THREE PHASE SYSTEMS

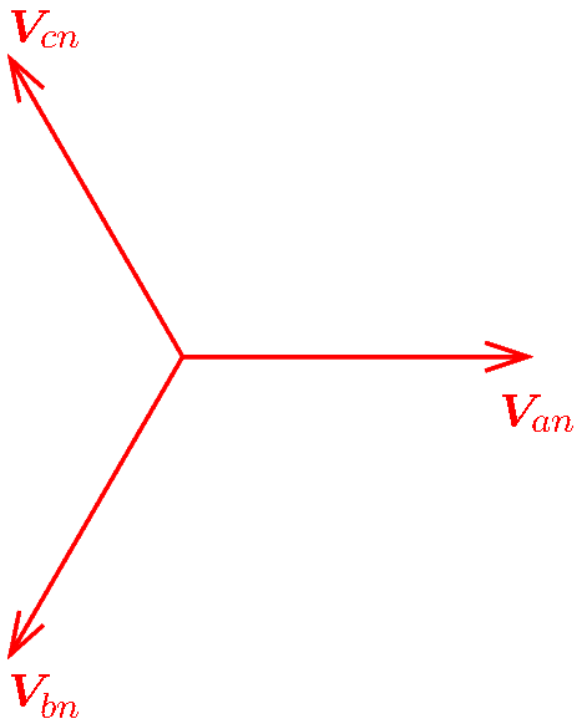
An Introduction

Introduction

- Power is usually transmitted in AC form using three phases.
- The voltage of each phase with respect to the common reference (called *neutral*) is the same, excepting the phase angle.
- Denoting by a, b, and c the three phases:

$$v_{an}(t) = V_m \cos(\omega t)$$
$$v_{bn}(t) = V_m \cos(\omega t - 120^\circ)$$
$$v_{cn}(t) = V_m \cos(\omega t + 120^\circ)$$

- In a phasor diagram, the three voltages are three vectors at 120° each from the other.

