AC Power—Part 2

RMS Values, Apparent Power, and Power Factor

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RMS Values—Review

- Let $u(t)$ be a current or a voltage.
- The *root mean square* value of $u(t)$, also known as *rms* value or *effective* value is informally defined as

$$U_{rms} = \sqrt{Mean\ of\ u(t)^2}$$

- Formally,

$$U_{rms} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t)^2 dt}$$

- If $u(t)$ is periodic and has the period T, the formula can be simplified to

$$U_{rms} = \sqrt{\frac{1}{T} \int_{-T/2}^{T/2} u(t)^2 dt}$$

RMS Values

- In general, there are simpler ways to find rms values.
- If $u(t) = U_m \cos(\omega t + \alpha)$, then $U_{rms} = U_m/\sqrt{2}$.
- If $u(t) = U_0 + \sum_k (U_k \cos(\omega_k t + \alpha_k) + U_k' \sin(\omega_k t + \alpha_k))$ and *all frequencies ω_k are distinct*, then

$$U_{rms} = \sqrt{U_0^2 + \sum_k \left(\frac{U_k^2}{2} + \frac{{U_k'}^2}{2} \right)}$$

- The formula above states that the rms value is the square root of the sum of the squares of the rms values of the components of $u(t)$.

RMS Values—Examples

Example: Find the effective value of $v(t) = 3 - 2\cos(100t) + 4\cos(150t)\ V.$

$$V_{rms} = \sqrt{3^2 + \frac{(-2)^2}{2} + \frac{4^2}{2}} = 4.36\ V$$

Example: Find the effective value of $v(t) = -3 + 2\cos(100t) - 5\sin(150t)\ V.$

$$V_{rms} = \sqrt{(-3)^2 + \frac{2^2}{2} + \frac{(-5)^2}{2}} = 4.85\ V$$

Example: Find the rms value of $v(t) = 2\cos(50t + 10°) - 4\sin(50t + 10°) - 5\sin(60t)\ V.$

$$V_{rms} = \sqrt{\frac{2^2}{2} + \frac{(-4)^2}{2} + \frac{(-5)^2}{2}} = 4.74\ V$$

RMS Values—Examples

Example: Find the effective value of $v(t) = 3\cos(50t) + 5\cos(50t + 180°)\ V.$

- *The two components of* $v(t)$ *do not have the form*

$$U\cos(\omega t + \alpha) + U'\sin(\omega t + \alpha)$$

- *Therefore, the formula cannot be applied immediately. We will see that*

$$V_{rms} \neq \sqrt{\frac{3^2}{2} + \frac{5^2}{2}} = 4.12\ V.$$

- *Note that* $5\cos(50t + 180°) = -5\cos(50t)$.
- *To find* V_{rms}*, let's simplify* $v(t)$ *to* $3\cos(50t) - 5\cos(50t) = -2\cos(50t)\ \ V.$

$$V_{rms} = \frac{2}{\sqrt{2}} = 1.41\ V$$

RMS Values—Examples

Example: Find the rms value of $v_x(t) = 3\cos(50t + 20°) - 5\sin(50t - 120°)\,V.$

- *The two components of* $v_x(t)$ *do not have the form*

$$U\cos(\omega t + \alpha) + U'\sin(\omega t + \alpha)$$

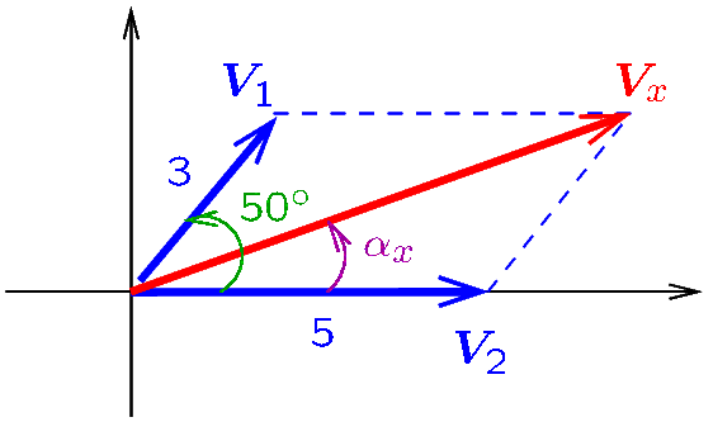
- *Therefore, the formula cannot be applied immediately.*
- *To find the result, we will need to combine the two components into a single sinusoidal function.*
    - *First, the two components will be represented in phasor form.*
    - *Then, the phasors will be added.*
    - *Finally, the result will be converted back to the time domain.*
- *Note that*

$$-5\sin(50t - 120°) = 5\sin(50t - 120° + 180°) = 5\sin(50t + 60°)$$
$$= 5\cos(50t + 60° - 90°) = 5\cos(50t - 30°)$$

