Transformers

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Introduction

- A transformer is an **AC** device that transfers energy between electrically isolated circuits by means of magnetically coupled inductors.
- Transformers have relatively low losses; energy is transferred efficiently.
- Typically, transformers transfer energy at different voltage levels; a transformer is capable of reducing or increasing the input voltage.

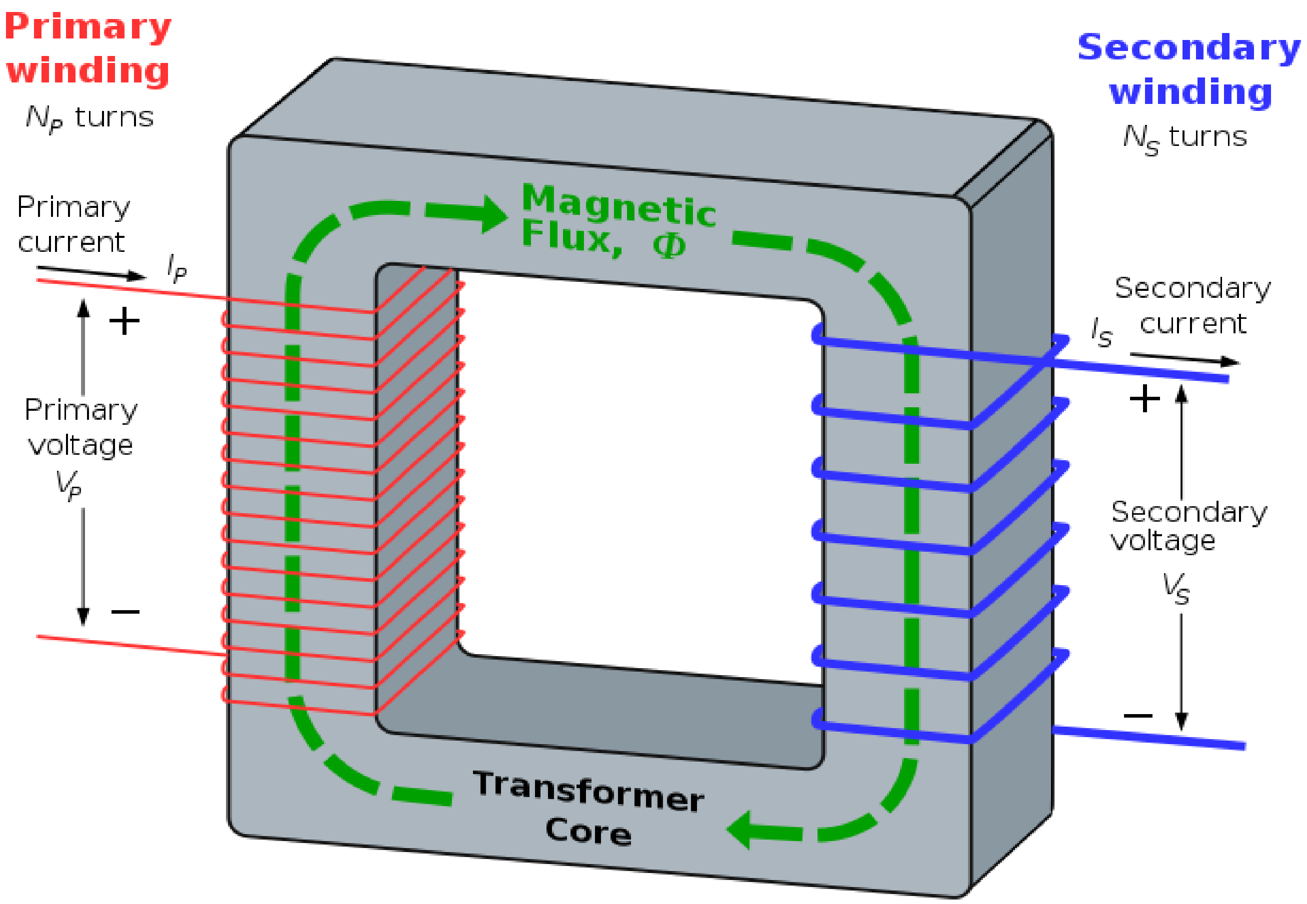


Figure downloaded from http://en.wikipedia.org/wiki/Transformer

Introduction

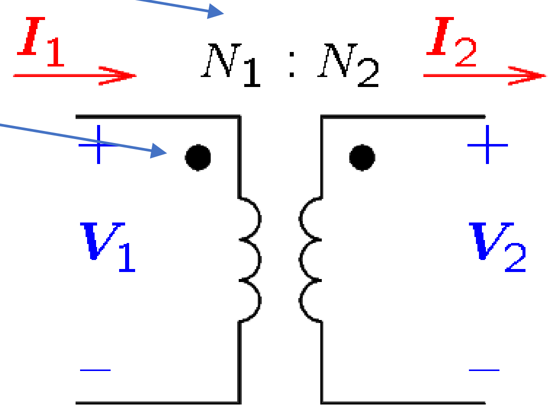
- The transformer coil to which power is applied is called *primary*.
- Any other coil of the transformer is a *secondary*.
