AC Power—Part 3

Apparent Power, Power Factor, and Power Factor Correction

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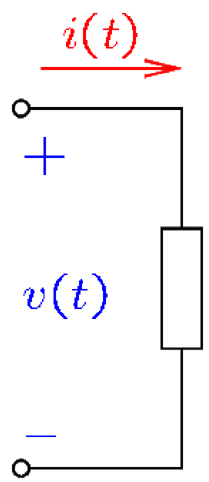
Power—Review



- Let $v(t) = V_m \cos(\omega t + \alpha_v)$ and $i(t) = I_m \cos(\omega t + \alpha_i)$.
- The instantaneous power $p(t)$ is the power at time t.
- The absorbed instantaneous power is:

$$p(t) = v(t)i(t)$$

- The apparent power is measured in $V \cdot A$ (Volt-Amperes) and has the formula:

$$S = V_{rms} I_{rms}$$

- The average of $p(t)$ is the average power:

$$P = S \cdot \cos(\alpha_v - \alpha_i)$$

- The power factor is $PF = \cos(\alpha_v - \alpha_i)$.
- When specifying the power factor, we indicate whether the current $i(t)$ is lagging or leading the voltage $v(t)$.

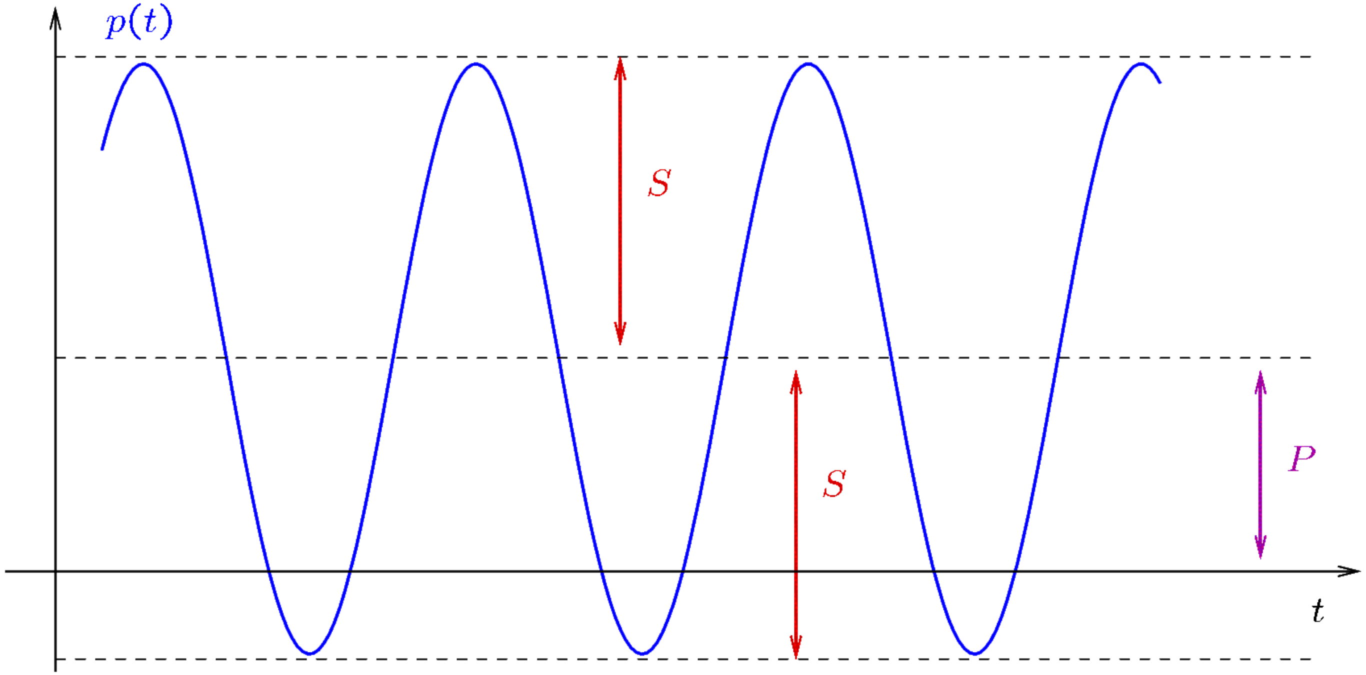
Power—Physical Meaning

- Let $v(t) = V_m \cos(\omega t + \alpha_v)$ and $i(t) = I_m \cos(\omega t + \alpha_i)$.
- The instantaneous power $p(t) = v(t)i(t)$ can also be written as:

$$p(t) = \frac{V_m I_m}{2}\cos(\alpha_v - \alpha_i) + \frac{V_m I_m}{2}\cos(2\omega t + \alpha_v + \alpha_i)$$

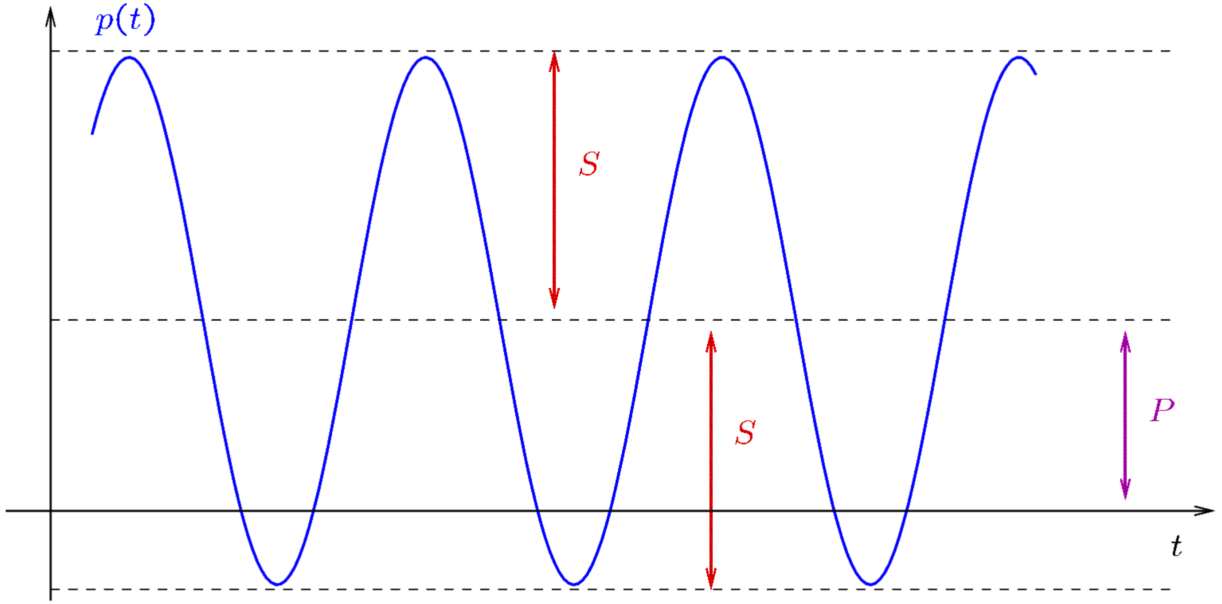
$$\Rightarrow p(t) = P + S \cdot \cos(2\omega t + \alpha_v + \alpha_i)$$

- P is the *average* of $p(t)$.
- S is the *amplitude* of $p(t)$.



