Transformers—Part 2

Examples

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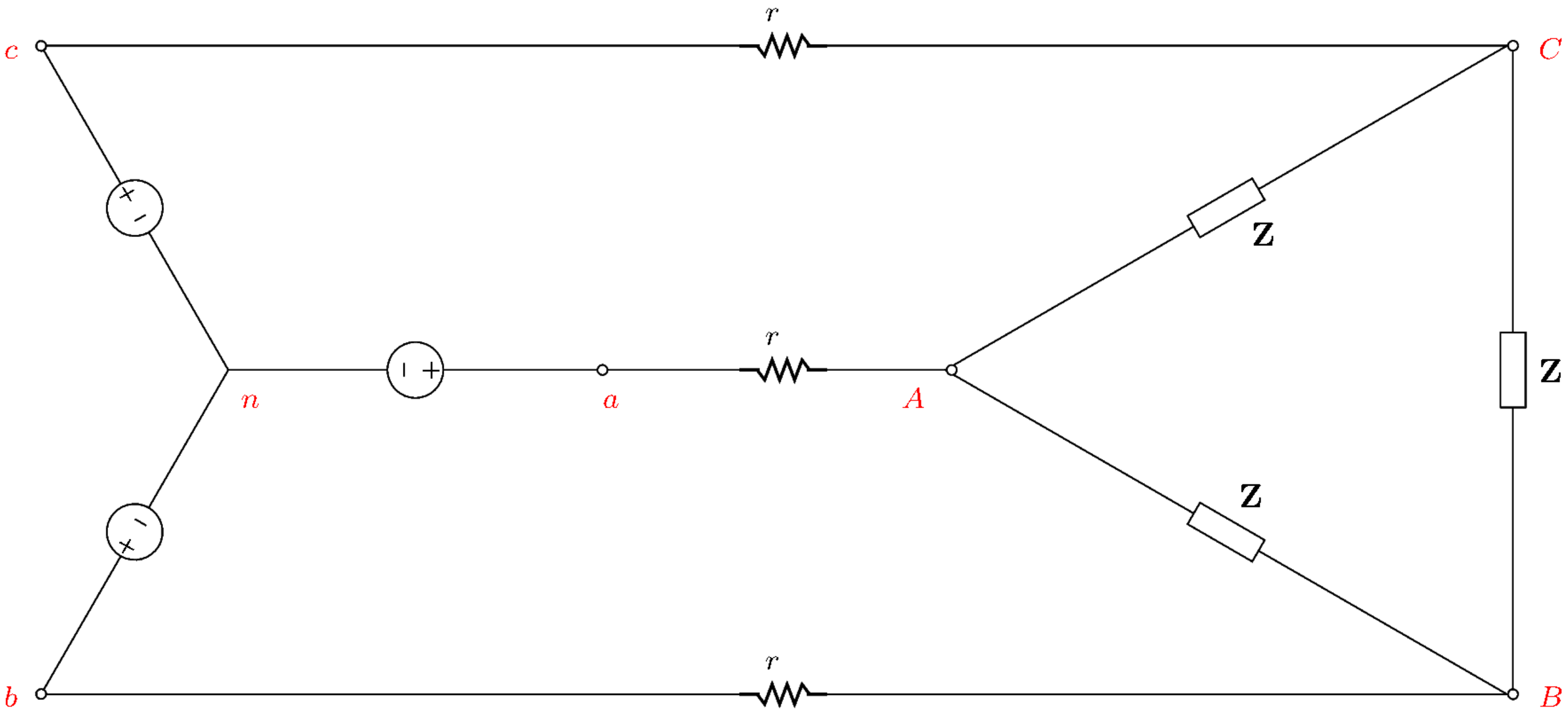
Transformers in Electric Power Distribution

- Having finished the topics of our class, we will focus the remaining lectures on examples.
- Note that transformers play an important role in electric power distribution, as they lower considerably losses.
- Power line losses are due to the resistance of the cables.
- Losses are proportional to the square of the rms current and the resistance.
- To operate power lines at lower currents, power is transmitted at very high voltage levels.
  - Transformers are used to step up voltage near the source of power.
  - Transformers are used to step down the voltage at the consumers.
- The following example will illustrate the role of transformers.

