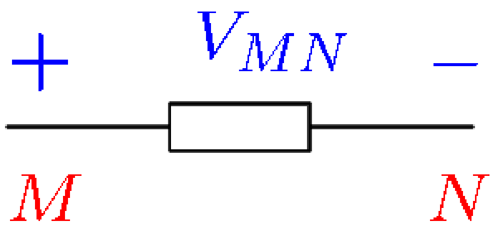
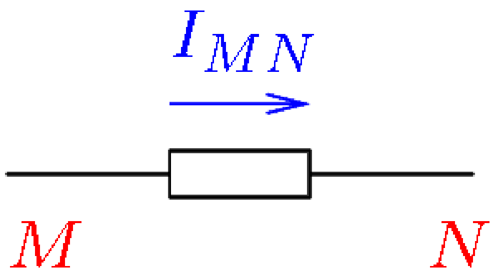
THREE PHASE SYSTEMS—PART 2

Notation

- *Let* M *and* N *denote the terminals of a circuit element.*
- V_{MN} *will be the voltage between* M *and* N *with* $+$ *at* M *and* $-$ *at N.*

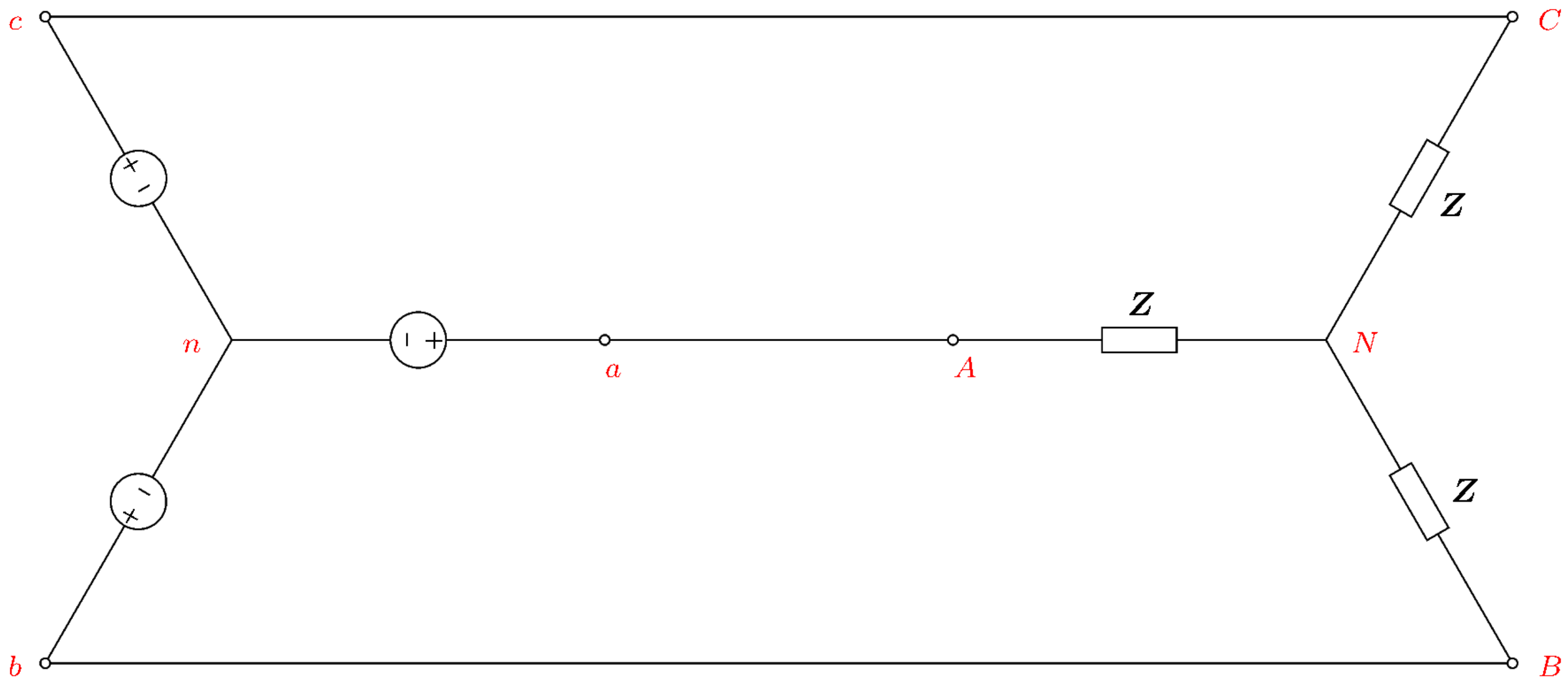


- I_{MN} *will be the current flowing from* M *to* N*.*



Example 1

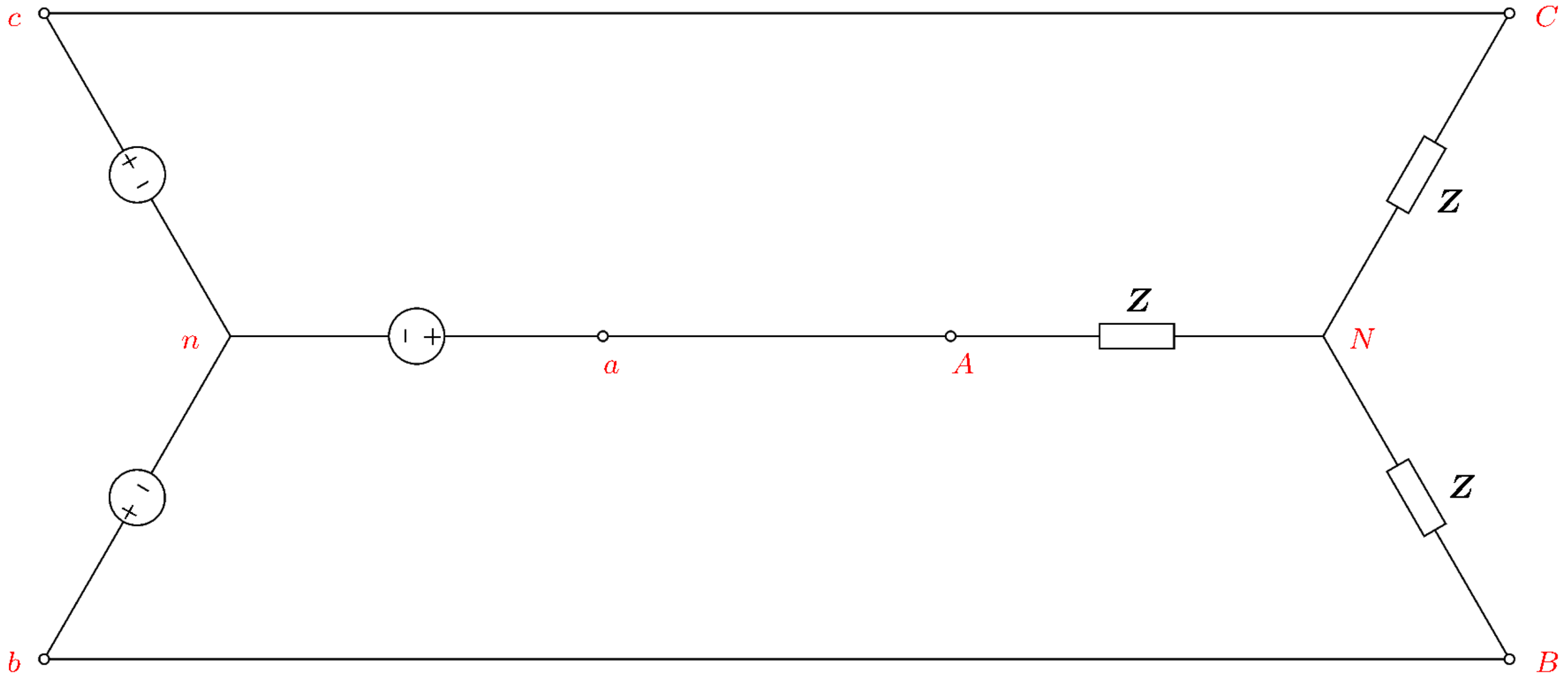
Assume $v_{an}(t) = 200\cos(\omega t)$ V *and* $\boldsymbol{Z} = 25\angle 20°\ \Omega$. *Find the voltage* $v_{ac}(t)$ *and the current* $i_{bB}(t)$.