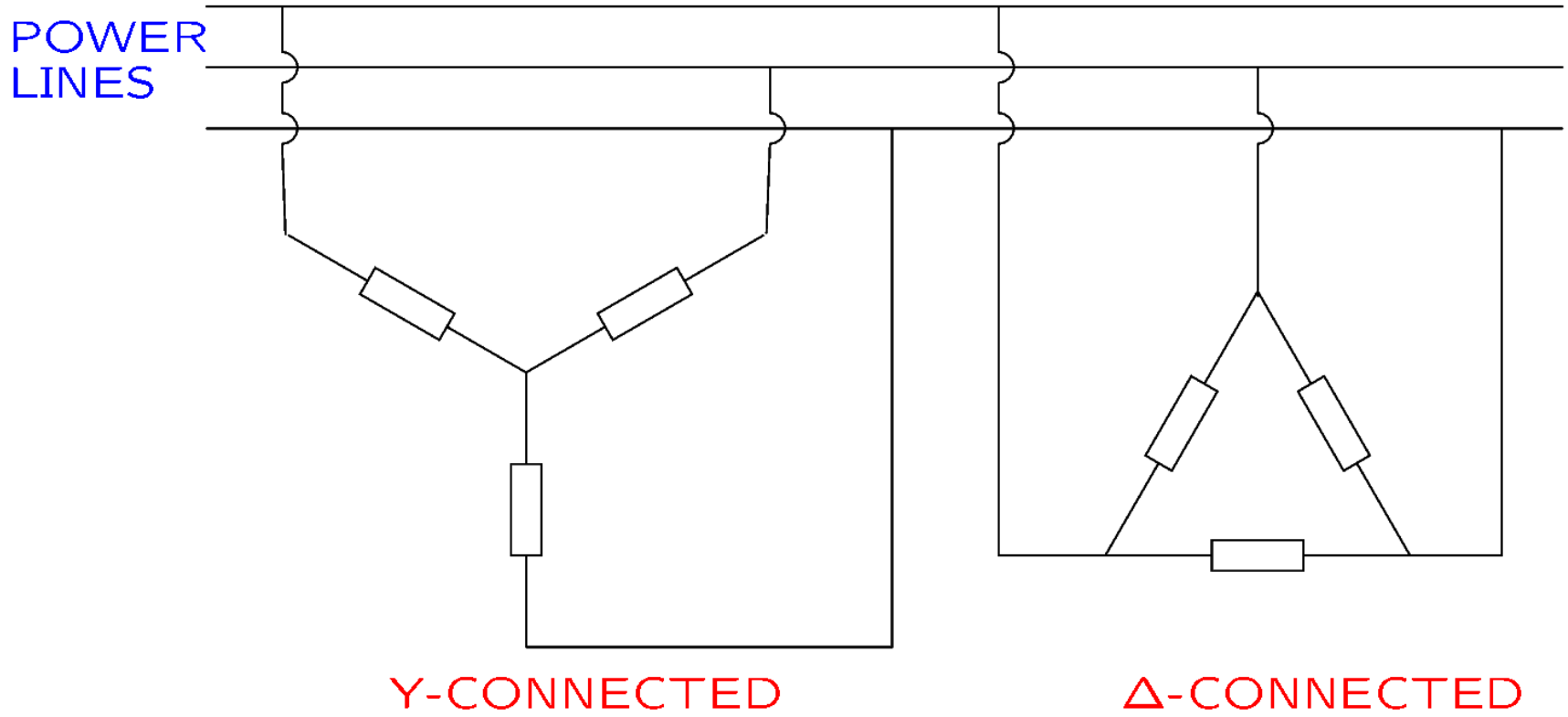


Introduction

- From a practical standpoint, three phase voltage is easily generated.
  - In a synchronous generator, coils at $120°$ from each other will naturally generate three phase voltage.
- For the same amount of power, a three phase power transmission system needs less copper than a single phase system.
  - This means that a three phase power transmission system is cheaper than a single phase system.

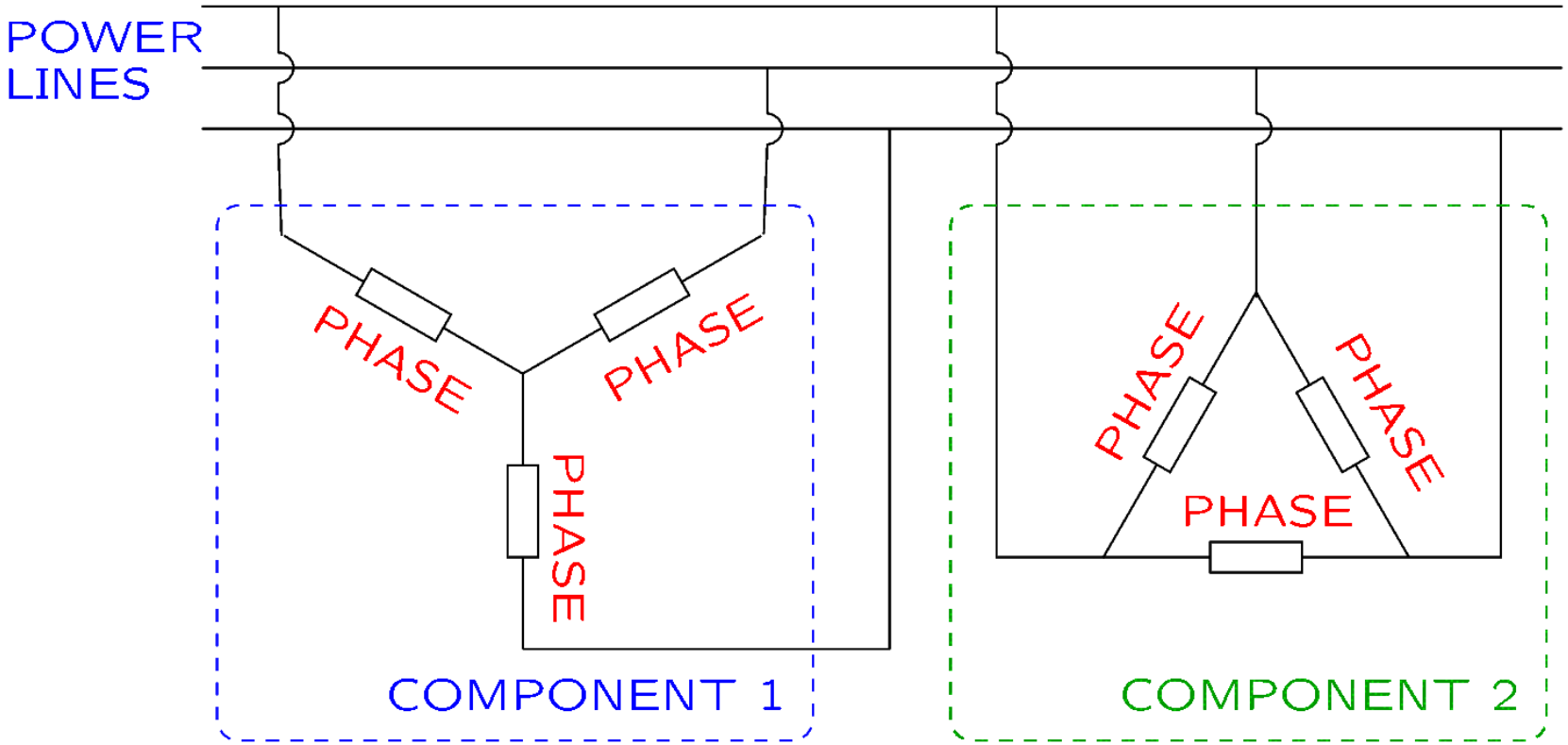
Definitions

- In a three phase system, sources and loads are connected in the *wye (Y) configuration* or the *delta (Δ) configuration*.