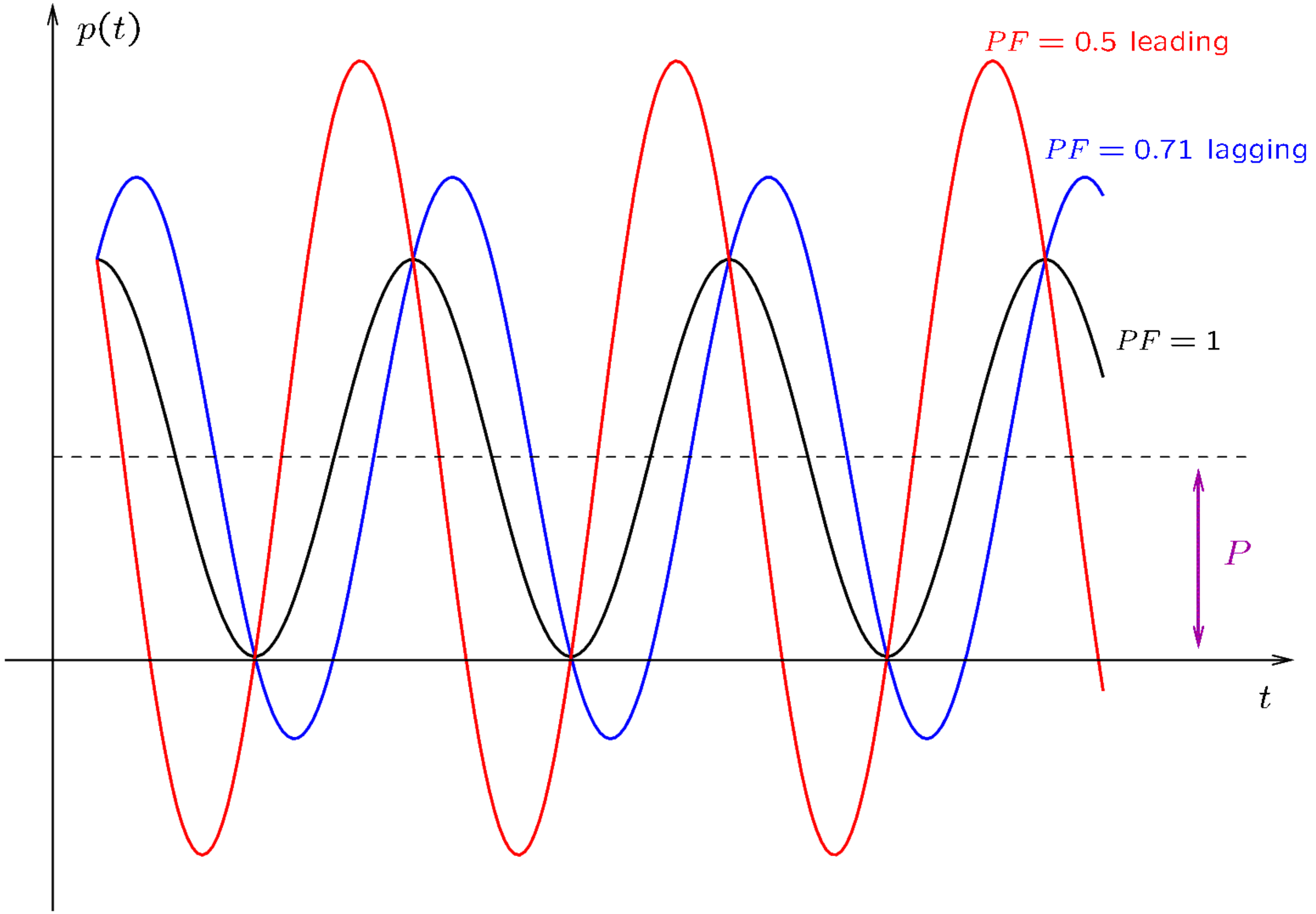
Power—Physical Meaning

- When $p(t) > 0$, energy is transferred from the source to the load.
- When $p(t) < 0$, energy is transferred backwards, from the load to the source.
- On the average, energy is transferred from the source to the load at the rate P.
- Practical systems cannot transfer energy without losses.
- A system in which energy keeps going back and forth between the source and the load is wasteful!