Example 1 (Continued)

By KVL:

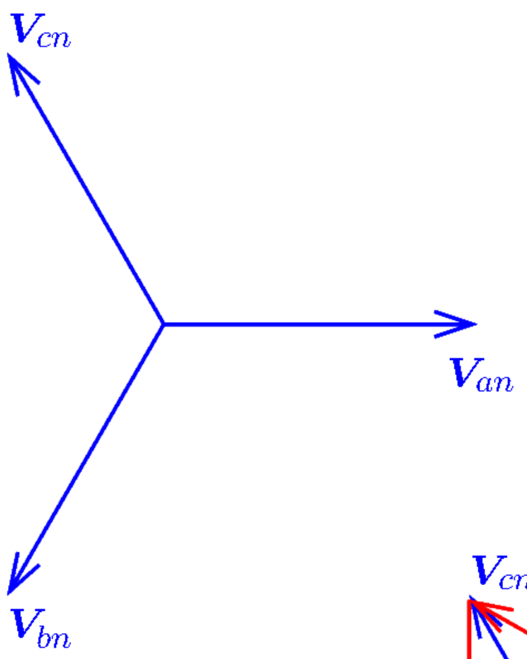
$$v_{ab} = v_{an} - v_{bn}$$
$$v_{ca} = v_{cn} - v_{an}$$
$$v_{bc} = v_{bn} - v_{cn}.$$



Example 1 (Continued)

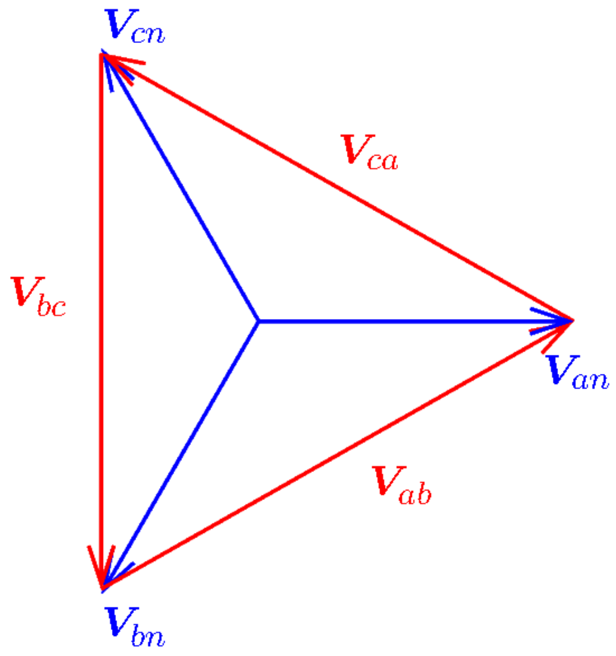
The phase voltage phasors are:

$$\boldsymbol{V_{an}} = 200\angle 0^\circ\ V$$
$$\boldsymbol{V_{bn}} = 200\angle -120^\circ\ V$$
$$\boldsymbol{V_{cn}} = 200\angle 120^\circ\ V.$$



The line voltage phasors $\boldsymbol{V_{ab}}, \boldsymbol{V_{bc}}, \boldsymbol{V_{ca}}$ *are:*

$$\boldsymbol{V_{ab}} = \boldsymbol{V_{an}} - \boldsymbol{V_{bn}}$$
$$\boldsymbol{V_{bc}} = \boldsymbol{V_{bn}} - \boldsymbol{V_{cn}}.$$
$$\boldsymbol{V_{ca}} = \boldsymbol{V_{cn}} - \boldsymbol{V_{an}}$$



The line voltages can be easily obtained graphically.