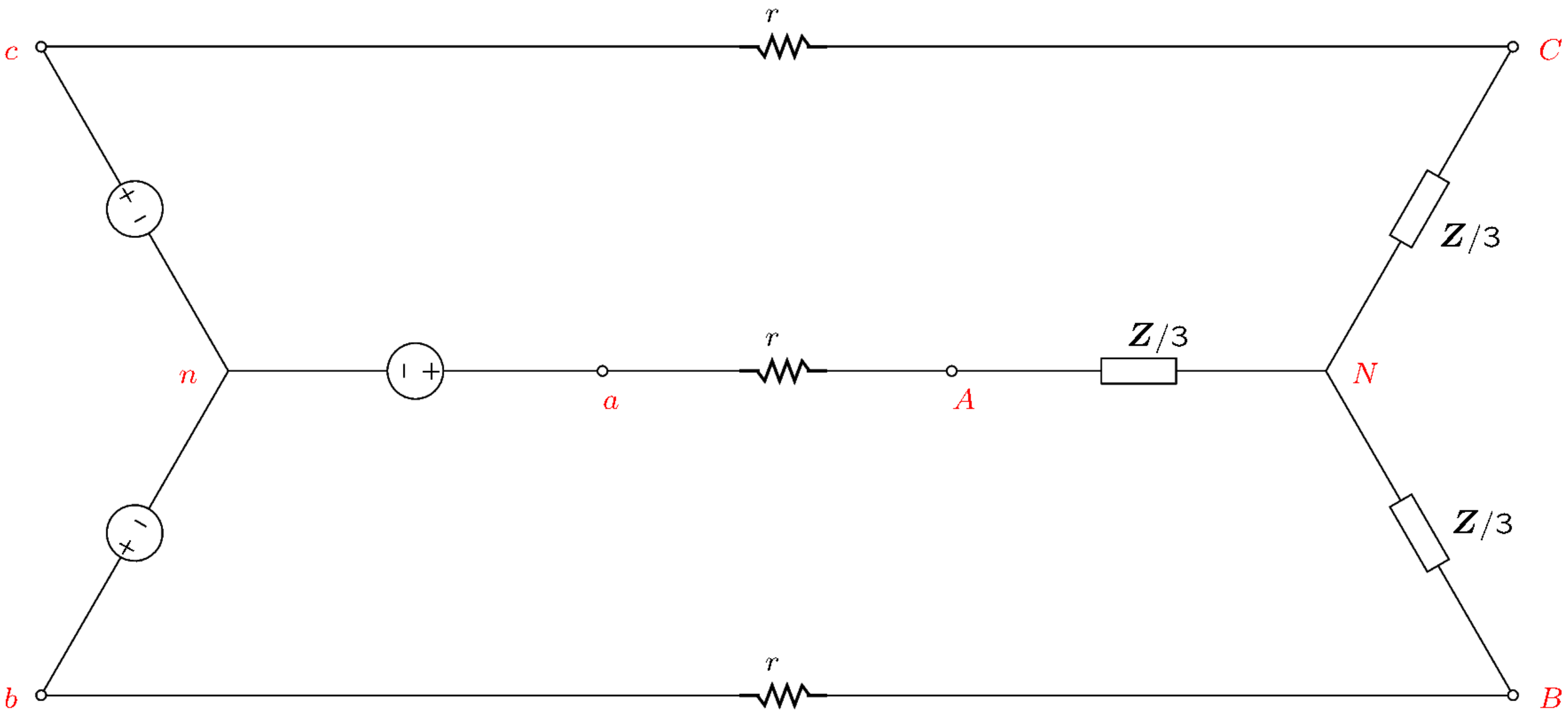
Example

Assume $v_{ab}(t) = 500\cos(\omega t)\ V$ *and* $\boldsymbol{Z} = 4\angle 30°\ \Omega$*. Assume that the power lines have each a resistance* $r = 2\ \Omega$*. Find the efficiency of the system.*