Definitions

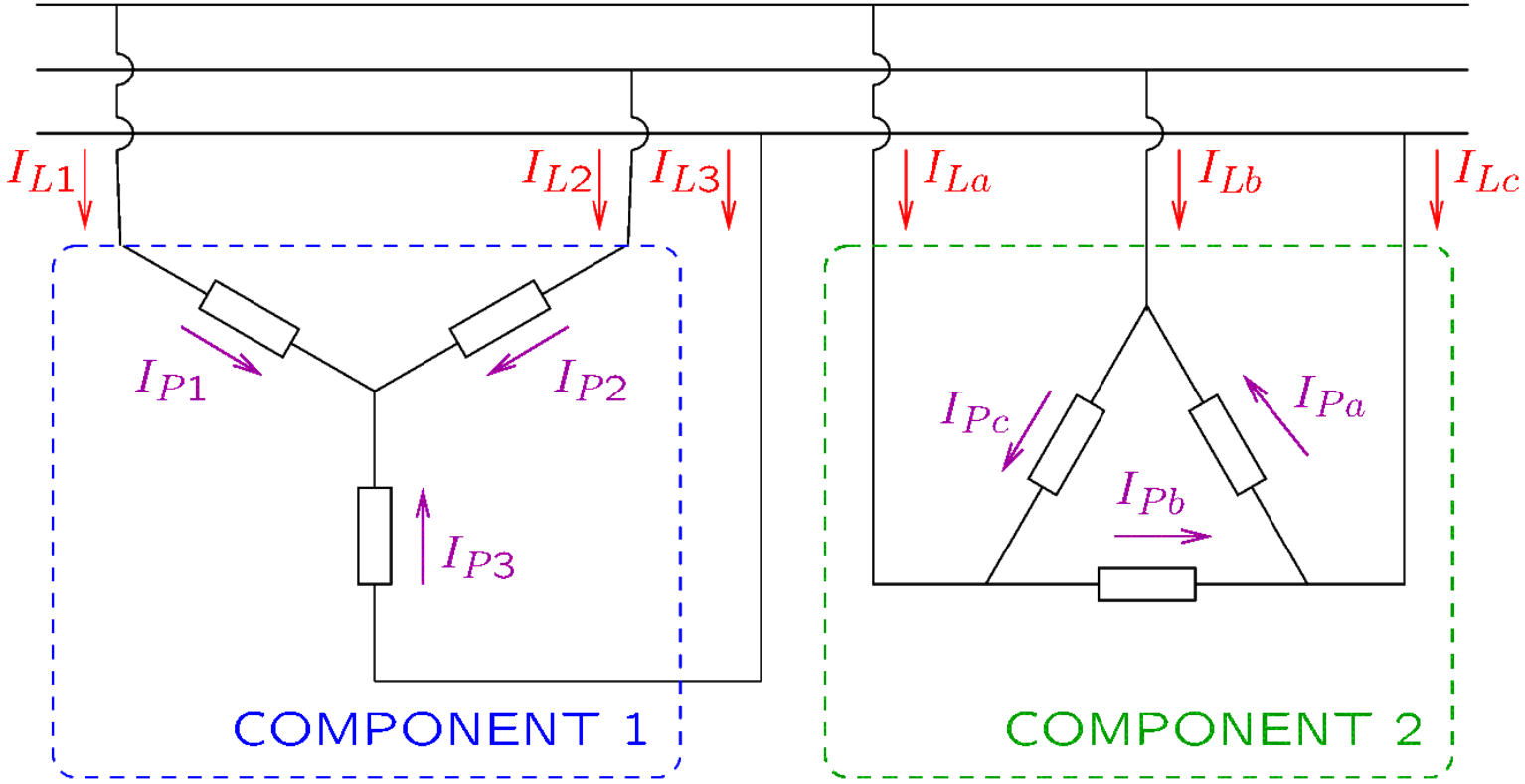
- A three phase component can be represented with three subcomponents, called *phases*, connected in Y or Δ.



Definitions—Currents

- The current flowing through a phase is called *phase current*.
- The current flowing through a conductor that powers a component is called *line current*.

- In the figure, the phase currents are:
  - I_{P1}, I_{P2}, I_{P3}
  - I_{Pa}, I_{Pb}, I_{Pc}
- The line currents are:
  - I_{L1}, I_{L2}, I_{L3}
  - I_{La}, I_{Lb}, I_{Lc}



Definitions—Currents

- *For a Y-connected component, phase and line currents are identical.*
- *For a Δ-connected component, the rms values are related by $I_L = \sqrt{3} I_P$.*