Example 1 (Continued)

Graphically, we can see that:

$$\boldsymbol{V_{ab}} = 200\sqrt{3}\angle 30^\circ\ V$$

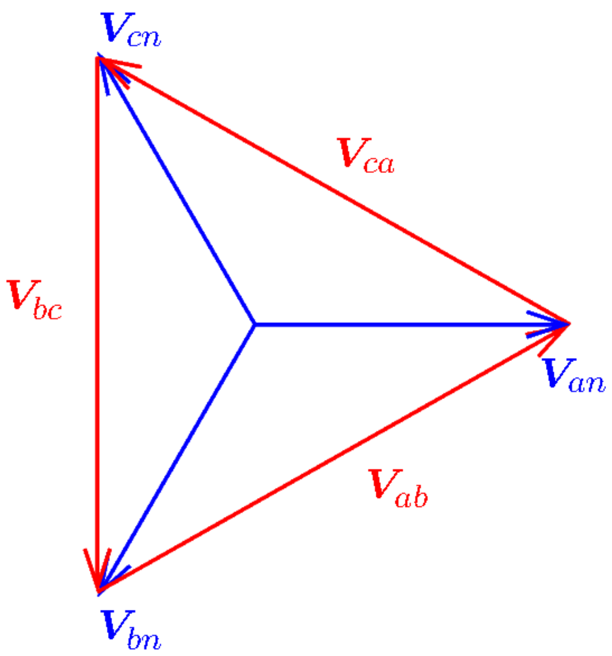
$$\boldsymbol{V_{ca}} = 200\sqrt{3}\angle 150^\circ\ V$$

$$\boldsymbol{V_{bc}} = 200\sqrt{3}\angle -90^\circ\ V.$$

Therefore,

$$v_{ac}(t) = -v_{ca}(t) = -200\sqrt{3}\cos(\omega t + 150^\circ)\, V$$

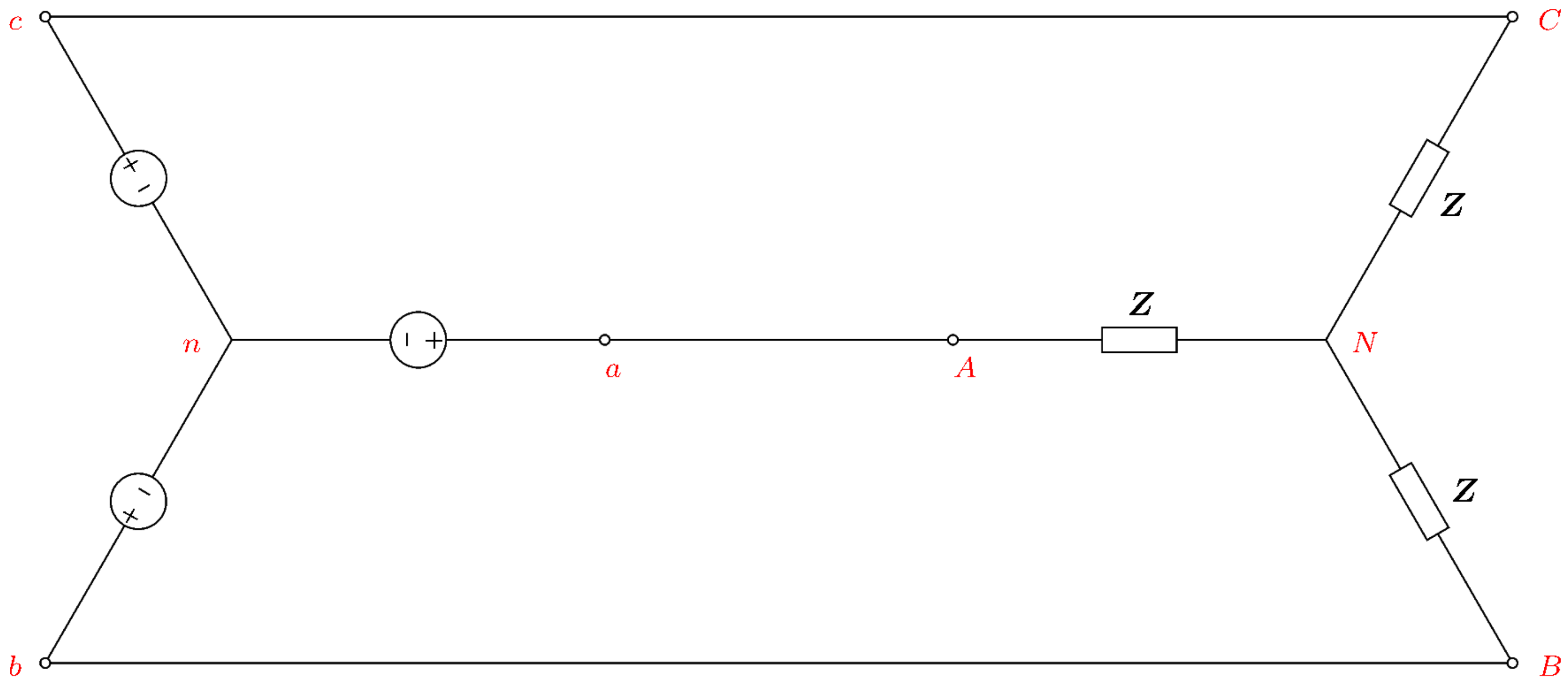
$$v_{ac}(t) = 346.41\cos(\omega t - 30^\circ)\, V\,.$$