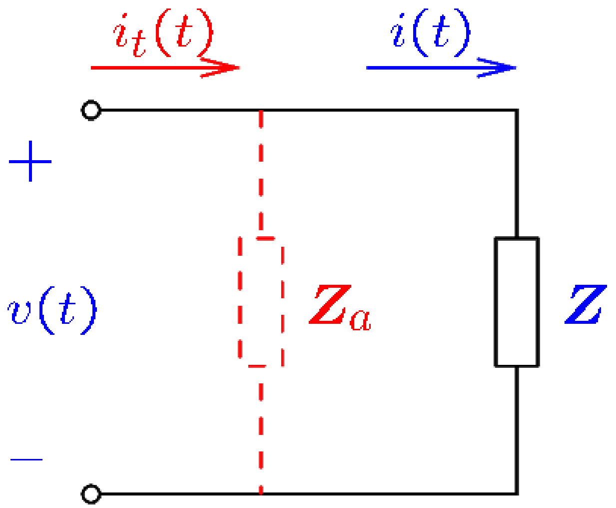
Power—Physical Meaning

- A given average power P can be delivered at any non-zero power factor.
- The lower the power factor, the bigger the amplitude of $p(t)$.
- A big amplitude of $p(t)$ indicates a big current.
- Transmission losses are proportional to the square of the current.
- Clearly, the best curve is the one with $PF = 1$.
- When $PF = 1$, the energy goes from the source to the load and never backwards.



Power Factor Correction

- In power factor correction, an additional component $\boldsymbol{Z_a}$ is connected in parallel to a load $\boldsymbol{Z}$ so as to ensure an overall $PF = 1$.
- Power factor correction minimizes the current: the total current i_t will be smaller than i and will have the least value when $PF = 1$.
- Because power factor correction reduces losses, it is important in practice.

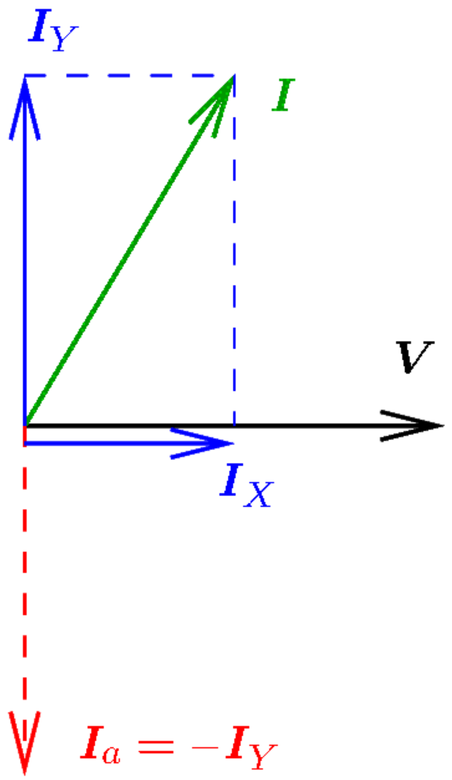


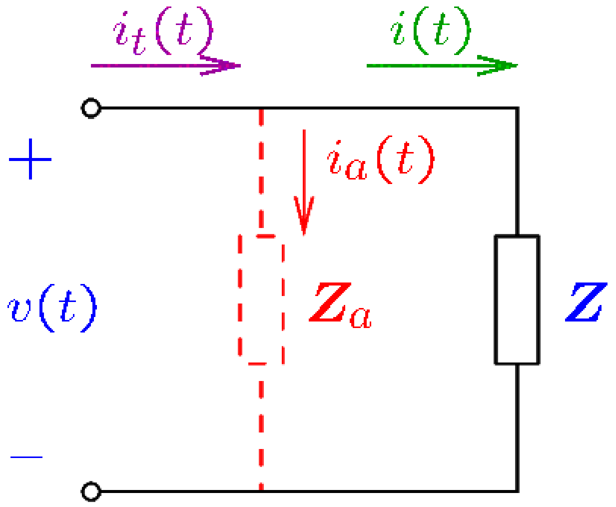
Power Factor Correction Methods

Method 1: Using $\boldsymbol{V}$ as a reference, sketch a phasor diagram. Select $\boldsymbol{Z_a}$ so that $\boldsymbol{I_Y} = -\boldsymbol{I_a}$.