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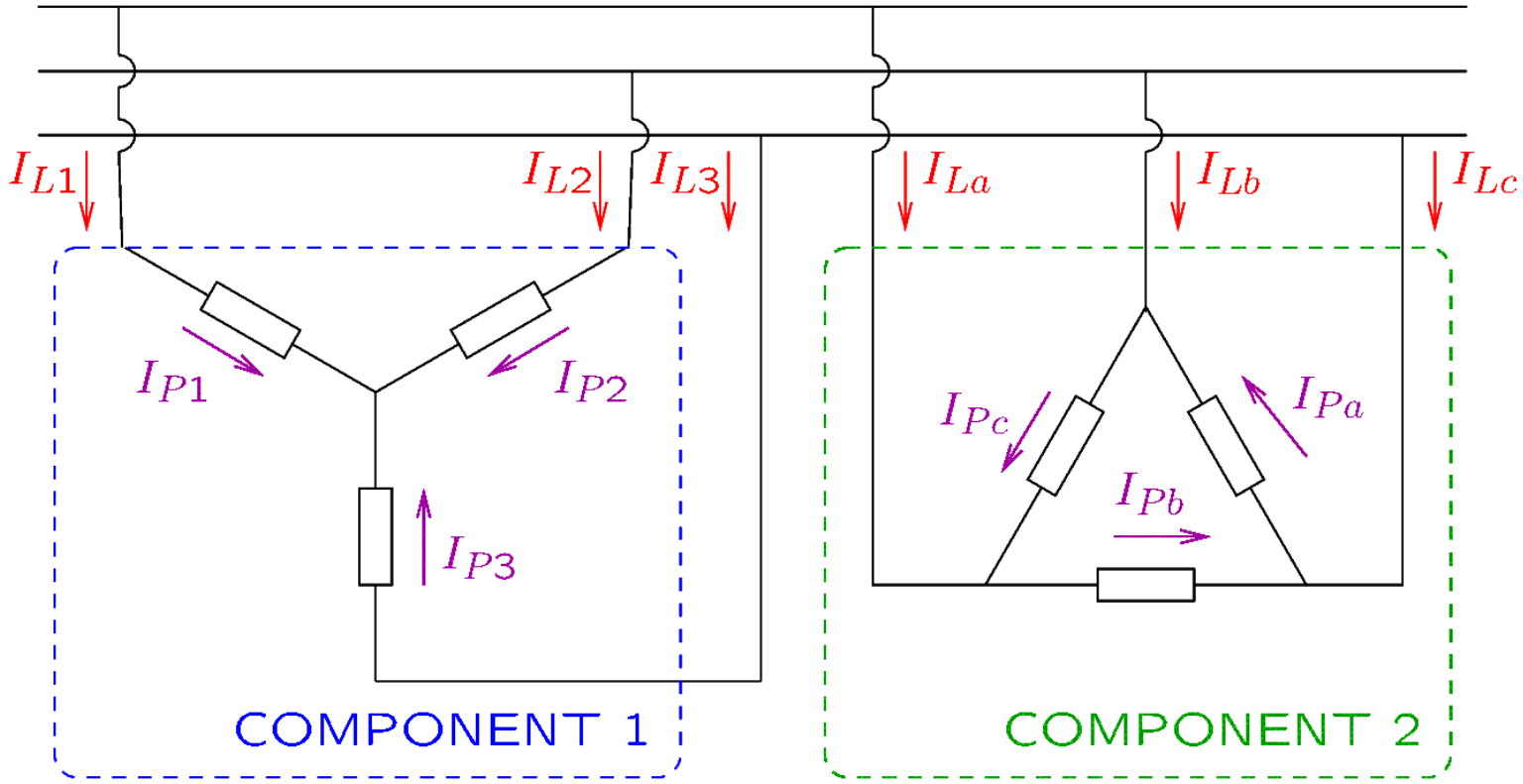

Definitions—Voltages

- The voltage on a phase is called *phase voltage*.
- The voltage between two power lines is called *line-to-line voltage,* or shortly, *line voltage*.

