AC Power

Instantaneous Power, Average Power, RMS Values.

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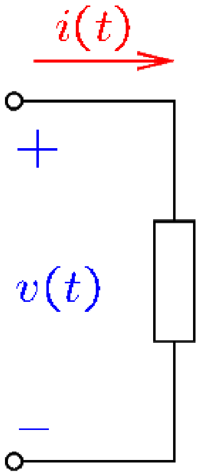
Instantaneous Power



- The instantaneous power $p(t)$ is the power at time t.
- It is calculated the same way as for DC circuits:

$$p(t) = v(t)i(t)$$

- In the figure, since $i(t)$ enters the component at the terminal where $v(t)$ has the $+$ sign, $v(t)i(t)$ is *absorbed* power.
- In the figure, the generated power is $-v(t)i(t)$.

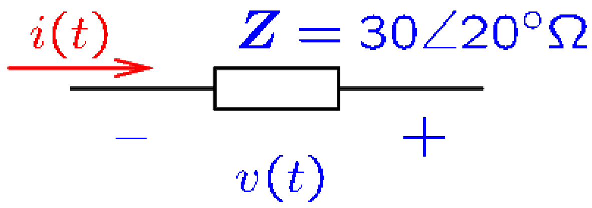
Example: $v(t) = 30\,cos(\omega t)\;\; V$ *and* $i(t) = 10\,cos(\omega t + 30°)\;\; A.$

If $\omega = 400\; rad/s$, *then the absorbed power at time* $t = 1.2\; s$ *is*

$$p(1.2) = 30\,cos(400 \cdot 1.2) \cdot 10cos\left(400 \cdot 1.2 + \frac{\pi}{6}\right) = 234\; W.$$

Instantaneous Power—Example

Example: $i(t) = 10\,cos(\omega t + 30°)$ A. *Find the instantaneous generated power at time t = 2 s. Assume* $\omega = 500\ rad/s$.



- $\boldsymbol{V} = -\boldsymbol{ZI}$, *where the minus sign is due to the fact that* $i(t)$ *enters the component at the terminal where* $v(t)$ *has the* $-$ *sign.*
- $\boldsymbol{V} = -30\angle 20° \cdot 10\angle 30° = 300\angle -130°\ V$.
- $\Rightarrow v(t) = 300\cos(\omega t - 130°)\ V$.
- *The absorbed power is* $p(t) = -v(t)i(t)$.
- *The minus sign is because* $i(t)$ *enters the component at the terminal where* $v(t)$ *has the* $-$ *sign.*

Instantaneous Power—Example

- *The absorbed power is* $p(t) = -v(t)i(t)$.
- *The generated power is*

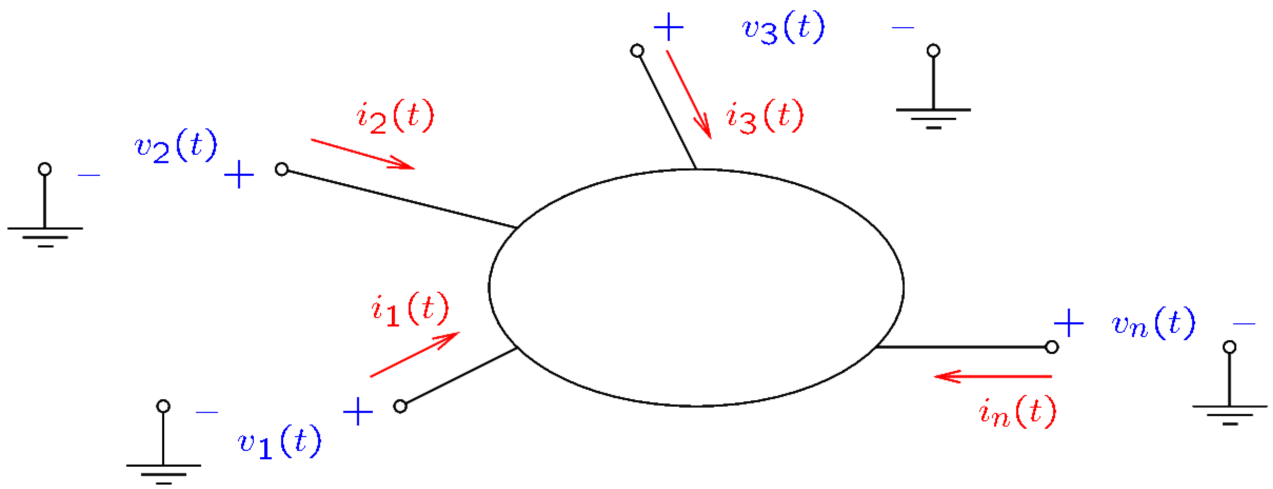
$$-p(t) = 300\cos(\omega t - 130°) \cdot 10\cos(\omega t + 30°)$$

- *Substituting* $t = 2\ s$ *and* $\omega = 500\ rad/s$,

$$-p(2) = 3000\cos\left(500 \cdot 2 - 130° \cdot \frac{\pi}{180°}\right)\cos\left(500 \cdot 2 + \frac{\pi}{6}\right)$$

- *Therefore, the generated power is* $p_g(2) = 60\ W$.
- *In other words, the absorbed power is* $p(2) = -60\ W$.

