

# Complex Power

# Notation

- Let  $\mathbf{M}$  be a phasor representing current or voltage.
- The rms phasor is defined as:

$$\mathbf{M}_{rms} = \frac{\mathbf{M}}{\sqrt{2}}$$

- If  $\mathbf{M} = M\angle\alpha$ , then  $M = |\mathbf{M}|$  denotes the magnitude of  $\mathbf{M}$ .
- If  $\mathbf{M} = M\angle\alpha = A + jB$ , then the complex conjugate of  $\mathbf{M}$  is

$$\mathbf{M}^* = M\angle -\alpha = A - jB$$

- Let us recall that

$$\mathbf{M} \cdot \mathbf{M}^* = |\mathbf{M}|^2$$

# Complex Power—Definition

- The absorbed **complex power** is defined as:

$$\mathbf{S} = V_{rms} \mathbf{I}_{rms}^*$$

- Assume that  $V_{rms} = V_{rms} \angle \alpha_v$  and  $I_{rms} = I_{rms} \angle \alpha_i$ .

- Then  $\mathbf{I}_{rms}^* = I_{rms} \angle -\alpha_i$  and

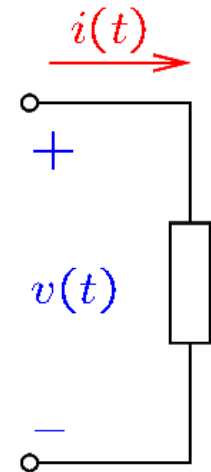
$$\mathbf{S} = V_{rms} I_{rms} \angle \alpha_v - \alpha_i$$

- In other words,

$$\mathbf{S} = S \angle \alpha_v - \alpha_i$$

where  $S = V_{rms} I_{rms}$  is the **apparent power**.

- The **unit** of  $\mathbf{S}$  is Volt-Ampere [ $V \cdot A$ ], the same as the unit of  $S$ .



# Reactive Power—Definition

- Representing  $\mathbf{S} = V_{rms}I_{rms}\angle\alpha_v - \alpha_i$  in the rectangular form,

$$\mathbf{S} = P + jQ$$

where:

- $P = V_{rms}I_{rms} \cos(\alpha_v - \alpha_i)$  is the average power.
- $Q = V_{rms}I_{rms} \sin(\alpha_v - \alpha_i)$  is the reactive power.
- The unit of  $Q$  is Volt-Ampere Reactive [VAR].
- Note that  $Q$  may be positive or negative, depending on whether the current lags or leads the voltage.

# Physical Meaning

- $P$  is associated with the resistors of the network.
- $P$  is the average power dissipated by the resistors of the network.
- $Q$  is associated with the *reactive components* of the network, that is, the capacitors and the inductors of the network.
- During each period of the source of voltage, energy goes back and forth to/from a reactive component.
- On the average, an ideal reactive component does not dissipate power.
- $|Q|$  is the maximum rate at which energy is transferred to/from the equivalent reactance of the network.

# Physical Meaning

- $P$  is the actual power dissipated by a load.
- $Q$  also has to be accounted for:
  - Excessive currents may result if  $Q$  is not controlled.
  - There are power systems that constrain the reactive power of a load.
  - Traditional power systems use synchronous generators. These can handle loads with any reactive power  $Q$ .
  - Induction generators (useful for wind turbines) require  $Q < 0$ .
  - Some electronic power converters require  $Q = 0$ .
- The complex power provides a convenient way of grouping two different concepts ( $P$  and  $Q$ ) into a single variable.