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  - V_{P1}, V_{P2}, V_{P3}
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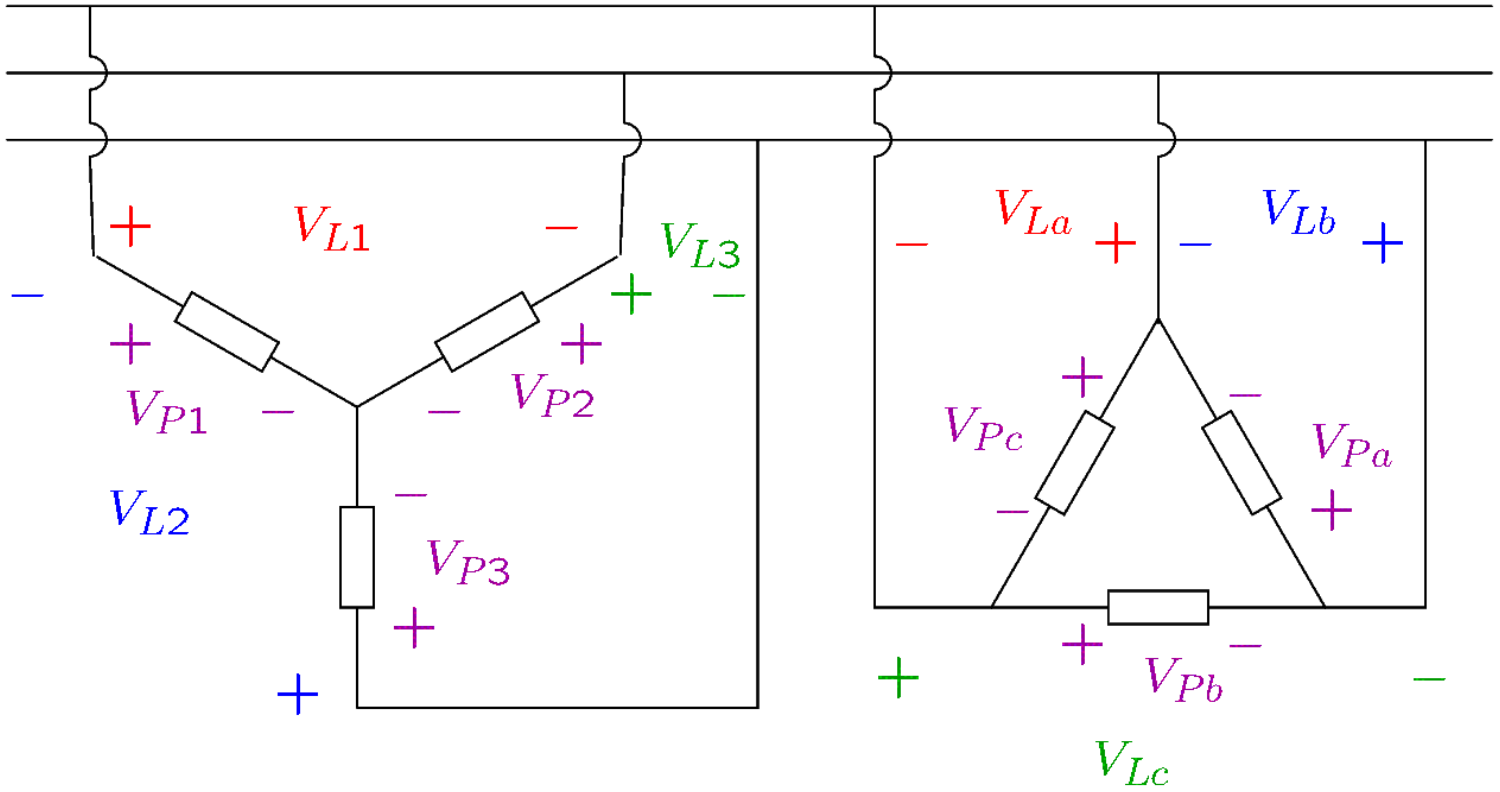

Definitions—Voltages

- *For a Δ-connected component, phase and line voltages are identical.*
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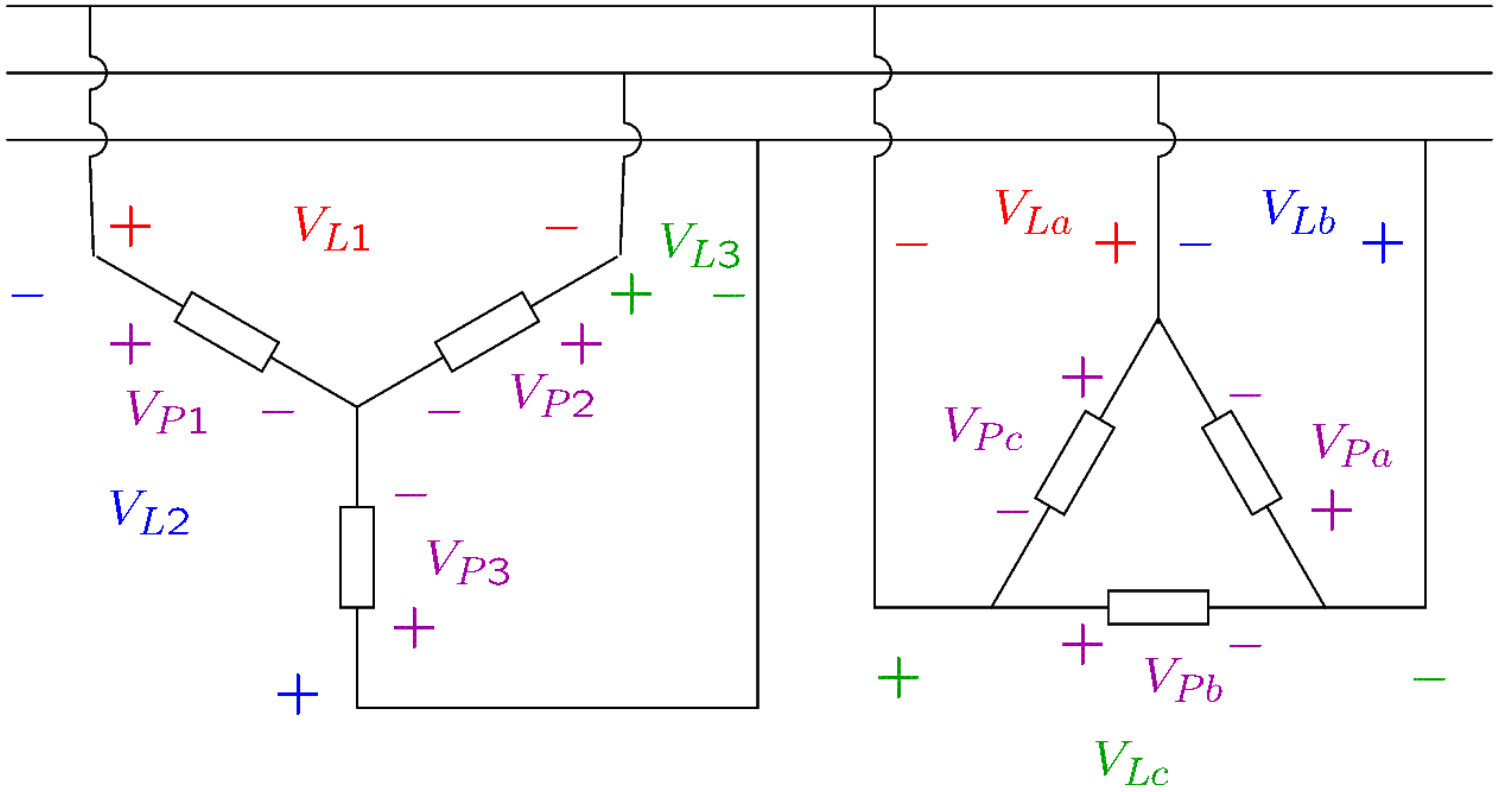