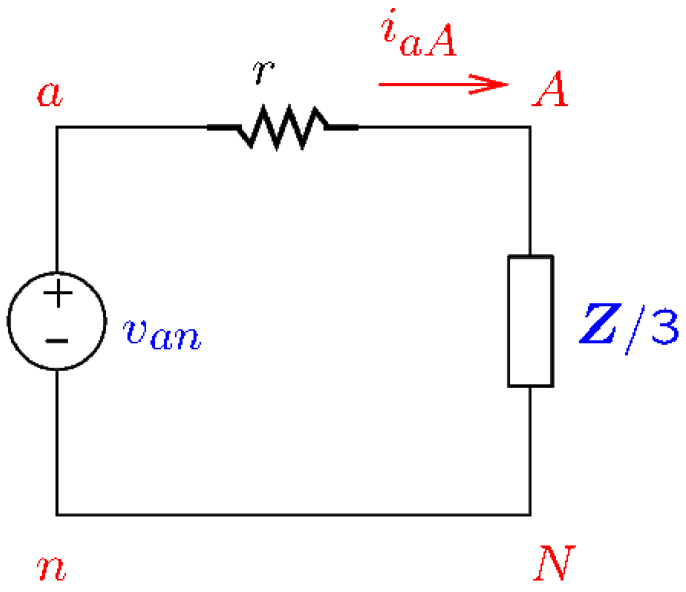
Example (Continued)

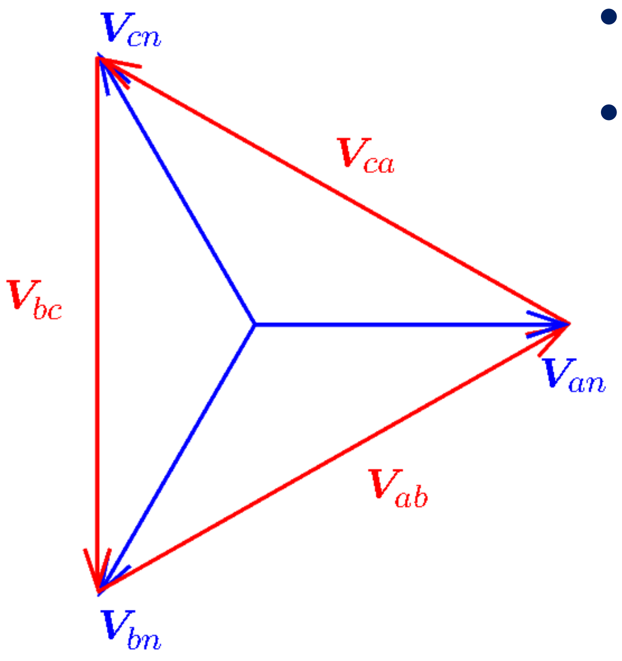
- *Let us first convert the Δ-connected load to a Y-connected load.*
- *The circuit is balanced, so we can draw an equivalent circuit per phase.*



Example (Continued)

- *The equivalent circuit per phase could show any of the three phases; let's show the first phase (this is a random choice).*
- *The problem does not give $v_{an}(t)$ but $v_{ab}(t)$.*
- *A phasor diagram showing the phase and line voltages can help establish the relationship between the two voltages.*





- *Note that $v_{ab}(t)$ leads $v_{an}(t)$ by $30°$.*
- *The magnitudes also are different by a factor of $\sqrt{3}$, since $v_{ab}(t)$ is a line voltage.*
  - *Therefore,* $v_{ab}(t) = 500\cos(\omega t)\,V \Rightarrow$

