AC Analysis of Circuits—Part 2

Mesh Analysis. The Thevenin Equivalent.

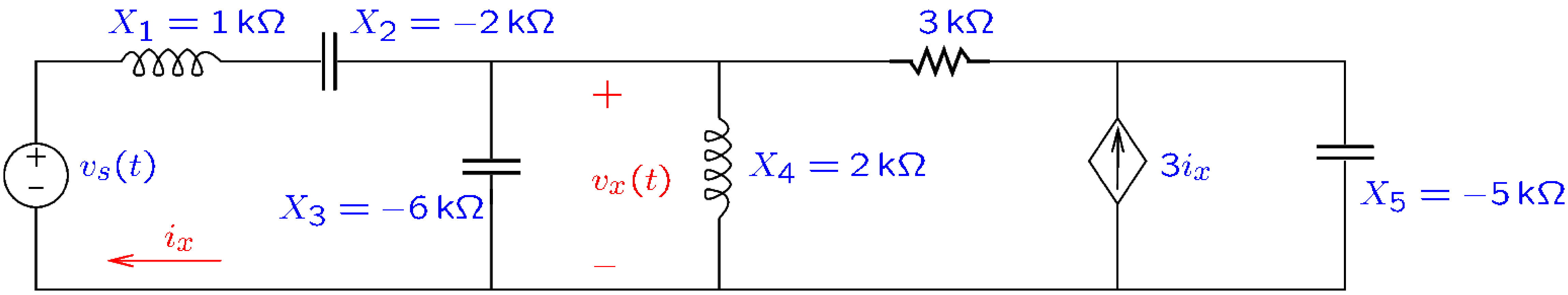
M.V. Iordache, *EEGR2053 Electric Circuits*, Spring 2020, LeTourneau University

Analysis Methods

- The following apply only in the frequency domain.
  - *Nodal analysis*
  - *Mesh analysis*
  - *Voltage division*
  - *Current division*
  - *Source transformations*
- Superposition can be used in both frequency and time domains.
- The Thevenin equivalent is normally used in the frequency domain, but works also in the time domain.
- *Analysis methods are applied exactly the same way as for DC circuits.*
  - *The only difference is that we work with complex numbers.*

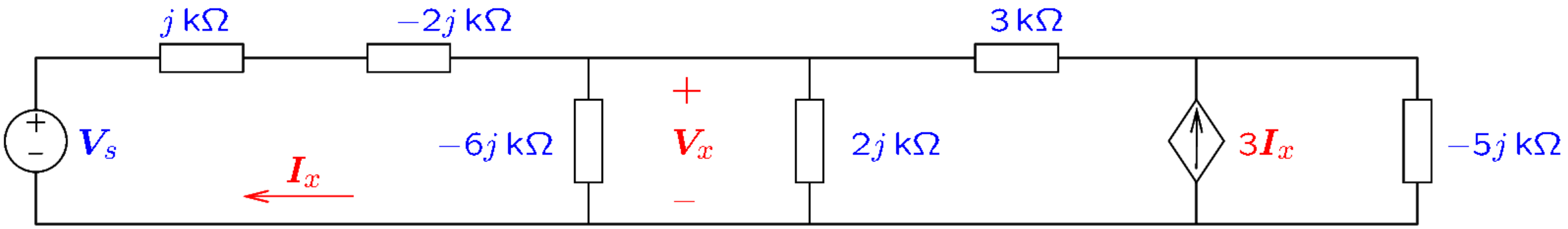
Mesh Analysis

Example: *Find the voltage* $v_x(t)$ *using mesh analysis. Assume* $v_s(t) = 5\cos(\omega t + 30°)\ V$. *Reactance values are marked for each inductor and capacitor.*



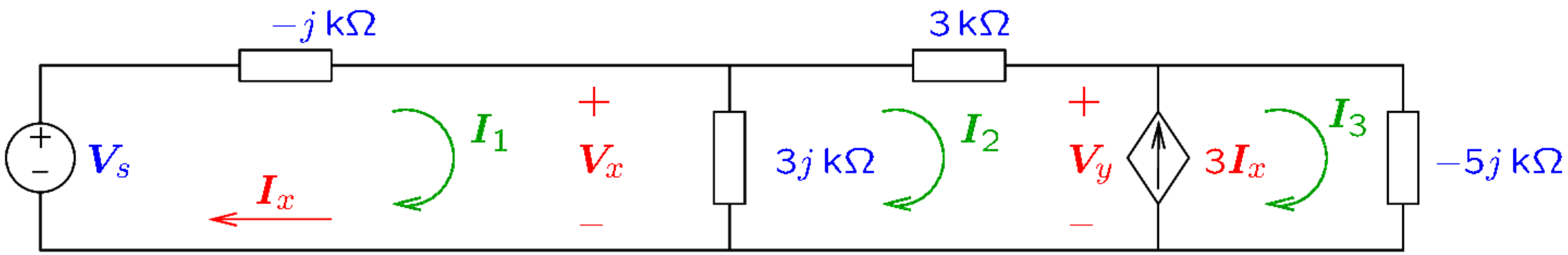
- *In general, the first step is calculating inductor and capacitor impedances.*
- *This step is not carried out here, since the reactances are given.*
- *We will first draw the circuit in the frequency domain.*