The current i_{bB} can be obtained immediately from a per-phase equivalent.

Example 1 (Continued)

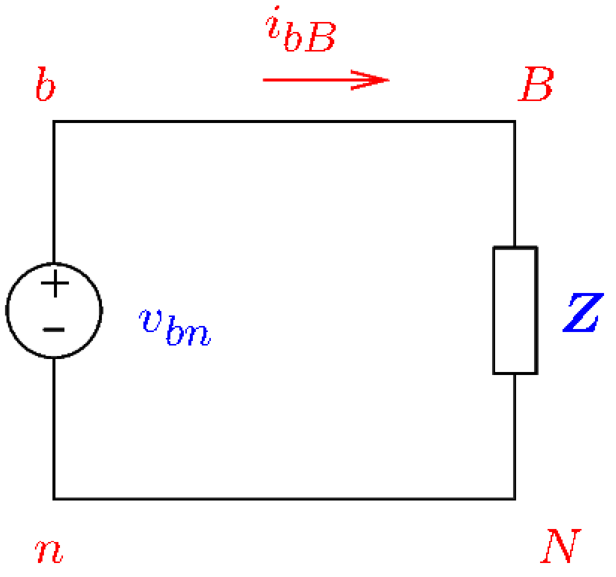
- *Since the system is balanced, the middle points n and N are at the same potential.*
- *Therefore, a per-phase equivalent may be used.*



Example 1 (Continued)

$$\boldsymbol{I_{bB}} = \frac{\boldsymbol{V_{bn}}}{\boldsymbol{Z}} = \frac{200\angle -120°}{25\angle 20°} = 8\angle -140°\ A$$