- Consider a transformer with a single secondary operating, of course, with AC.
- Ideally,

$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}$$

  where
  - V_P and V_S are the rms values of the primary and secondary voltages.
  - I_P and I_S are the rms values of the primary and secondary currents.
  - N_P and N_S are the number of turns of the primary and secondary coils.
- The equations can be written also in terms of phasor voltages.
- However, we need to define first the *dot* notation, so that we write the signs right.

Symbol

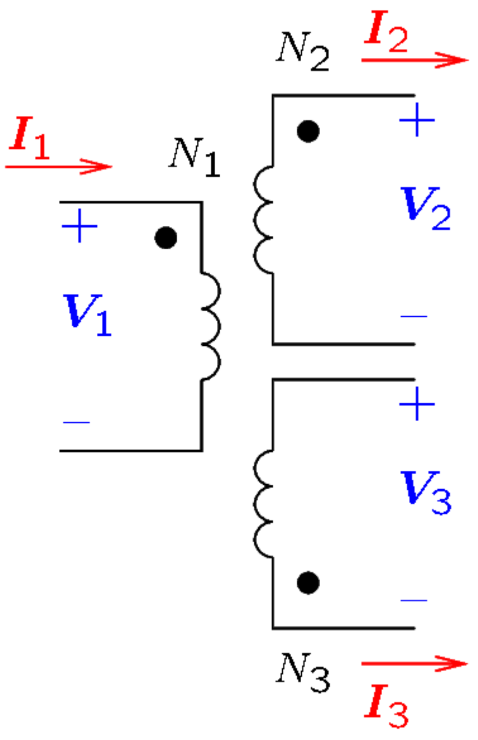
- The symbol of a transformer
  - Specifies the turns ratio
  - Has a dot next to one of the terminals of each coil.