# Relation to Instantaneous Power

- $v(t) = V_m \cos(\omega t + \alpha_v)$  and  $i(t) = I_m \cos(\omega t + \alpha_i)$ .
- The absorbed **instantaneous power**  $p(t)$  is the absorbed power at time  $t$ .

$$p(t) = v(t)i(t)$$

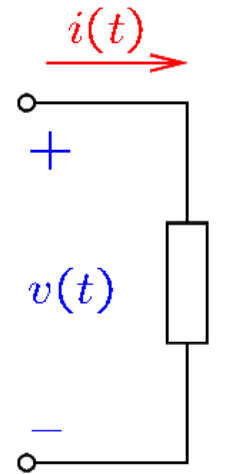
- By using trigonometric identities:

$$\begin{aligned} p(t) &= P + S \cdot \cos(2\omega t + \alpha_v + \alpha_i) \\ &= P + S \cdot \cos((2\omega t + 2\alpha_i) + \alpha_v - \alpha_i) \\ &= P + S \cdot \cos(\alpha_v - \alpha_i) \cdot \cos(2\omega t + 2\alpha_i) - S \cdot \sin(\alpha_v - \alpha_i) \cdot \sin(2\omega t + 2\alpha_i) \end{aligned}$$

- This shows that

$$p(t) = P + P \cdot \cos(2\omega t + 2\alpha_i) - Q \cdot \sin(2\omega t + 2\alpha_i)$$

- Let  $Z = R + jX$  be the impedance of the load.
  - $P + P \cdot \cos(2\omega t + 2\alpha_i)$  is the instantaneous power absorbed by  $R$ .
  - $-Q \cdot \sin(2\omega t + 2\alpha_i)$  is the instantaneous power absorbed by  $X$ .

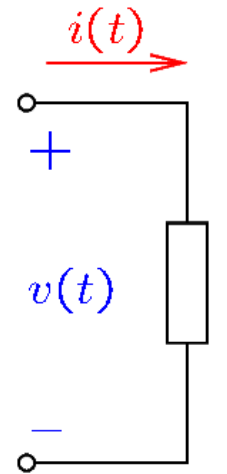


# Relation to Instantaneous Power

- The instantaneous power is

$$p(t) = P + P \cdot \cos(2\omega t + 2\alpha_i) - Q \cdot \sin(2\omega t + 2\alpha_i)$$

- Let  $Z = R + jX$  be the impedance of the load.
  - $P + P \cdot \cos(2\omega t + 2\alpha_i)$  is the instantaneous power absorbed by  $R$ .
  - $-Q \cdot \sin(2\omega t + 2\alpha_i)$  is the instantaneous power absorbed by  $X$ .
  - On the average,  $R$  dissipates the power  $P$ .
  - On the average,  $X$  dissipates zero power.
  - Energy is transferred to  $R$  at a rate  $0 \dots 2P$ , with an average rate of  $P$ .
  - Energy goes back and forth to/from  $X$  with a rate  $0 \dots |Q|$ , the maximum rate being  $|Q|$ .
- It is important to note that  $S = S \angle \alpha_v - \alpha_i$  is a complex number, not a phasor.
- $p(t)$  never equals  $S \cos(\omega t + \alpha_v - \alpha_i)$ , which is a meaningless expression.