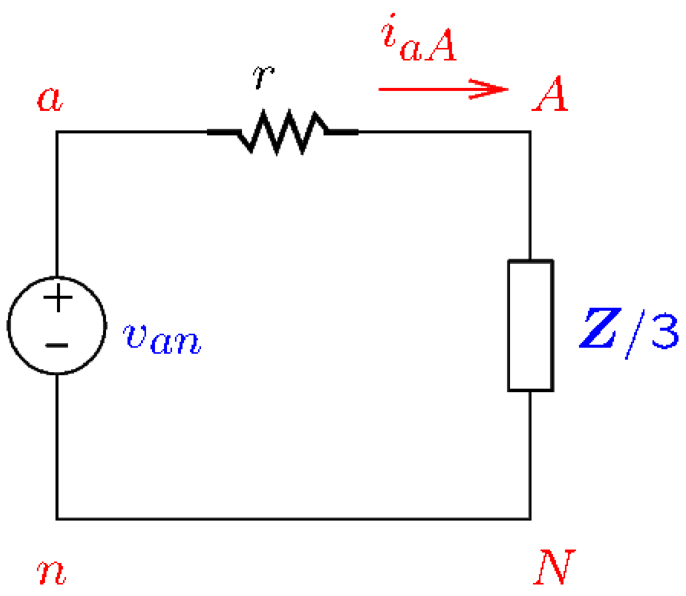
$$v_{an}(t) = \frac{500}{\sqrt{3}}\cos(\omega t - 30°)\,V.$$

Example (Continued)

- *Note that:*

$$\boldsymbol{V}_{an} = \frac{500}{\sqrt{3}}\angle -30°\ V$$

$$\boldsymbol{I}_{aA} = \frac{\boldsymbol{V}_{an}}{r + \boldsymbol{Z}/3} = 89.53\angle\ -41.93°\ \ A$$



- *The power generated by the source, per phase, is*

$$P_{in} = V_{an,rms} I_{aA,rms} \cos(\alpha_v - \alpha_i)$$

$$= \frac{500}{\sqrt{3}\sqrt{2}} \cdot \frac{89.53}{\sqrt{2}} \cdot \cos\big(-30° - (-41.93°)\big)$$

$$P_{in} = 12.643\ kW$$

- *To find the power dissipated by the load, we need to find first* $\boldsymbol{V}_{AN}$.

Example (Continued)

- *By voltage division:*

$$\boldsymbol{V}_{AN} = \boldsymbol{V}_{an} \cdot \frac{\boldsymbol{Z}/3}{r + \boldsymbol{Z}/3}$$

$$\boldsymbol{V}_{AN} = 119.37\angle -11.93° \ \ V$$

- *The power dissipated by the load, per phase, is:*

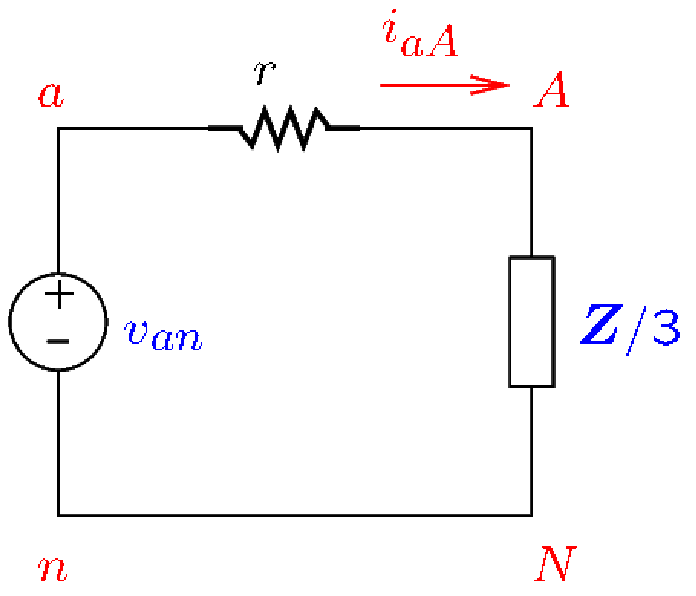
$$P_{out} = V_{AN,rms} I_{aA,rms} \cos(\alpha_v - \alpha_i)$$