Instantaneous Power



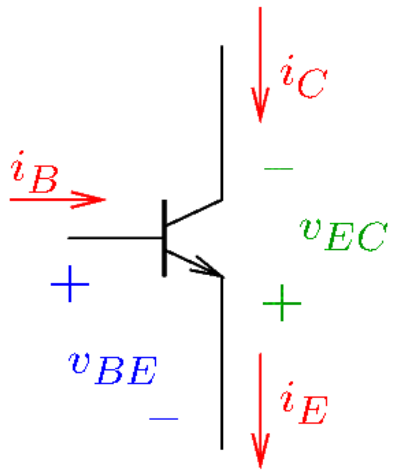
- For components with multiple terminals, the absorbed instantaneous power is

$$p(t) = \sum_{k=0}^{n} v_k(t) i_k(t)$$

- The product $v_k(t)i_k(t)$ has the plus sign when the current i_k enters the component at the terminal where v_k has the $+$ sign.
- Note that all voltages $v_k(t)$ are with respect to the *same* GND reference.

Instantaneous Power—Example

Example: The component shown in the figure is a bipolar transistor operating at $i_E = 1.04 - 0.502\,cos(\omega t)\ A$, $i_B = 40 - 20\,cos(\omega t)\ mA$, $v_{BE} = 0.7\ V$, *and* $v_{EC} = -5\,cos(\omega t) - 10\ V$. *Find the dissipated power when* i_E *is maximum.*



- *Using the lower terminal as a reference, the instantaneous power is*

$$p(t) = i_B v_{BE} - i_C v_{EC}.$$

- *By KCL,*

$$i_C = i_E - i_B = 1 - 0.5\cos(\omega t)\ \ A.$$

- i_E *is maximum when* $\cos(\omega t) = -1$, *which results in an absorbed power*

$$p = 0.06 \cdot 0.7 - 1.5 \cdot (-5) = 7.54\ W.$$

Average Power

- Assuming $v(t) = V_m \cos(\omega t + \alpha_v)$ and $i(t) = I_m \cos(\omega t + \alpha_i)$, the instantaneous power is:

$$p(t) = V_m \cos(\omega t + \alpha_v)\, I_m \cos(\omega t + \alpha_i)$$

- Applying the formula $\cos(x)\cos(y) = \frac{1}{2}\cos(x - y) + \frac{1}{2}\cos(x + y)$:

$$p(t) = \frac{V_m I_m}{2} \cos(\alpha_v - \alpha_i) + \frac{V_m I_m}{2} \cos(2\omega t + \alpha_v + \alpha_i)$$