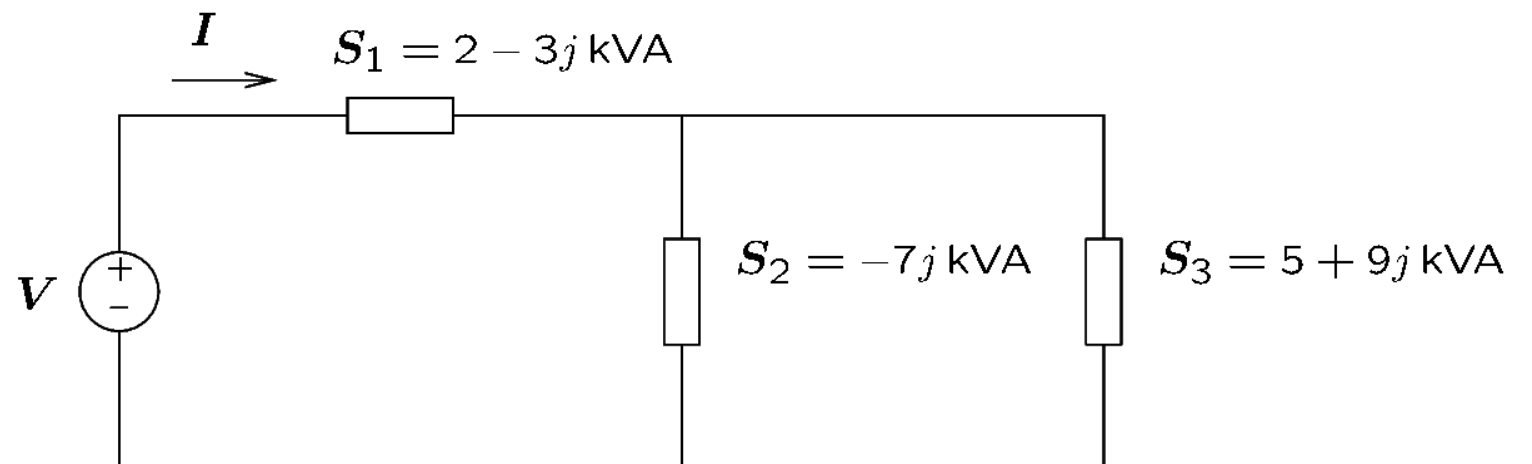
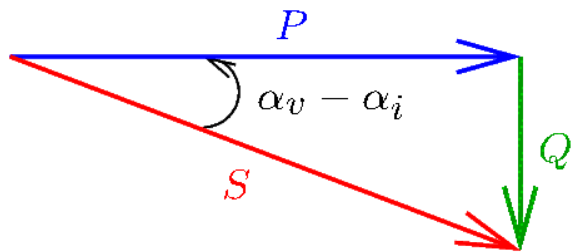
# Total Power

- The total instantaneous power can be found by adding the instantaneous power of each component.
- The same is true about the total average power, the total reactive power, and the total complex power.
- *Apparent powers cannot be added together!*
- *Power factors cannot be added together!*

# Example 1

The figure shows the power absorbed by each component. Find the power factor of the source. If the source has 300 V rms, what is the effective source current?

- The power generated by the source must equal the total power dissipated by the loads:  $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 7 - j \text{ kVA}$ . Thus,  $P = 7 \text{ kW}$  and  $Q = -1 \text{ kVAR}$ .
- Since  $\mathbf{S} = S \angle \alpha_v - \alpha_i = P + jQ$ , it follows that  $S = \sqrt{P^2 + Q^2} = 7.071 \text{ kVA}$  and  $\alpha_v - \alpha_i = \tan^{-1} \frac{Q}{P} = -8.13^\circ$ .
- The power factor is  $PF = \cos(-8.13^\circ) = 0.99 \text{ leading}$  ( $\alpha_v - \alpha_i < 0 \Rightarrow \text{leading}$ ).



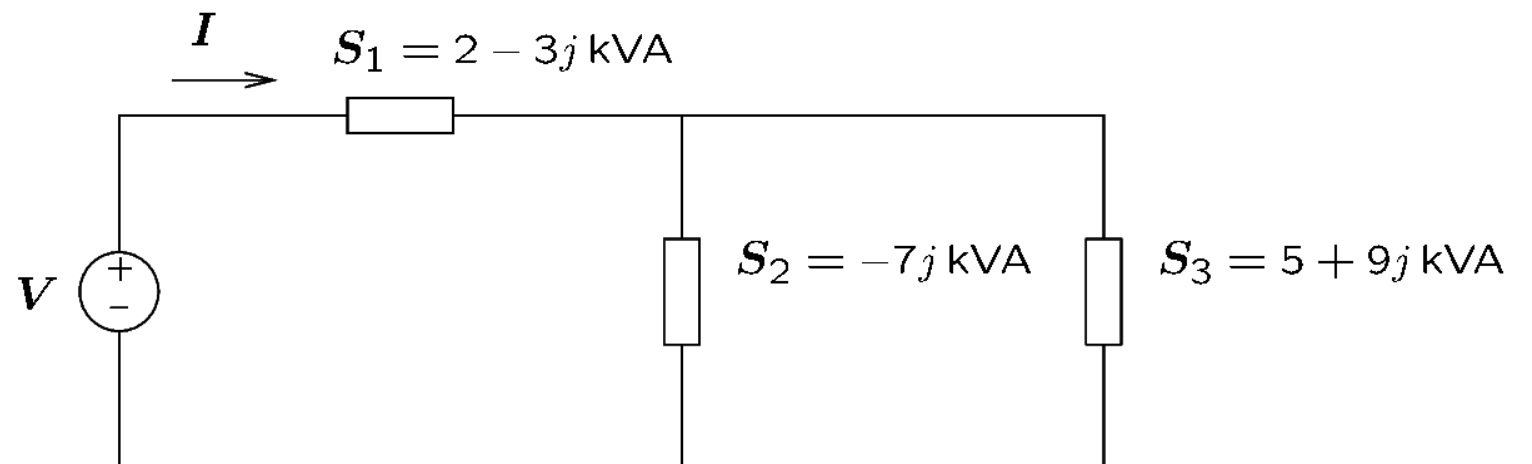
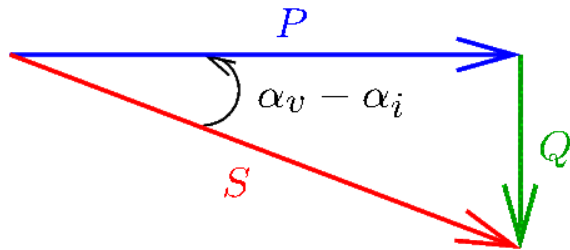
# Example 1 (Continued)

