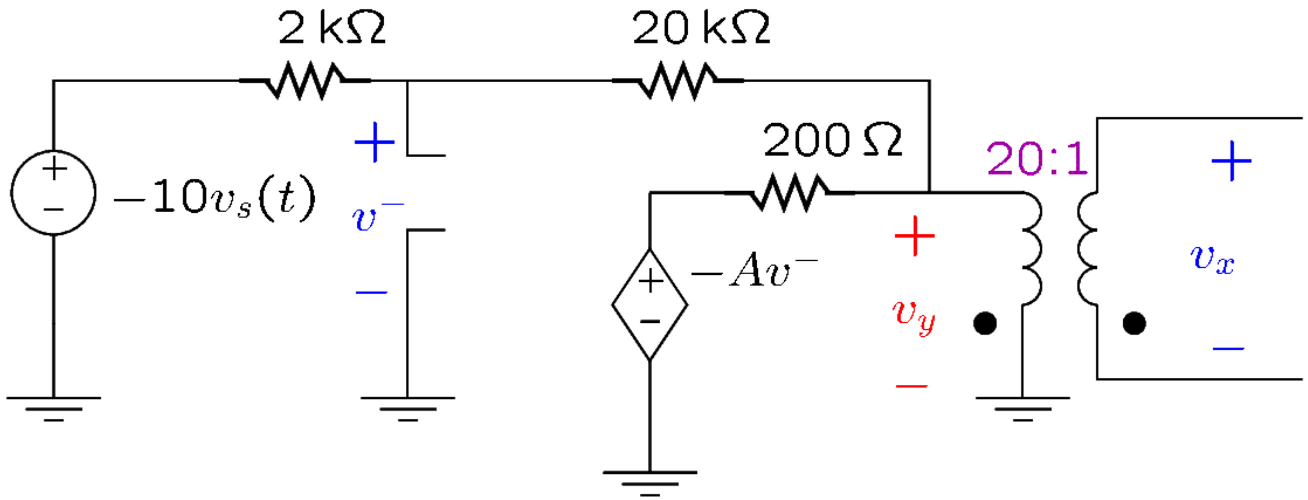
Example 1 (Continued)

- *We have simplified the circuit to the form shown below.*
- *We can further simplify it by combining the* $1\ k\Omega$ *resistors.*
- *Let us also notice that* $v^+ = 0$.

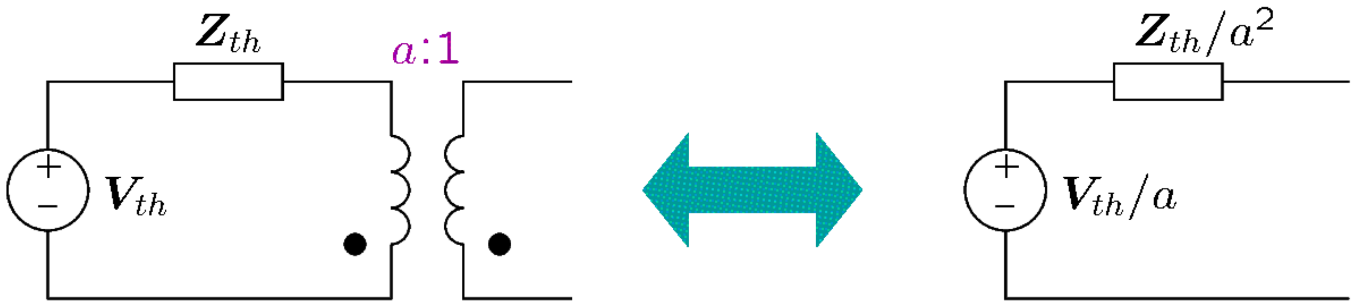


Example 1 (Continued)