$$= \frac{119.37}{\sqrt{2}} \cdot \frac{89.53}{\sqrt{2}} \cdot \cos(30°)$$

$$P_{out} = 4.6277 \ kW$$

- *The efficiency is:*

$$\eta = \frac{P_{out}}{P_{in}} = \frac{4.6277}{12.643} = 36.6\%$$

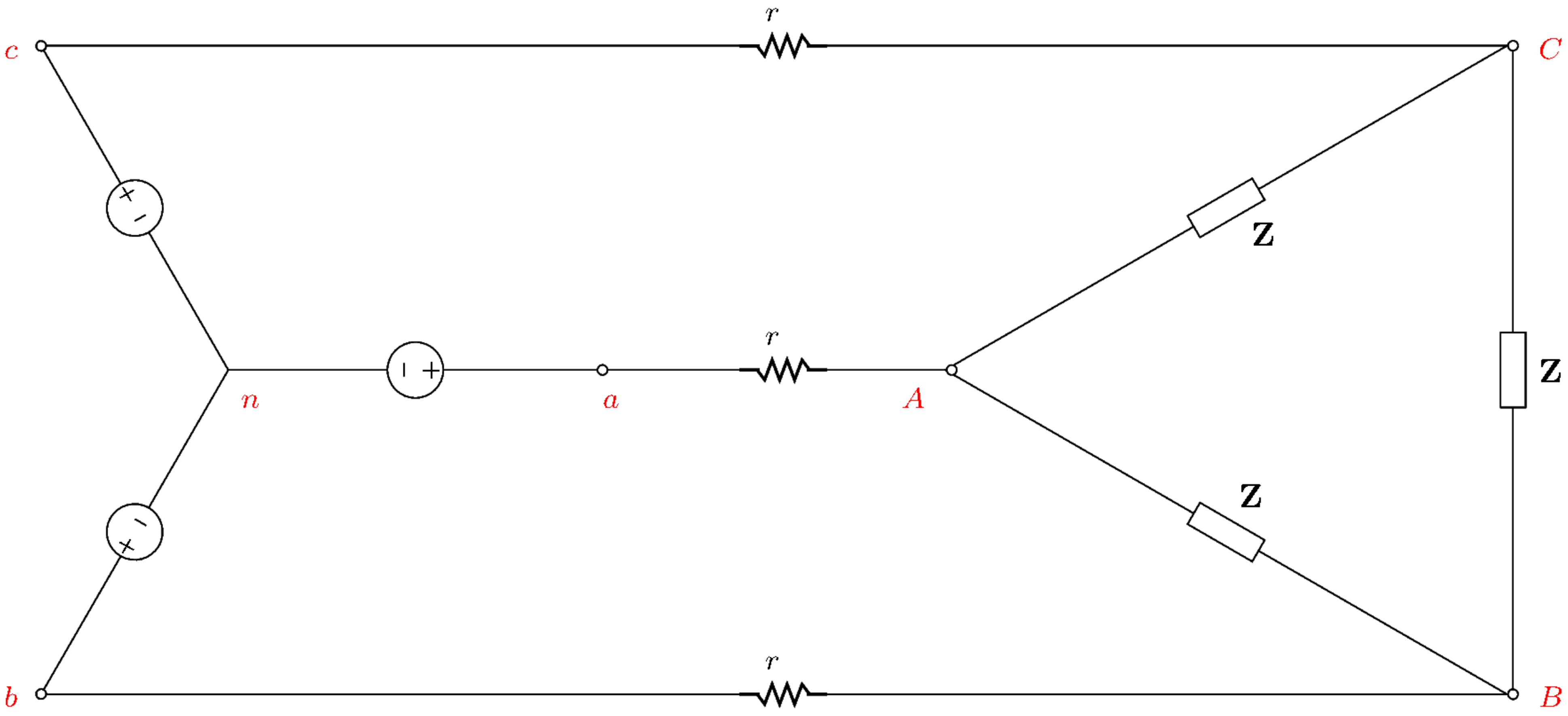


Remarks

- In our previous example, only 36.6% of the power generated by the source was dissipated by the load.
- 63.4% (almost twice as much!) was dissipated on the power lines.
- A possible way of improving efficiency is by correcting the power factor.
- This will reduce the line current, and in this way the losses on the power lines.
- However, this improvement will not eliminate power losses.
- Note that power factor correction must be done at the load, after the power lines, not at the generator, before the power lines.
- Otherwise, the current through the power lines will not be reduced.