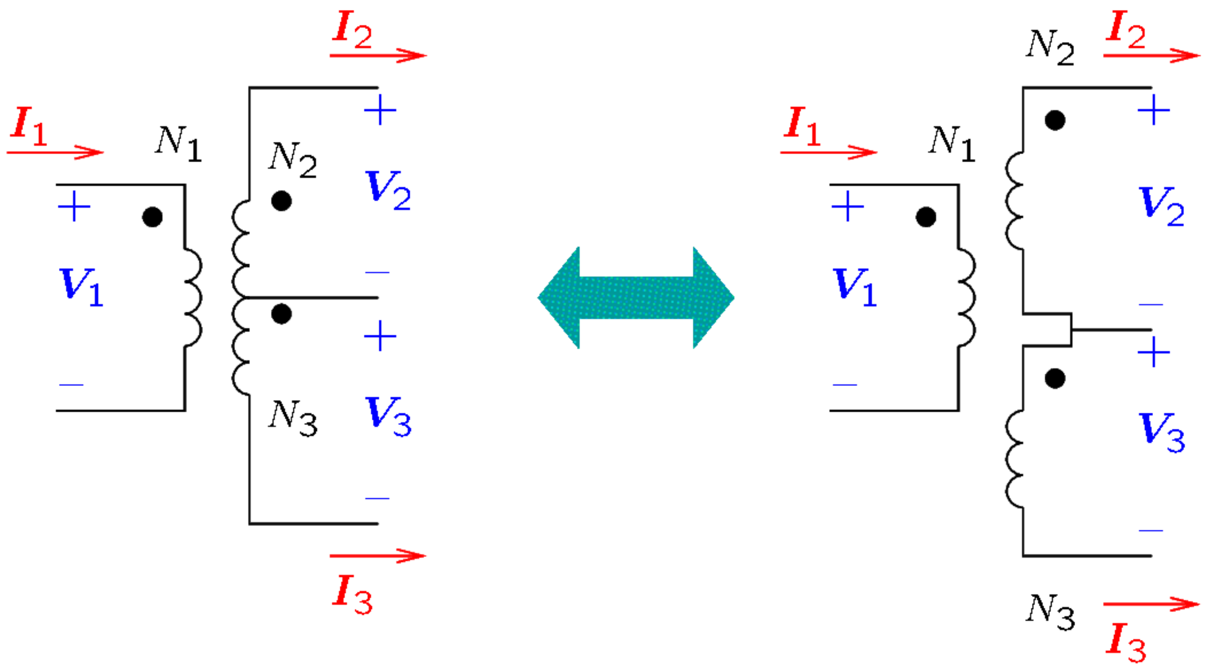


- For example, a $3:2$ turns ratio could mean:
  - $N_1 = 300$ and $N_2 = 200$
  - $N_1 = 600$ and $N_2 = 400$
  - $N_1 = 3000$ and $N_2 = 2000$
  - …
- Practical transformers resemble ideal transformers when they have a sufficiently large number of turns.

Symbol

- A transformer may have more than two windings.
- When more than two windings are present, we specify the number of turns for each coil.
- A coil may have more than one terminal.
- For example a coil with three terminals can be seen as consisting of two coils connected together.





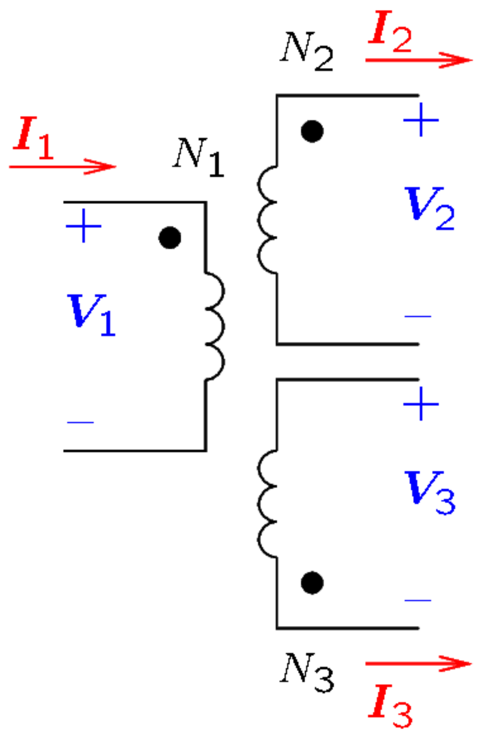
Equations

- Ideally, the voltage ratio is the turns ratio.
- *If the + sign of the voltage is at the dot, write the voltage with the + sign; otherwise, write it with the − sign.*
- Examples:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