Method 2: Use $\boldsymbol{Z_a} = -\frac{1}{jB}$, where B is the susceptance of $\boldsymbol{Z}$.

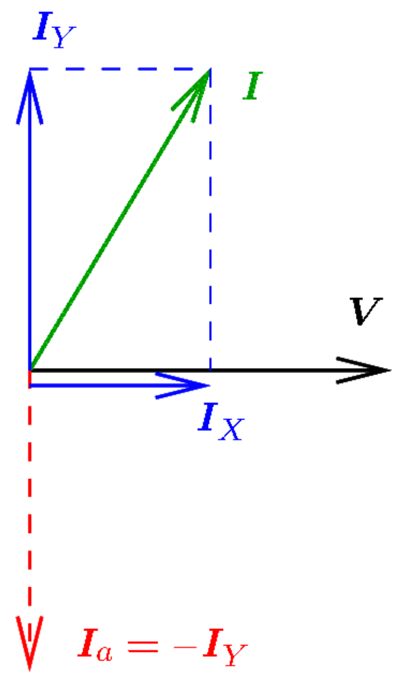
Method 3: Select $\boldsymbol{Z_a}$ so as to cancel the *reactive power* of $\boldsymbol{Z}$. This method will be studied in a future lecture.





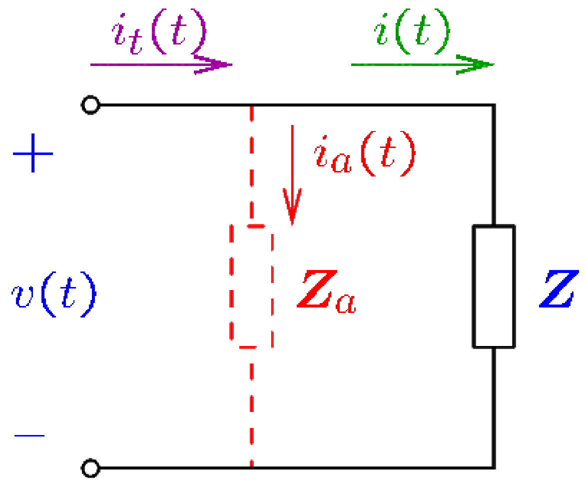
Example 1

Assume $v(t) = 200\cos(\omega t)\ V$ and $i(t) = 20\cos(\omega t + 60°)\ A$. The frequency is 60 Hz. Find $\boldsymbol{Z_a}$ that corrects the power factor.



- *Note that* $\boldsymbol{I} = 20\angle 60°$.
- *Therefore,* $\boldsymbol{I_X} = 20\cos 60° = 10\ A$ *and* $\boldsymbol{I_Y} = 20\sin 60°\ \angle 90° = 10\sqrt{3}\angle 90°\ A$.
- $\boldsymbol{I_a} = -\boldsymbol{I_Y} = 10\sqrt{3}\angle -90°\ \ A$.
- $\boldsymbol{Z_a} = \frac{\boldsymbol{V}}{\boldsymbol{I_a}} = \frac{200\angle 0°}{10\sqrt{3}\angle -90°} = \frac{20j}{\sqrt{3}}$.
- $\boldsymbol{Z_a}$ *corresponds to an inductor of value*

$$L = \frac{\boldsymbol{Z_a}}{j\omega} = \frac{20}{2\pi 60\sqrt{3}} = 30.63\ mH.$$



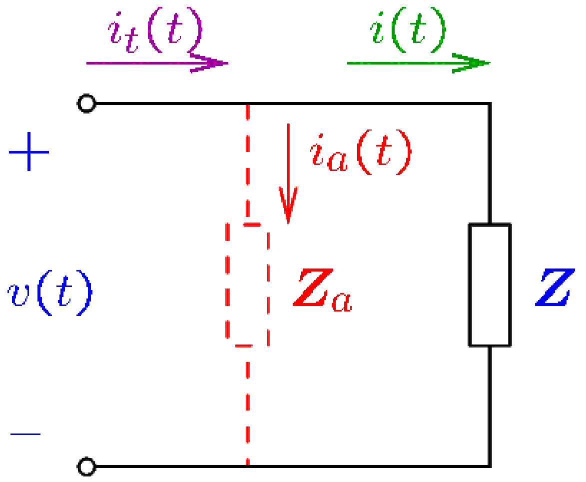
- *By power factor correction, the total current was reduced to* $i_t(t) = i_x(t) = 10\cos(\omega t)\ A$*, two times smaller than* $i(t)$*!*