- *We have simplified the circuit to the form below.*

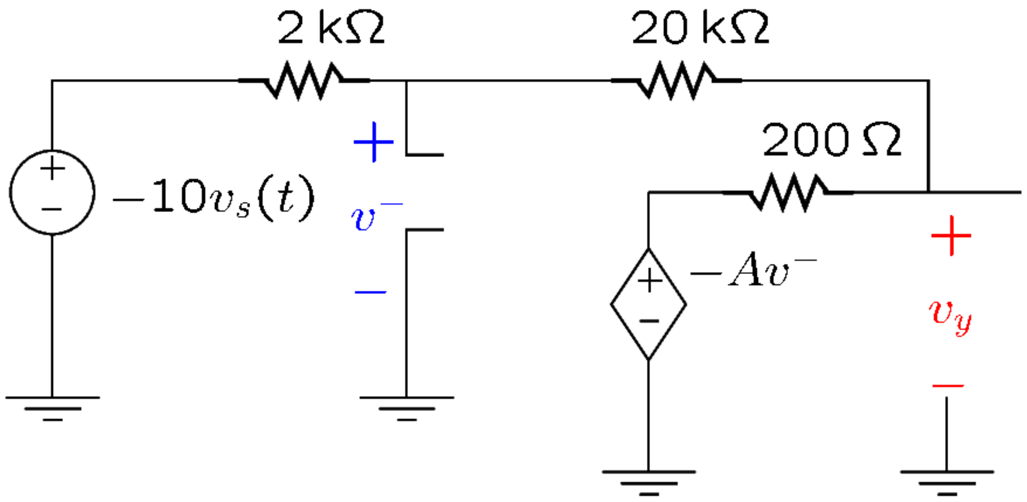


- *Now if we find the Thevenin equivalent between the nodes of v_y, we can obtain immediately the Thevenin equivalent between the nodes of v_x!*



Example 1 (Continued)

- *Therefore, we will find first the Thevenin equivalent of this simpler circuit:*



- *The nodal equations are:*

$$\frac{\boldsymbol{V_y} - (-A\boldsymbol{V}^-)}{200} + \frac{\boldsymbol{V_y} - \boldsymbol{V}^-}{20\ k\Omega} = 0 \Rightarrow \boldsymbol{V_y} = -\boldsymbol{V}^- \frac{100A - 1}{101}$$

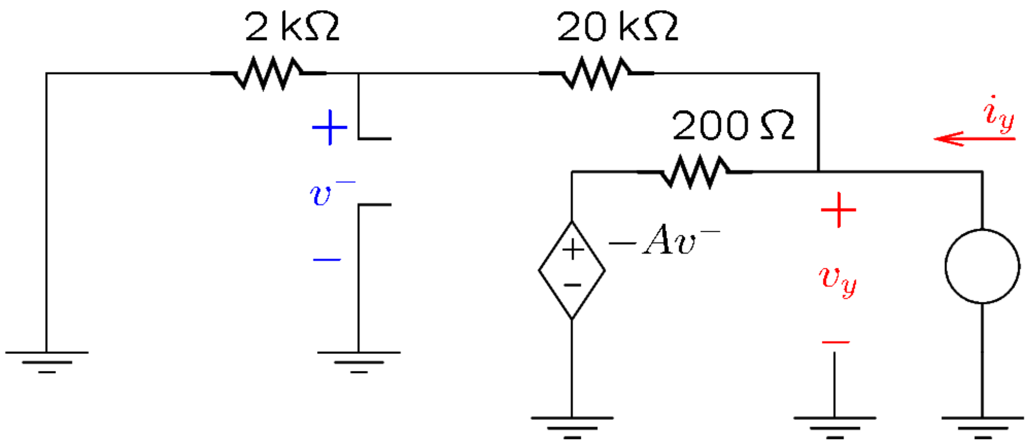
$$\frac{\boldsymbol{V}^- - (-10\boldsymbol{V_s})}{2\ k\Omega} + \frac{\boldsymbol{V}^- - (-A\boldsymbol{V}^-)}{20.2\ k\Omega} = 0 \Rightarrow \boldsymbol{V}^- = -\boldsymbol{V_s} \frac{101}{A + 11.1}$$

Example 1 (Continued)

- *We conclude that*

$$\boldsymbol{V_{th}} = \boldsymbol{V_y} = \boldsymbol{V_s}\frac{100A - 1}{A + 11.1} = 900\angle 0^{\circ}\ mV$$