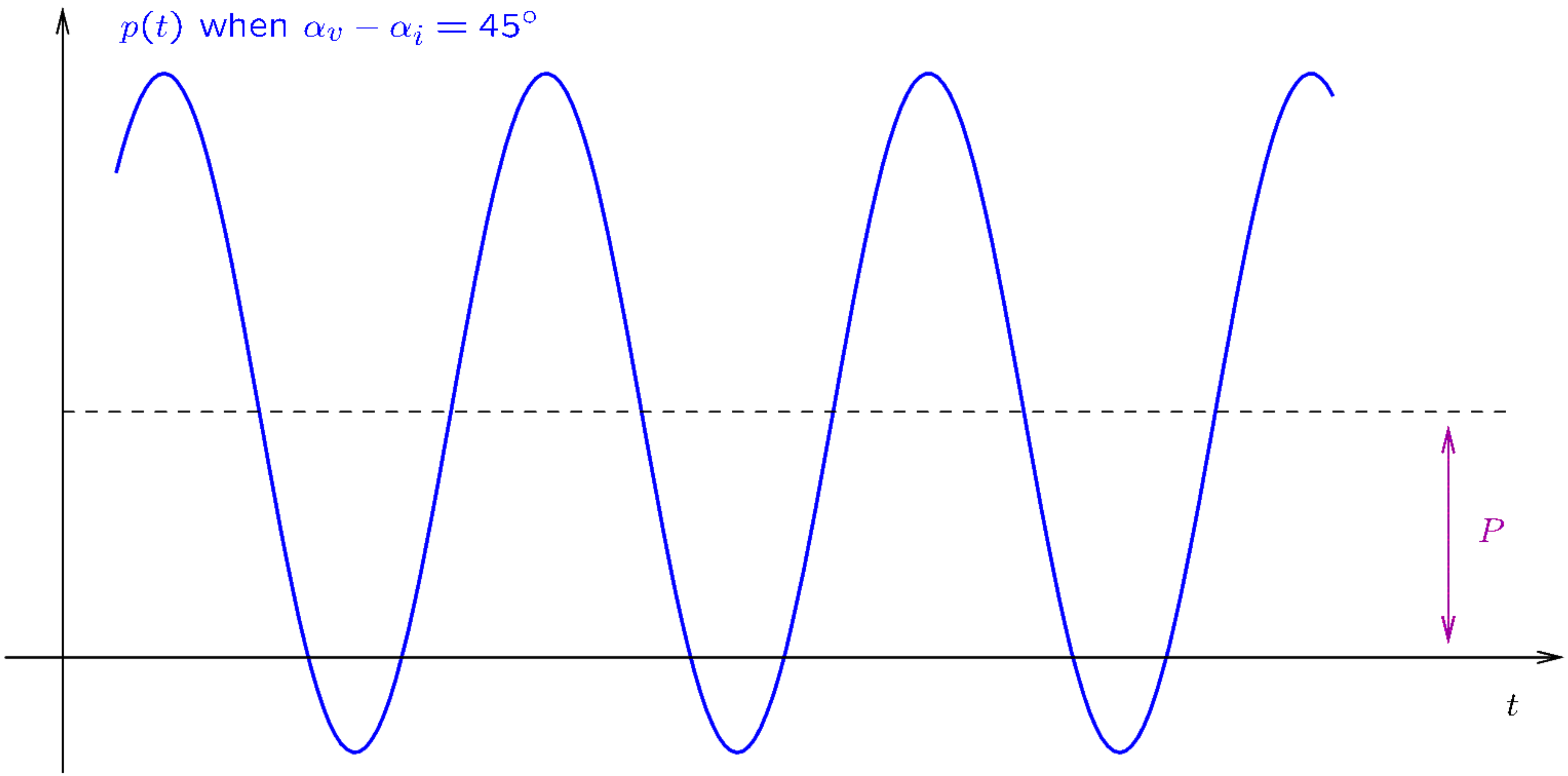
- We could write $p(t)$ as

$$p(t) = P + \frac{V_m I_m}{2} \cos(2\omega t + \alpha_v + \alpha_i)$$

where $P = \frac{V_m I_m}{2} \cos(\alpha_v - \alpha_i)$ is the average of $p(t)$ and is called average power.

Average Power

- The instantaneous power $p(t)$ may have both positive and negative values.
- P is its average of $p(t)$.



Average Power

- In the case of signals that are not periodic, the average power can be calculated with the formula

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t)dt$$

- In the case of periodic signals of period T, the formula can be simplified to

$$P = \frac{1}{T} \int_{-T/2}^{T/2} p(t)dt$$

- We have shown that if $v(t) = V_m \cos(\omega t + \alpha_v)$ and $i(t) = I_m \cos(\omega t + \alpha_i)$, the average power is:

$$P = \frac{V_m I_m}{2} \cos(\alpha_v - \alpha_i)$$

  - This formula is simplest when written in terms of *rms voltages and currents*.

RMS Voltages and Currents

- Let $u(t)$ be a current or a voltage.
- The *root mean square* value of $u(t)$, also known as *rms* value or *effective* value is informally defined as

$$U_{rms} = \sqrt{Average\ of\ u(t)^2}$$

- Formally,

$$U_{rms} = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} u(t)^2 dt}$$

- If $u(t)$ is periodic and has the period T, the formula can be simplified to

$$U_{rms} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} u(t)^2 dt}$$

- If $u(t) = U_m \cos(\omega t + \alpha)$, then $U_{rms} = U_m/\sqrt{2}$.

RMS Voltages and Currents

- Let V_{rms} be the rms value of $v(t) = V_m \cos(\omega t + \alpha_v)$ and I_{rms} the rms value of $i(t) = I_m \cos(\omega t + \alpha_i)$. Then:

$$V_{rms} = \frac{V_m}{\sqrt{2}} \text{ and } I_{rms} = \frac{I_m}{\sqrt{2}}$$

- The average power is:

$$P = \frac{V_m I_m}{2} \cos(\alpha_v - \alpha_i) = V_{rms} I_{rms} \cos(\alpha_v - \alpha_i)$$