Mesh Analysis—Example



- *In the frequency domain:*
  - *Each resistor, inductor, and capacitor is replaced by an impedance.*
  - $v_s(t) = 5\cos(\omega t + 30°)\ V$ *becomes* $\boldsymbol{V_s} = 5\angle 30°\ V$.
  - $i_x(t)$ *becomes* $\boldsymbol{I_x}$.
- *Before applying mesh analysis, let's simplify the circuit as much as possible.*
  - *Let's combine the* $j\ k\Omega$ *and* $-2j\ k\Omega$ *impedances into* $\boldsymbol{Z_1} = j + (-2j) = -j\ k\Omega$.
  - *Let's combine the* $-6j\ k\Omega$ *and* $2j\ k\Omega$ *impedances into* $\boldsymbol{Z_2} = (-6j)||(2j) = 3j\ k\Omega$.

Mesh Analysis—Example



- *KVL on the first mesh is:*

$$-\boldsymbol{V_s} + \boldsymbol{I_1}(-j) + (\boldsymbol{I_1} - \boldsymbol{I_2})3j = 0$$

- *KVL on the supermesh is:*

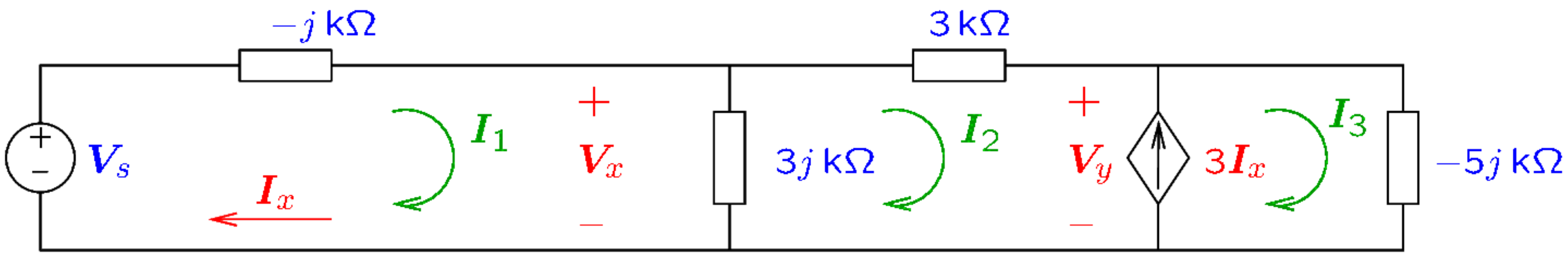
$$(\boldsymbol{I_2} - \boldsymbol{I_1})3j + \boldsymbol{I_2}3 + \boldsymbol{I_3}(-5j) = 0$$

- *The dependent source adds the constraint:*

$$\boldsymbol{I_3} - \boldsymbol{I_2} = 3\boldsymbol{I_x}$$

where $\boldsymbol{I_x} = \boldsymbol{I_1}$.