$$i_{bB}(t) = 8\cos(\omega t - 140°)\ \ A$$



Line and Phase Voltages

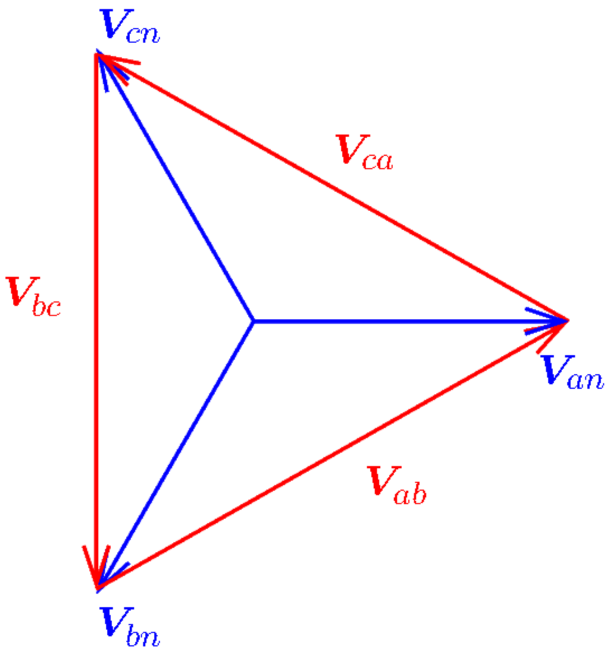
The general relationship between line and phase voltages is

$$\boldsymbol{V}_{\boldsymbol{line}} = \boldsymbol{V}_{\boldsymbol{phase}} \cdot \sqrt{3}\angle\varphi$$

where the angle φ depends on which line and phase voltages we compare.

The possible values of φ are:

$$\varphi = \pm 30°, \pm 90°, \pm 150°$$

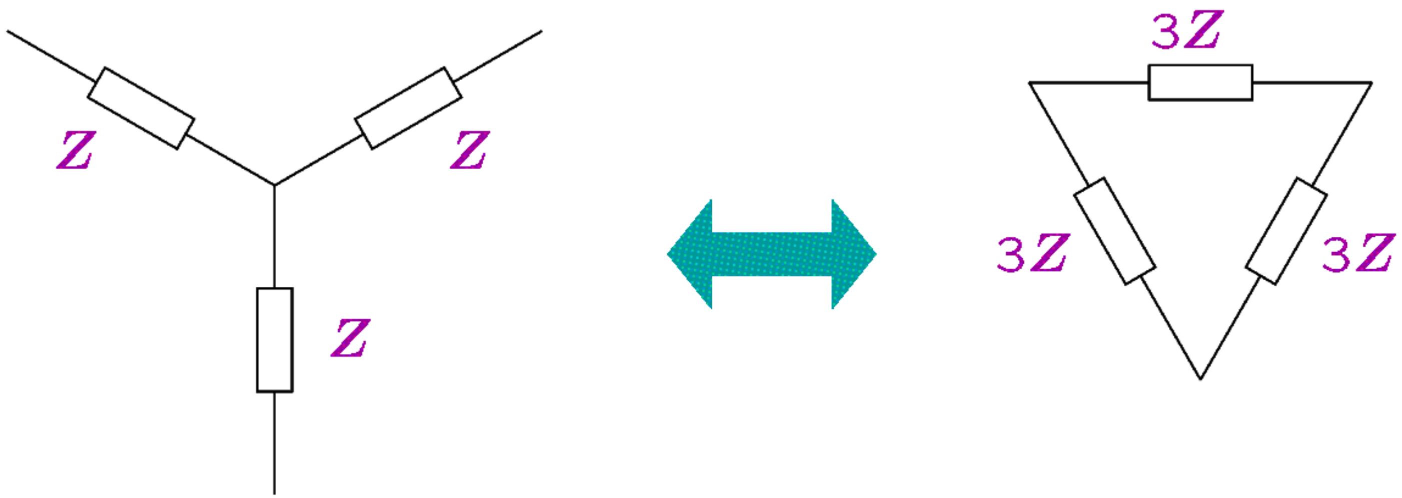


Examples:

$$\boldsymbol{V}_{\boldsymbol{ac}} = \boldsymbol{V}_{\boldsymbol{an}} \cdot \sqrt{3}\angle - 30°$$
$$\boldsymbol{V}_{\boldsymbol{ab}} = \boldsymbol{V}_{\boldsymbol{an}} \cdot \sqrt{3}\angle + 30°$$
$$\boldsymbol{V}_{\boldsymbol{cb}} = \boldsymbol{V}_{\boldsymbol{an}} \cdot \sqrt{3}\angle + 90°$$
$$\boldsymbol{V}_{\boldsymbol{bc}} = \boldsymbol{V}_{\boldsymbol{an}} \cdot \sqrt{3}\angle - 90°$$