Example 2

Assume $v(t) = 200\cos(\omega t)\ V$ *and* $i(t) = 20\cos(\omega t - 60°)\ A$*. The frequency is 60 Hz. Find* $\boldsymbol{Z_a}$ *that corrects the power factor using the second method.*

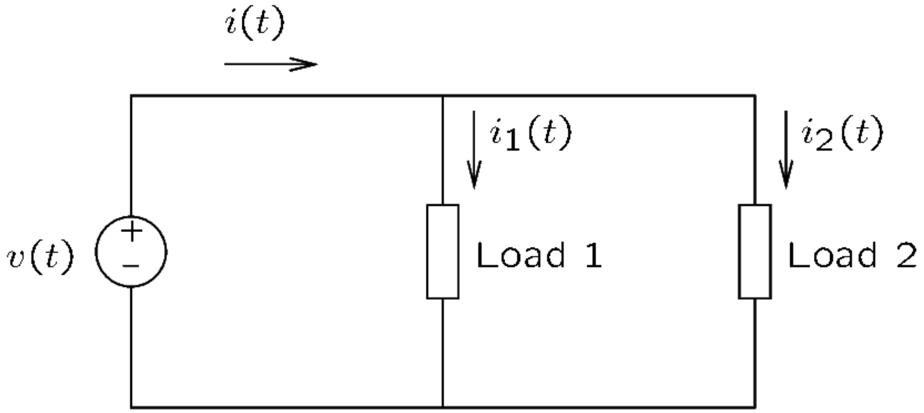
- *Note that* $\boldsymbol{I} = 20\angle - 60°$.
- *The admittance is* $\boldsymbol{Y} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{20\angle{-60°}}{200\angle 0°} = 0.1\angle - 60°\ \Omega^{-1}$.
- *The susceptance is* $B = 0.1\sin(-60°)\ \Omega^{-1}$.
- $\boldsymbol{Z_a} = -\frac{1}{jB} = \frac{1}{j0.1\sqrt{3}/2}\Omega$.
- $\boldsymbol{Z_a}$ *corresponds to a capacitor of value*

$$C = \frac{1}{j\omega \boldsymbol{Z_a}} = \frac{0.1\sqrt{3}/2}{2\pi 60} = 229.72\ \mu F.$$



Example 3

Assume $v(t) = 200\cos(\omega t)\ V$ and $i_1(t) = 20\cos(\omega t - 45°)\ A$. The second load absorbs $5\ kW$ at a power factor of 0.5 lagging. What is the power factor of the source?



- $\frac{|\boldsymbol{I_2}|}{\sqrt{2}} = \frac{5kW}{\frac{200\ V}{\sqrt{2}} \cdot 0.5} \Rightarrow |\boldsymbol{I_2}| = 100\ A.$
- *The angle of $\boldsymbol{I_2}$ is derived from*

  $$\alpha_v - \alpha_{i2} = \cos^{-1} 0.5 = \pm 60°.$$

- *Since $\boldsymbol{I_2}$ is lagging, $\alpha_v - \alpha_{i2} = +60°$.*
- *Thus, $\boldsymbol{I_2} = 100\angle - 60°\ A$.*
- $\boldsymbol{I} = \boldsymbol{I_1} + \boldsymbol{I_2} = 20\angle - 45° + 100\angle - 60° = 119.43\angle\ - 57.52°\ A.$
- *The power factor is*

$$PF = \cos(57.52°) = 0.537\ \textit{lagging}.$$