- Since  $S = \sqrt{P^2 + Q^2} = 7.071 \text{ kVA}$  and  $V_{rms} = 300 \text{ V}$ , we can find

$$I_{rms} = \frac{S}{V_{rms}} = 23.57 \text{ A}$$

- While the problem does not ask for the instantaneous current, if we assume  $\alpha_v = 0$ , we have sufficient information to find it:

$$i(t) = I_{rms}\sqrt{2} \cos(\omega t + \alpha_i) = 33.33 \cos(\omega t - 8.13^\circ) \text{ A}$$



## Example 2

*A certain load absorbs 2 kW at a power factor of 0.3 lagging. The source has 300 V rms and 60 Hz. Find the impedance of the load and an additional component correcting the power factor when connected in parallel to the load.*

- $P = V_{rms} I_{rms} \cos(\alpha_v - \alpha_i) = 300 I_{rms} \cdot 0.3 \Rightarrow I_{rms} = \frac{200}{9} \text{ A}.$

- $PF = 0.3 \Rightarrow \alpha_v - \alpha_i = \pm \cos^{-1} 0.3 = \pm 72.54^\circ.$

- *Lagging implies  $\alpha_v > \alpha_i$ . Thus,  $\alpha_v - \alpha_i = +72.54^\circ$ .*

- *The impedance is:*

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle \alpha_v}{I_m \angle \alpha_i} = \frac{V_{rms}}{I_{rms}} \angle \alpha_v - \alpha_i = \mathbf{13.5 \angle 72.54^\circ \Omega}$$

- *We have already discussed two power factor correction methods.*

- *A third method will be introduced here.*

## Example 2

- The reactive power is  $Q = P \cdot \tan(\alpha_v - \alpha_i) = 6.36 \text{ kVAR}$ .
- We need a component with a reactive power  $Q_a$  that cancels  $Q$ :  
$$Q_a = -Q$$
- $Q_a = V_{rms} I_{a,rms} \sin(\alpha_v - \alpha_{a,i}) = -6.36 \text{ kVAR}$ .
- Since the component does not absorb power ( $P_a = 0$ ),  $\alpha_v - \alpha_{a,i} = \pm 90^\circ$ .
- For  $Q_a < 0$ , we need  $\alpha_v - \alpha_{a,i} = -90^\circ$ .
- $I_{a,rms} = \frac{6.36 \text{ kVAR}}{300 \text{ V}} \Rightarrow \mathbf{Z_a} = \frac{V_{rms}}{I_{a,rms}} \angle -90^\circ = -14.15j \Omega$ .
- This impedance corresponds to a capacitor:  $\mathbf{Z_a} = \frac{1}{j\omega C}$ .
- Given that  $\omega = 2\pi f$  and  $f = 60 \text{ Hz}$ ,  $\mathbf{C = 187.44 \mu F}$ .