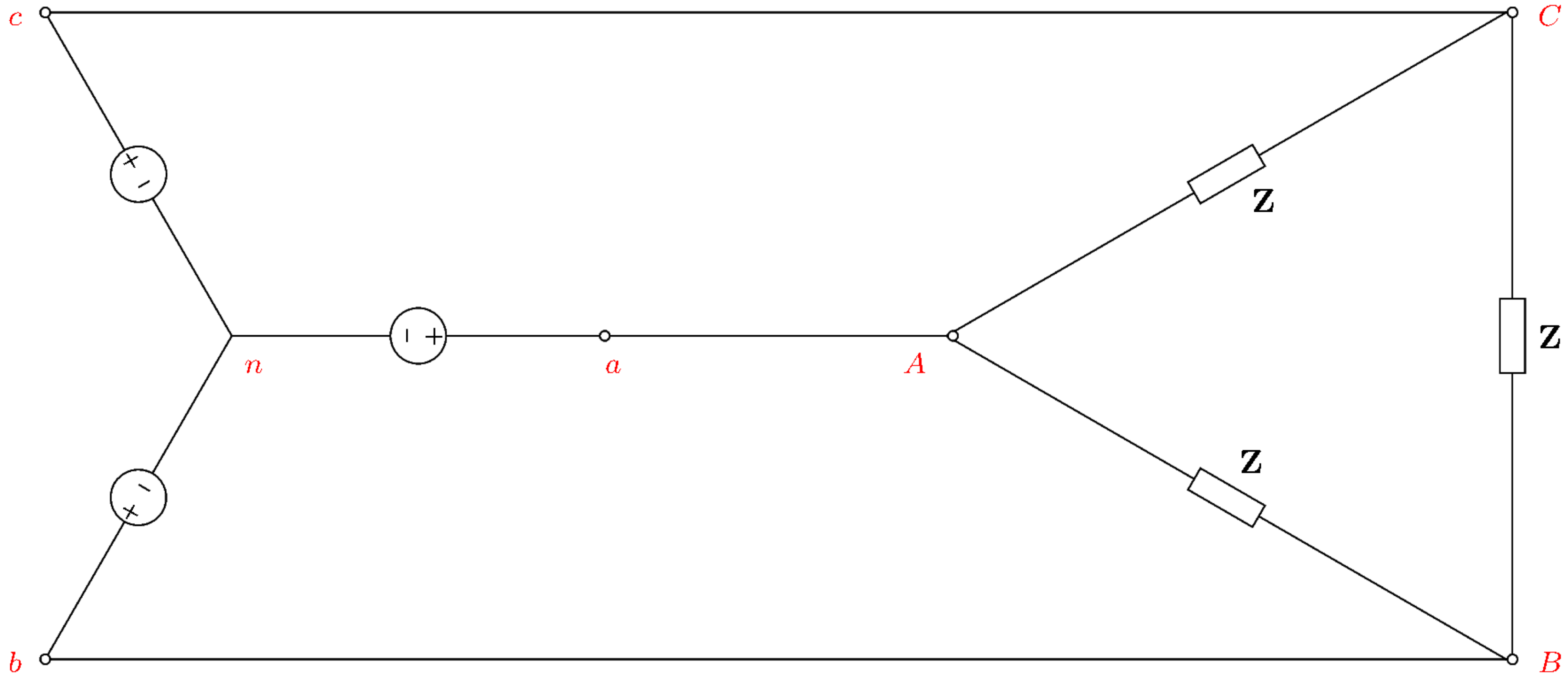
Conversions between $\boldsymbol{Y}$ and $\boldsymbol{\Delta}$

- It is sometimes convenient to substitute a Y-connected load with a Δ-connected load, or vice-versa.
- For balanced loads, the impedance per phase of the equivalent Δ-connected load is triple.