Definitions—Balanced System

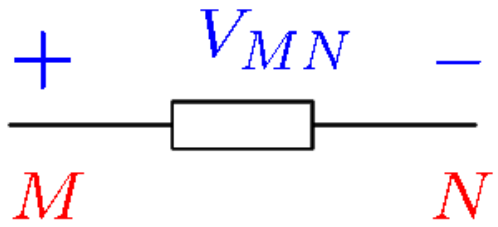
A three phase system is balanced if all of the following apply:

- The phases of the source are 120° out of phase.
- The phases of the source are identical, except for being 120° out of phase.
- The phases of the load are identical.

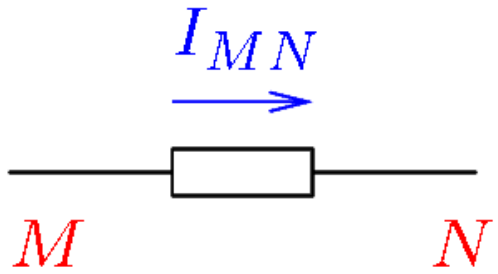
We will normally assume balanced systems.

Notation

- *Let M and N denote the terminals of a circuit element.*
- *V_{MN} will be the voltage between M and N with $+$ at M and $-$ at N.*



- *I_{MN} will be the current flowing from M to N.*

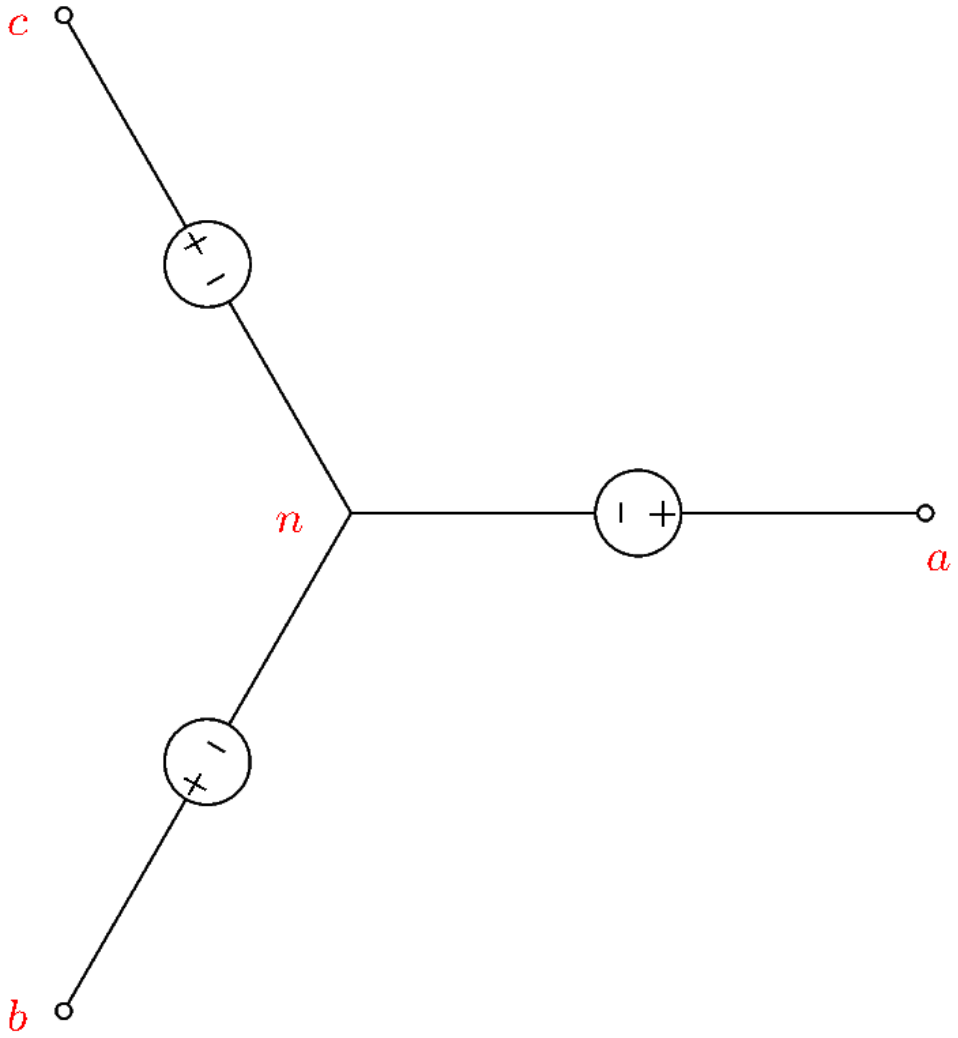


Convention

- For three phase source of voltage, we will use the convention:

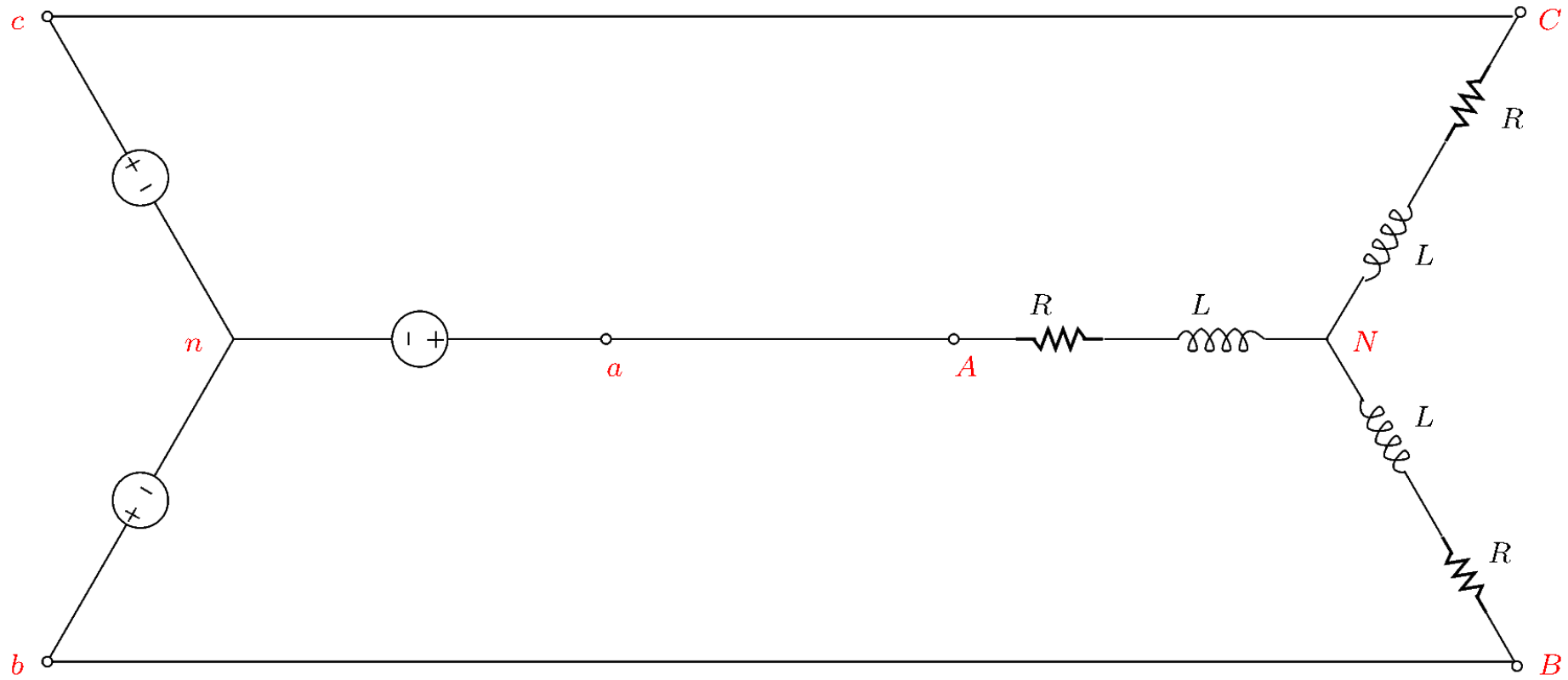
$$v_{an}(t) = V_m \cos(\omega t)$$
$$v_{bn}(t) = V_m \cos(\omega t - 120°)$$
$$v_{cn}(t) = V_m \cos(\omega t + 120°)$$

- This naming convention is known as the *positive phase sequence*.
- The negative phase sequence has $b \leftrightarrow c$.
- For a Y-connected source, the node n is called *neutral*.