Example 2

Assume $v_{ab}(t) = 500\cos(\omega t)\ V$, $\boldsymbol{Z} = 4\angle 30°\ \Omega$, *and* $r = 2\ \Omega$. *Find the efficiency of the system after power factor correction.*



Example (Continued)

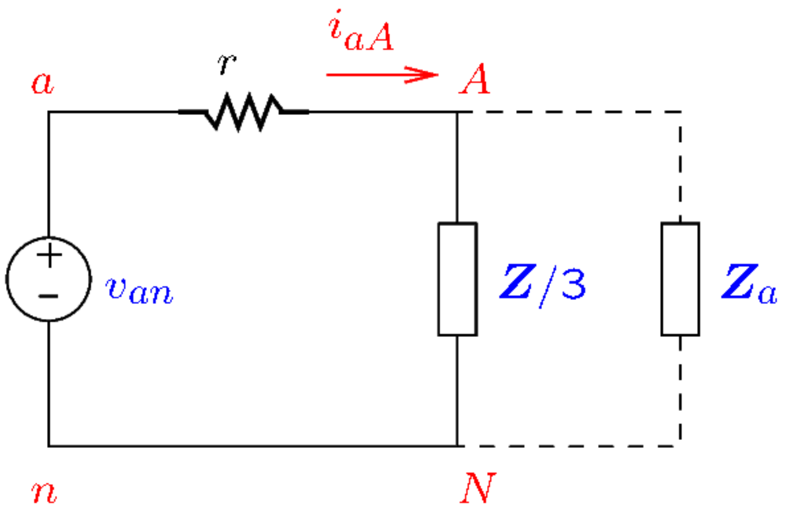
- *Power factor correction at the load is equivalent to an additional component* $\boldsymbol{Z_a}$ *added in parallel to each phase of the load.*
- *The role of* $\boldsymbol{Z_a}$ *is to cancel the susceptance of the load.*
- *Recall that the susceptance of* $\boldsymbol{Z}/3$ *is the imaginary part of* $1/(\boldsymbol{Z}/3)$.
- *The equivalent load impedance is*

$$\boldsymbol{Z_L} = \left(\frac{\boldsymbol{Z}}{3}\right) || \boldsymbol{Z_a}$$

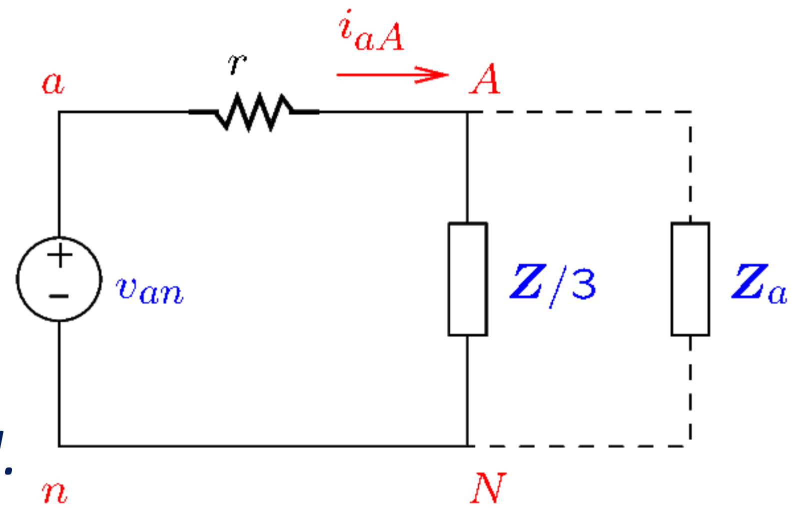
$$\frac{1}{\boldsymbol{Z_L}} = \frac{3}{\boldsymbol{Z}} + \frac{1}{\boldsymbol{Z_a}} = 0.6495 - 0.375j + \frac{1}{\boldsymbol{Z_a}}$$

- *Power factor correction cancels the imaginary part, so*

$$\frac{1}{\boldsymbol{Z_L}} = 0.6945\ S \Rightarrow \boldsymbol{Z_L} = 1.54\angle 0^\circ\ \Omega$$



Example (Continued)