RMS Values—Example (Continued)

- *In phasor form,* $\boldsymbol{V_x} = 3\angle 20° + 5\angle -30°\ V.$
- *Let us draw a phasor diagram showing* $\boldsymbol{V_1} = 3\angle 20°$, $\boldsymbol{V_2} = 5\angle -30°$, *and* $\boldsymbol{V_x}$.
- *Let* $\boldsymbol{V_2} = 5\angle -30°$ *be the reference (this is an arbitrary choice).*
- *We will draw the reference along the x axis.*
- *By the law of cosines*

$$|\boldsymbol{V_x}| = \sqrt{3^2 + 5^2 - 2 \cdot 3 \cdot 5 \cdot \cos 130°} = 7.3\ V$$



- *Applying again the law of cosines,*

$$\cos \alpha_x = \frac{|\boldsymbol{V_x}|^2 + 5^2 - 3^2}{2 \cdot 5 \cdot |\boldsymbol{V_x}|} \Rightarrow \alpha_x = 18.35°.$$

- *Since* α_x *is with respect to* $\boldsymbol{V_2} = 5\angle -30°$,

$$\boldsymbol{V_x} = 7.3\angle\alpha_x + (-30°) = 7.3\angle -11.65°\ V.$$

RMS Values—Example (Continued)

- *We have found that* $v_x(t) = 7.3\cos(50t - 11.65°)\ V$.
- *At this point we can calculate the rms value:*

$$V_{x,rms} = \frac{7.3}{\sqrt{2}} = 5.16\ V$$

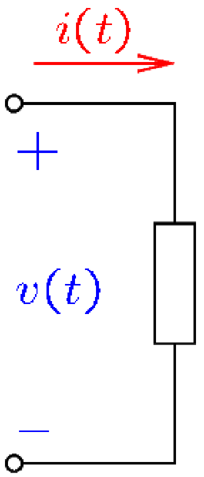
Power—Review

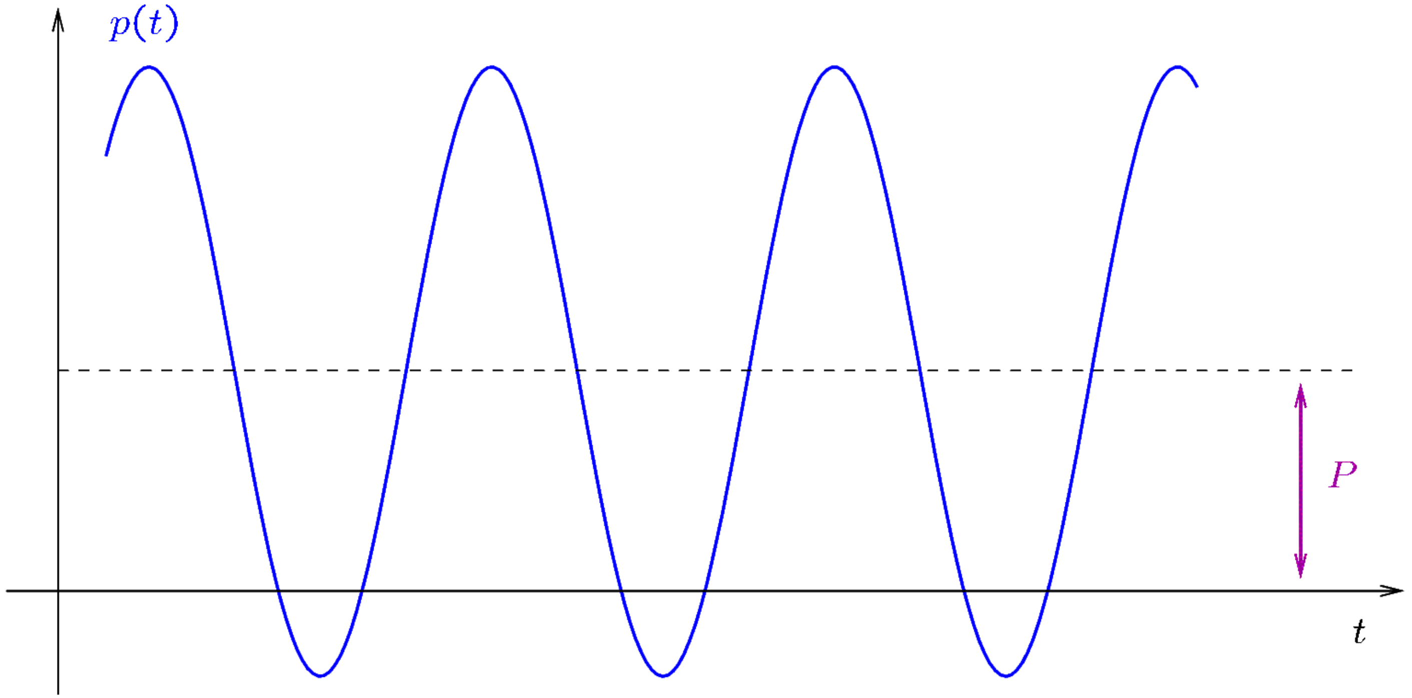
- The instantaneous power $p(t)$ is the power at time t.
- The absorbed instantaneous power is:

$$p(t) = v(t)i(t)$$

- If $v(t) = V_m \cos(\omega t + \alpha_v)$ and $i(t) = I_m \cos(\omega t + \alpha_i)$, the average of $p(t)$ is:

$$P = V_{rms} I_{rms} \cos(\alpha_v - \alpha_i)$$





Power

- The average power is

$$P = V_{rms} I_{rms} \cos(\alpha_v - \alpha_i)$$

- By definition, the apparent power is

$$S = V_{rms} I_{rms}$$

- To indicate that a number represents the apparent power and not the average power, a different unit is used.
- The unit of the apparent power is $V \cdot A$ (Volt Ampere).
- By definition, the power factor is

$$PF = \cos(\alpha_v - \alpha_i)$$

Power

- By definition, the power factor is

$$PF = \cos(\alpha_v - \alpha_i)$$

- Therefore,

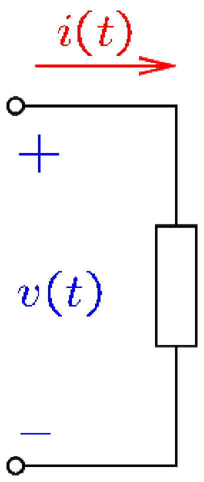
$$\alpha_v - \alpha_i = \pm\cos^{-1} PF$$

- To determine $\alpha_v - \alpha_i$ it is not sufficient to know the power factor; it is necessary to know also whether the current leads or lags the voltage.
- When specifying the power factor, we also indicate whether the current leads or lags the voltage.

  Example: If a load has a power factor of 0.7 *leading, then* $PF = 0.7$ *and* $\alpha_i > \alpha_v$*, because the current is leading the voltage.*

  Example: If a load has a power factor of 0.5 *lagging, then* $PF = 0.5$ *and* $\alpha_i < \alpha_v$*.*

Power—Examples

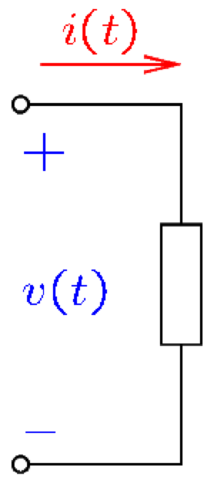


A load connected to a source of voltage $v(t) = 300\,cos(\omega t)\ V$ operates at a power factor of 0.5 lagging. If the apparent power of the load is $3\ kVA$, what is the current $i(t)$?

- $\alpha_v - \alpha_i = \pm\cos^{-1} 0.5 = \pm 60°$.
- *A power factor of 0.5 lagging indicates that the current lags the voltage, that is, $\alpha_i < \alpha_v$.*
- *Therefore, $\alpha_v - \alpha_i = +60°$.*
- $v(t) = 300\cos(\omega t) \Rightarrow \alpha_v = 0 \Rightarrow \alpha_i = -60°$.
- $I_{rms} = \frac{S}{V_{rms}} = \frac{3000}{300/\sqrt{2}} = 10\sqrt{2}\ A$.
- *The amplitude of the current is $I_m = I_{rms}\sqrt{2} = 20\ A$.*
- *We obtain $i(t) = I_m \cos(\omega t + \alpha_i) = 20\cos(\omega t - 60°)\ \ A$.*

Power—Examples

A load connected to a source of voltage $v(t) = 300\,cos(\omega t + \alpha_v)\ V$ operates at a power factor of 0.707 leading. If the apparent power of the load is $3\ kVA$, what is the impedance of the load?



- $\alpha_v - \alpha_i = \pm\cos^{-1} 0.707 = \pm 45°$.
- *A power factor of 0.707 leading indicates that the current leads the voltage, that is,* $\alpha_i > \alpha_v$.
- *Therefore,* $\alpha_v - \alpha_i = -45°$.
- $I_{rms} = \dfrac{S}{V_{rms}} = \dfrac{3000}{300/\sqrt{2}} = 10\sqrt{2}\ A$.
- *The amplitude of the current is* $I_m = I_{rms}\sqrt{2} = 20\ A$.
- *The impedance is* $\mathbf{Z} = \dfrac{V_m\angle\alpha_v}{I_m\angle\alpha_i} = \dfrac{V_m}{I_m}\angle\alpha_v - \alpha_i = \dfrac{300}{20}\angle - 45° = 15\angle - 45°\ \Omega$.