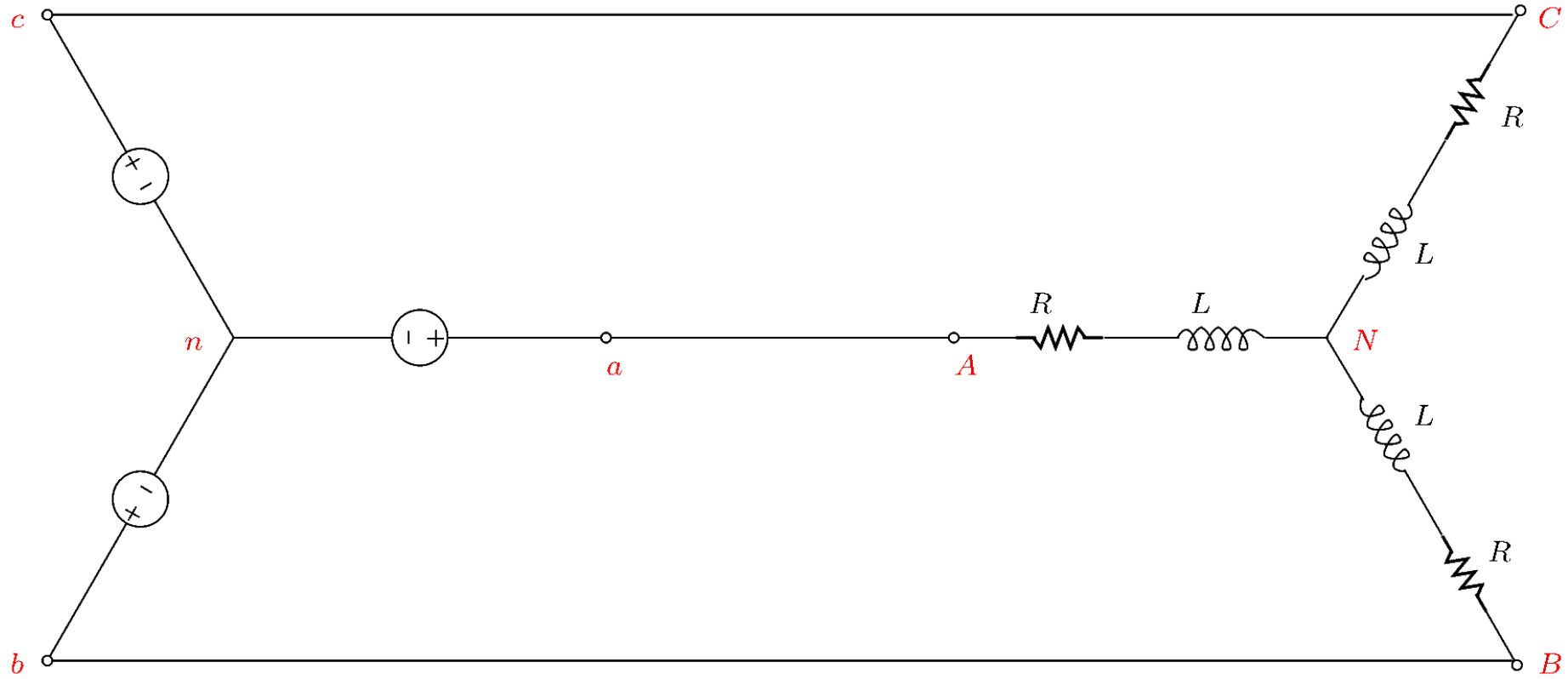
Example

The following balanced system has a line voltage of $500\ V$ *rms,* $R = 10\ \Omega$*, and inductors of reactance* $X_L = 20\ \Omega$*. Find the current* $i_{AN}(t)$*.*



Example (Continued)

- *Note that for a balanced system, the nodes n and N are at the same potential: $V_{nN} = 0$.*
- *For a balanced system, the equivalent circuit per phase may be used.*



Example (Continued)

- *In the equivalent circuit per phase:*
  - *The nodes n and N are connected.*
  - *We draw only one phase of the source and of the load.*
- *Let's find first the rms phase voltage.*

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}}\ V\ rms$$

- *Therefore, converting rms to peak amplitude:*

$$\boldsymbol{V_{AN}} = \frac{500\sqrt{2}}{\sqrt{3}} \angle 0°\ V$$

$$\Rightarrow \boldsymbol{I_{AN}} = \frac{\boldsymbol{V_{AN}}}{R + jX_L} = \frac{500\sqrt{2}/\sqrt{3}}{10 + 20j}$$

$$\Rightarrow i_{AN}(t) = 18.26\cos(\omega t - 63.43°)\ A$$