- *To find $\boldsymbol{Z_{th}}$, we could find the short-circuit current or apply an external source.*
- *The external source method will be illustrated here.*
- *We will set to zero the independent sources and connect an external source between the nodes of v_y.*



Example 1 (Continued)

- *Note that*

$$I_y = \frac{V_y}{2\ k\Omega + 20\ k\Omega} + \frac{V_y - (-AV^-)}{200\ \Omega}$$

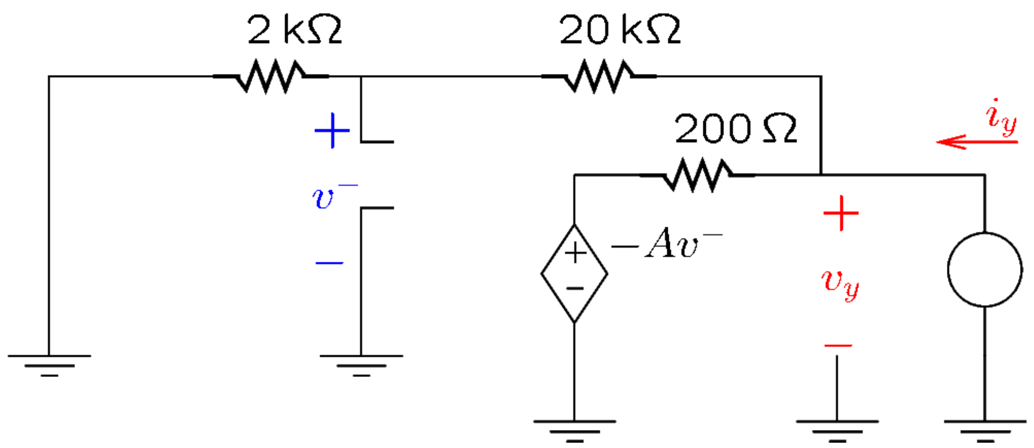
- *By voltage division:*

$$V^- = V_y \frac{2\ k\Omega}{2\ k\Omega + 20\ k\Omega}$$

- *Substituting* V^-*:*

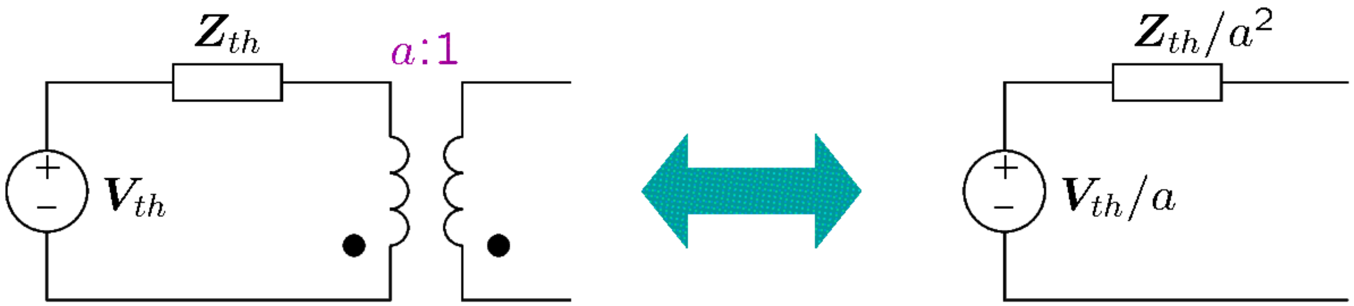
$$I_y = \frac{V_y(111 + 10A)}{22\ k\Omega}$$

$$\Rightarrow Z_{th} = \frac{V_y}{I_y} = \frac{22\ k\Omega}{111 + 10A} = 19.8\ \Omega$$



Example 1 (Continued)

- *At this point we need to take in account the last transformer in order to obtain the final result.*
- *As mentioned earlier, the effect of the transformer is:*



- *Since* $a = 20$*, the Thevenin equivalent between the nodes of* $\boldsymbol{V_x}$ *has*

$$\boldsymbol{V_{x,th}} = \frac{900}{20} = 45\angle 0°\ mV$$

$$\boldsymbol{Z_{x,th}} = \frac{19.8}{20^2}\ \Omega = 49.5\angle 0°\ \text{m}\Omega$$

Example 2

Repeat the previous example for $A \to \infty$.