Example 2

Assume $v_{an}(t) = 300\cos(\omega t)\ V$ *and* $\boldsymbol{Z} = 24\angle 20°\ \Omega$. *Find the phasor currents* $\boldsymbol{I}_{\boldsymbol{CB}}$ *and* $\boldsymbol{I}_{\boldsymbol{bB}}$.



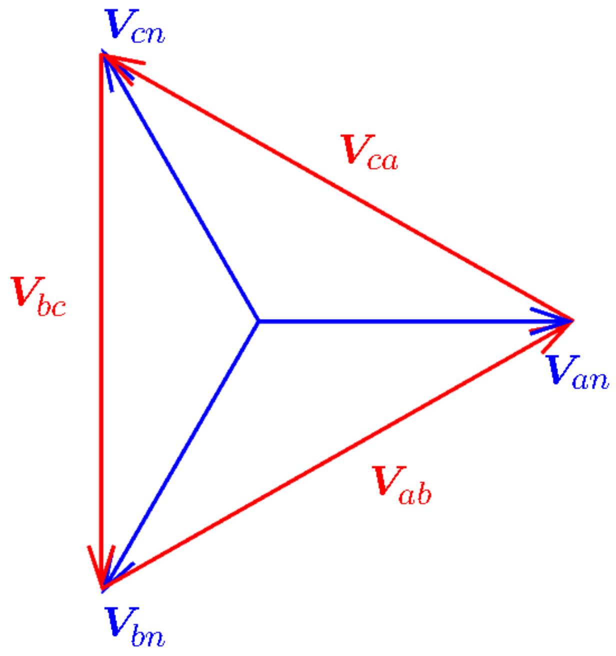
Example 2 (Continued)

From the figure, we can see that $\boldsymbol{V_{CB}} = 300\sqrt{3}\angle 90°$.

$$\boldsymbol{I_{CB}} = \frac{\boldsymbol{V_{CB}}}{\boldsymbol{Z}} = 21.65\angle 70°\ A$$

Converting the load to the equivalent Y*-connected load, the impedance per phase becomes* $\frac{\boldsymbol{Z}}{3} = 8\angle 20°\ \Omega$.

$$\boldsymbol{I_{bB}} = \frac{\boldsymbol{V_{bn}}}{\boldsymbol{Z}/3} = \frac{300\angle -120°}{24\angle 20°/3} = 37.5\angle -140°\ A$$



Example 3

Find additional circuit elements that correct the power factor in the previous example. Assume a 60 Hz frequency.

- *Since the system is balanced, the reactive power is the same on all three phases.*
- *Let's calculate the reactive power on one of the phases.*
- *The complex power on the impedance between B and C is:*

$$\boldsymbol{S_{CB}} = \boldsymbol{V_{CB,rms} I^*_{CB,rms}} = 300\sqrt{3}/\sqrt{2}\angle 90^\circ \left(21.65/\sqrt{2}\angle 70^\circ\right)^*$$

$$\boldsymbol{S_{CB}} = 5.625\angle 20^\circ\ kVA$$

$$Q_{CB} = 5.625 \sin(20^\circ) = 1.92\ kVAR$$

- *Thus, we need a balanced Y-connected or Δ-connected load that has a total reactive power* $Q = -3Q_{CB}$.

Example 3 (Continued)

- *Let's choose a* Δ*-connected load.*
- *Capacitors generate reactive power, so we will have identical capacitors on each phase of the load.*
- *Let* X_c *be the reactance of a capacitor,* $V_{c,rms}$ *its rms voltage, and* Q_c *the reactive power of the capacitor.*
- *Recall that* $X_c = -\frac{1}{C\omega}$.

$$Q_c = \frac{V_{c,rms}^2}{X_c} \Rightarrow C = -\frac{Q_c}{V_{c,rms}^2\omega}$$

- $V_{c,rms}$ *is the line voltage* $\frac{300\sqrt{3}}{\sqrt{2}}$ V, $Q_c = -Q/3$, *and* $\omega = 2\pi 60$, *so* $C = 72.73\ \mu F$.
- *If capacitors were connected in the* Y*-configuration, their value would be triple. Can you tell why?*