RMS Voltages and Currents

- *The rms value of a signal $u(t)$ is the DC value that would produce the same average power on a resistor as $u(t)$.*
  - *The rms value is the DC equivalent of an AC signal.*
- *The wall outlet voltage is 120 V rms!*
  - *The amplitude of the wall outlet voltage is therefore $120\sqrt{2} = 169.71\ V$.*
- *When measuring the value of AC signals, multimeters display the rms value.*

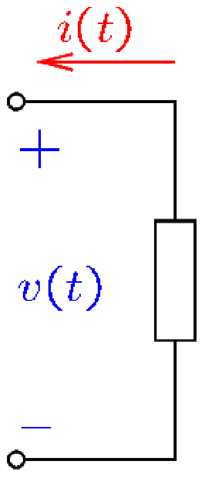
RMS Voltages and Currents—Examples

Example: Assume $v(t) = 50\,cos(\omega t + 30°)\ V$ *and* $i(t) = 4\,cos(\omega t + 75°)\ A$.

- *The rms value of* $i(t)$ *is* $I_{rms} = \frac{4}{\sqrt{2}} = 2.83\ A$.
- *The rms value of* $v(t)$ *is* $V_{rms} = \frac{50}{\sqrt{2}} = 35.35\ V$.
- *The average absorbed power is* $P = -V_{rms} I_{rms} \cos(\alpha_v - \alpha_i)$.
- *The minus sign is due to the fact that* $i(t)$ *enters the component at the terminal where* $v(t)$ *has the minus sign.*
- *Substituting* $\alpha_v = 30°$ *and* $\alpha_i = 75°$:

$$P = -\frac{50}{\sqrt{2}} \cdot \frac{4}{\sqrt{2}} \cdot \cos(30° - 75°) = -70.71\ W.$$

- *Thus, the component generates on the average* $P_g = -P = 70.71\ W$.



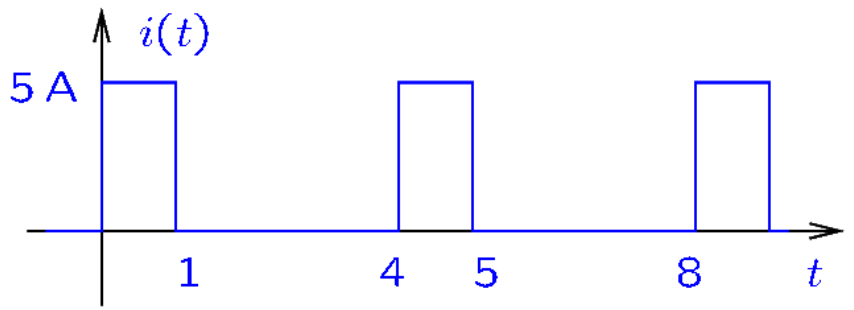
RMS Voltages and Currents—Examples

Example: Find the rms value of the DC voltage $v(t) = 3\ V$.

- $V_{rms} = \sqrt{Average\ of\ 3^2} = \sqrt{3^2} = 3\ V$.

Example: Find the rms value of the shown current.

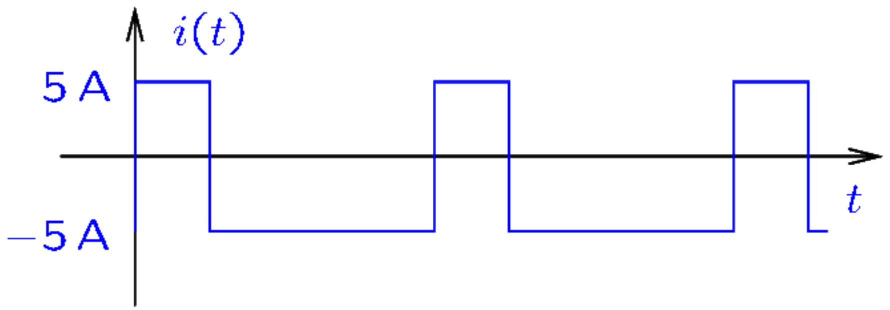
- $I_{rms} = \sqrt{Average\ of\ i^2} = \sqrt{5^2 \cdot 1/4} = 2.5\ A$.



Example: Find the rms value of the shown current.

- $I_{rms} = \sqrt{Average\ of\ i^2} = \sqrt{Average\ of\ 5^2}$

$$= \sqrt{5^2} = 5\ A.$$



RMS Voltages and Currents—Examples

Example: Find the rms value of the DC voltage $v(t) = 5\cos(\omega t) + 3\ V$.

- $V_{rms} = \sqrt{Average\ of\ (5\cos(\omega t) + 3)^2}$

$$= \sqrt{Average\ of\ 25\,(\cos(\omega t))^2 + 9 + 30\cos(\omega t)}$$

$$= \sqrt{\frac{25}{2} + 9 + 0} = 4.63\ V.$$