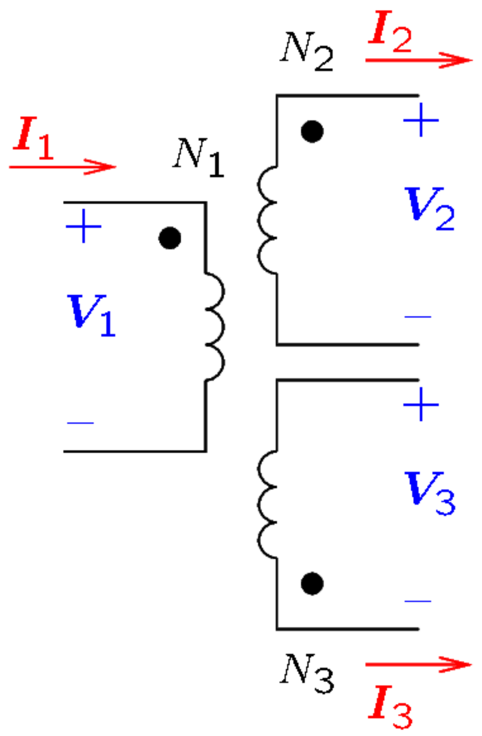
$$\frac{-V_3}{V_2} = \frac{N_3}{N_2}$$

$$\frac{V_1}{-V_3} = \frac{N_1}{N_3}$$



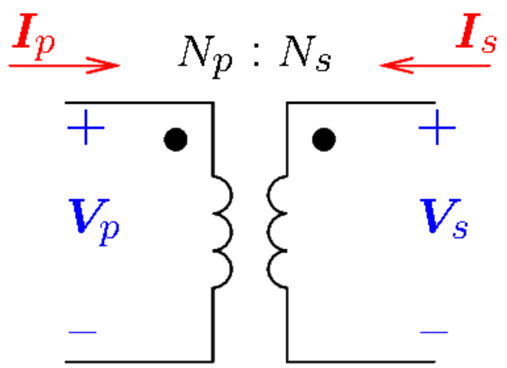
Equations

- Ideally, the algebraic sum of currents weighted by the number of turns is zero.
- *If a current enters the transformer at a dotted terminal, write it with the + sign; otherwise, write it with the − sign.*
- Examples:



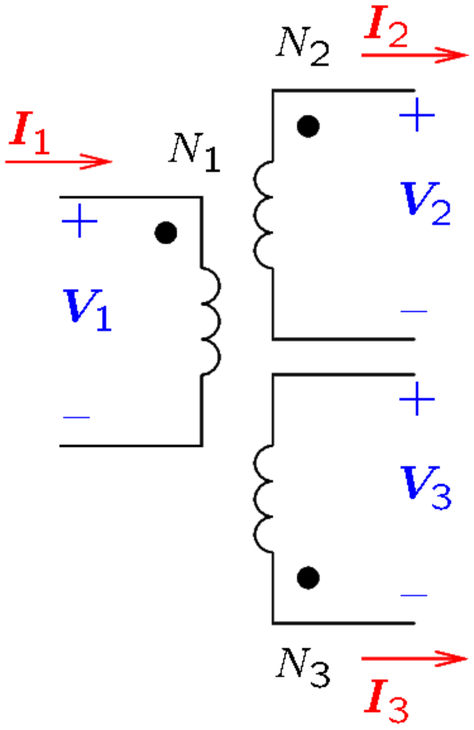
$$N_1 \boldsymbol{I_1} - N_2 \boldsymbol{I_2} - N_3 \boldsymbol{I_3} = 0$$

$$N_p \boldsymbol{I_p} + N_s \boldsymbol{I_s} = 0$$

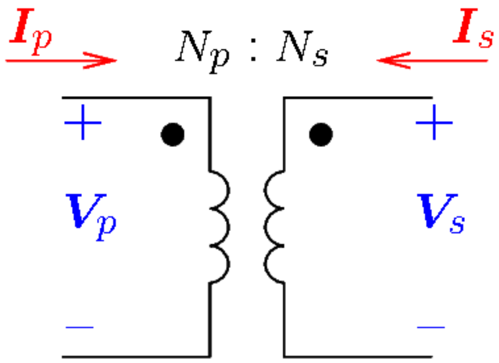


Power

- Ideally, a transformer does not absorb power.
- *This is can be seen easily from the ideal voltage and current equations of a transformer.*
- For example, assuming rms phasor currents and voltages, in terms of the absorbed complex power:



$$\boldsymbol{V}_1\boldsymbol{I}_1^* - \boldsymbol{V}_2\boldsymbol{I}_2^* - \boldsymbol{V}_3\boldsymbol{I}_3^* = 0$$



$$\boldsymbol{V}_p\boldsymbol{I}_p^* + \boldsymbol{V}_s\boldsymbol{I}_s^* = 0$$

Example

The transformer has $N_1 = 1000$, $N_2 = 500$, and $N_3 = 200$. The frequency is 60 Hz and the source has 120 V rms. Find $i_1(t)$.