- *By voltage division:*

$$\boldsymbol{V}_{AN} = \boldsymbol{V}_{an} \cdot \frac{\boldsymbol{Z}_L}{r + \boldsymbol{Z}_L}$$

$$\boldsymbol{V}_{AN} = 0.435 \cdot \boldsymbol{V}_{an}$$

- *Since* $\boldsymbol{Z}_L$ *has an angle of* $0°$*, the total impedance is real.*
- *Thus, all voltages and currents are in phase.*
- *This means that apparent power is identical to average power, since* $\alpha_v = \alpha_i$.
- *Therefore, the efficiency can be calculated as*

$$\eta = \frac{V_{AN,rms} I_{aA,rms}}{V_{an,rms} I_{aA,rms}} = \frac{V_{AN,rms}}{V_{an,rms}} = \frac{V_{AN}}{V_{an}} = 43.5\%$$

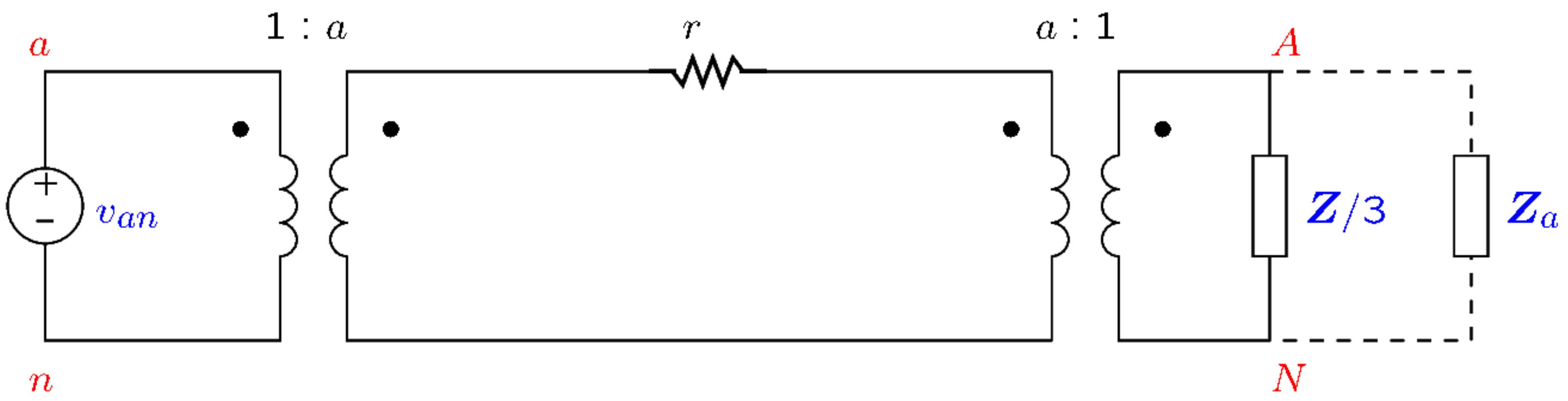
Remarks

- In our example, power factor correction has improved the efficiency of the system from 36.6% to 43.5%.
- This is a considerable improvement!
- However, even after power factor correction, more than half of the generated power is wasted!
- Losses can be further reduced with transformers.

Example 3

Consider the previous example. Assume that a three-phase transformer steps up the source voltage by a factor of $a = 100$. Assume that at the load, a three-phased transformer steps down the voltage by the same factor. Assume also that all transformers have the primary and secondary windings Y-connected. Find the efficiency of the system after power factor correction.

- *The equivalent circuit per phase is shown in the figure.*



Example 3 (Continued)

- *As shown in the previous example, the load has an impedance* $\boldsymbol{Z_L} = 1.54\angle 0^\circ\ \Omega$.
- *Thus, the load transformer with the* $\boldsymbol{Z_L}$ *load is equivalent to the impedance* $a^2\boldsymbol{Z_L}$.
- *Since* r *is in series with* $a^2\boldsymbol{Z_L}$*, proceeding as in the previous example:*

$$\eta = \frac{a^2\boldsymbol{Z_L}}{r + a^2\boldsymbol{Z_L}} = 99.99\%$$

- *We have assumed ideal transformers; real transformers are very efficient but do have losses, so they will reduce a little this efficiency.*