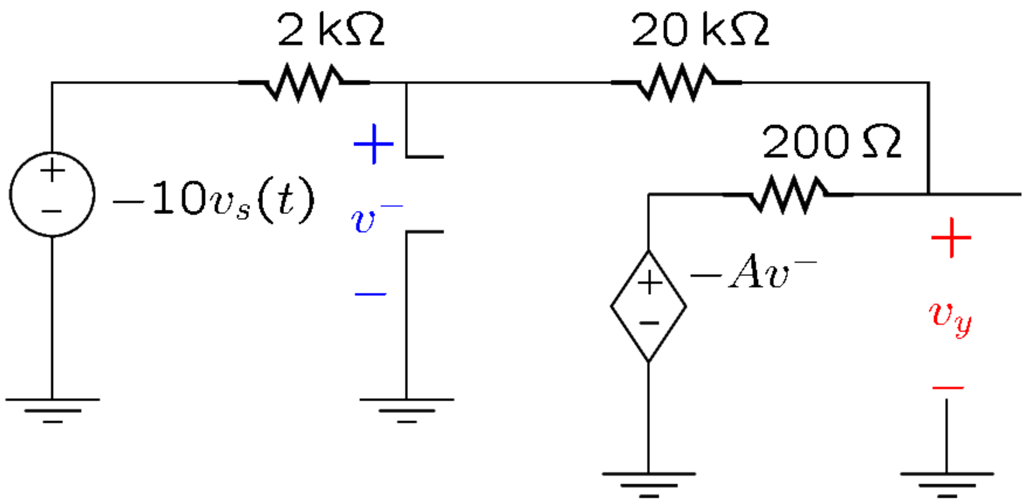
- *Since the calculations were done in terms of A, we can simply take the limit:*

$$\boldsymbol{V_{th}} = \lim_{A\to\infty} \boldsymbol{V_s} \frac{100A - 1}{A + 11.1} = 100\boldsymbol{V_s} \Rightarrow \boldsymbol{V_{x,th}} = \frac{\boldsymbol{V_{th}}}{20} = 50\angle 0^\circ\ mV$$

$$\boldsymbol{Z_{th}} = \lim_{A\to\infty} \frac{22\ k\Omega}{111 + 10A} \Rightarrow \boldsymbol{Z_{x,th}} = 0$$

- *However, the approximation $A \to \infty$ is used because it simplifies very much all calculations; it is unnecessary to solve for the unknowns in terms of A and then take the limit.*

Example 2 (Continued)



- *Note that* $A \to \infty$ *implies* $v^- \to 0$.
- *Accounting for* $\boldsymbol{V^-} = 0$*, the nodal equation of* $\boldsymbol{V^-}$ *is:*

$$\frac{\boldsymbol{V_y - 0}}{20\ k\Omega} + \frac{-10\boldsymbol{V_s} - 0}{2\ k\Omega} = 0 \Rightarrow \boldsymbol{V_{th}} = \boldsymbol{V_y} = 100\boldsymbol{V_s}$$

- *Therefore,*

$$\boldsymbol{V_{x,th}} = \frac{\boldsymbol{V_{th}}}{20} = 50\angle 0°\ mV$$

Example 2 (Continued)

- *Moreover, when finding the Thevenin impedance with an external source, we can note that by voltage division:*

$$V^- = V_y \frac{2\ k\Omega}{2\ k\Omega + 20\ k\Omega}$$

- *Therefore,* $V^- = 0 \Rightarrow V_y = 0 \Rightarrow Z_{th} = \frac{V_y}{I_y} = 0$

$$\Rightarrow Z_{x,th} = \frac{Z_{th}}{20^2} = 0$$