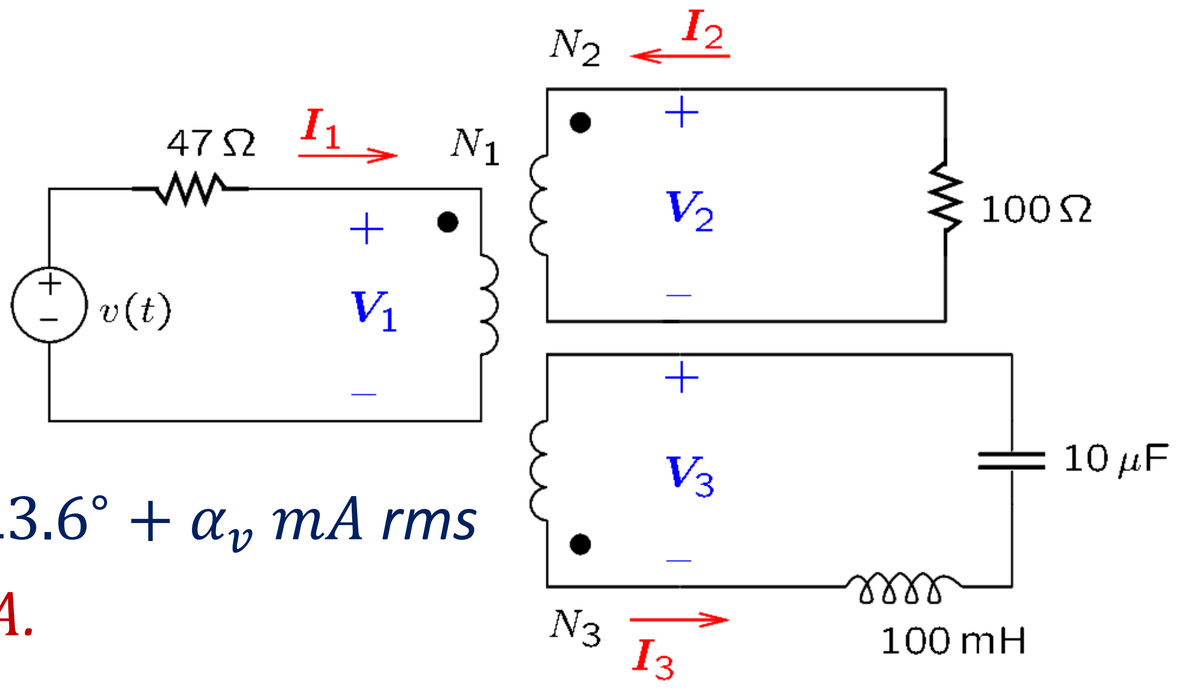
- *Let* $\boldsymbol{Z_2} = 100\ \Omega$ *be the impedance connected to the second winding.*
- *Let* $\boldsymbol{Z_3} = j\omega 100\ mH + \dfrac{1}{j\omega 10\ \mu F} = -227.56j\ \Omega$ *be the impedance connected to the third winding.*
- *The equations are:*

$$N_1\boldsymbol{I_1} + N_2\boldsymbol{I_2} - N_3\boldsymbol{I_3} = 0$$

$$\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = \frac{N_2}{N_1} \text{ and } \frac{-\boldsymbol{V_3}}{\boldsymbol{V_1}} = \frac{N_3}{N_1}$$

$$\boldsymbol{V_2} = -\boldsymbol{I_2}\boldsymbol{Z_2} \text{ and } \boldsymbol{V_3} = -\boldsymbol{I_3}\boldsymbol{Z_3}$$

$$\boldsymbol{V} = 47\boldsymbol{I_1} + \boldsymbol{V_1}$$



- *Solving for* $\boldsymbol{I_1}$*, the result is:* $\boldsymbol{I_1} = 269.11\angle 3.6° + \alpha_v\ mA\ rms$

$\Rightarrow i_1(t) = 380.58\cos(\omega t + 3.6° + \alpha_v)\ mA.$

Example—Remarks

- *Note that the current equation*

$$N_1\boldsymbol{I_1} + N_2\boldsymbol{I_2} - N_3\boldsymbol{I_3} = 0$$

  can be brought to the form

$$\boldsymbol{I_1} = \boldsymbol{V_1}\left(\frac{1}{\boldsymbol{Z_2 N_1^2/N_2^2}} + \frac{1}{\boldsymbol{Z_3 N_1^2/N_3^2}}\right)$$

- *In other words, the transformer circuit is equivalent to the circuit shown below.*
- $\boldsymbol{Z_2 N_1^2/N_2^2}$ *and* $\boldsymbol{Z_3 N_1^2/N_3^2}$ *are reflected impedances (from the secondary windings to the primary winding).*