Mesh Analysis—Example



- *The equations can be written in the form:*

$$\boldsymbol{I_1}2j - \boldsymbol{I_2}3j = 5\angle 30°$$

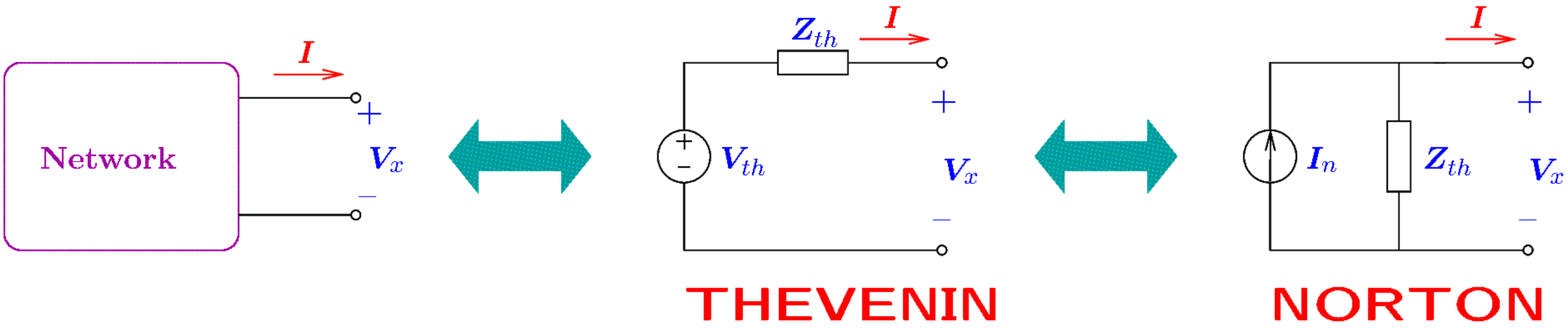
$$-\boldsymbol{I_1}3j + \boldsymbol{I_2}(3+3j) + \boldsymbol{I_3}(-5j) = 0$$

$$3\boldsymbol{I_1} + \boldsymbol{I_2} - \boldsymbol{I_3} = 0$$

- *Solving the equations and using* $\boldsymbol{V_x} = 3j(\boldsymbol{I_1} - \boldsymbol{I_2})$ *we obtain:*

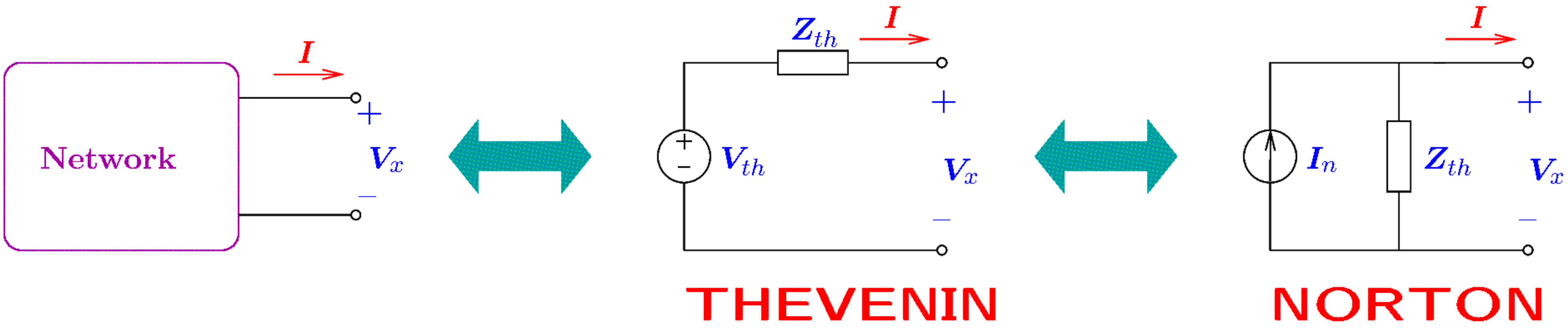
Thevenin and Norton Equivalents

- Are obtained in the frequency domain.
- Can be converted to the time domain.
  - However, the time domain representation is frequency dependent.
- The equivalents are obtained using the same methods as in DC.
  - *However, note that we have a Thevenin impedance $\boldsymbol{Z_{th}}$ in the place of a Thevenin resistance R_{th}.*

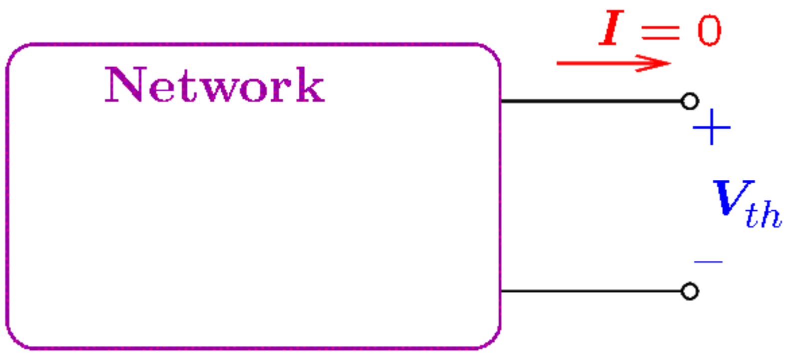


Thevenin/Norton Equivalents—Review

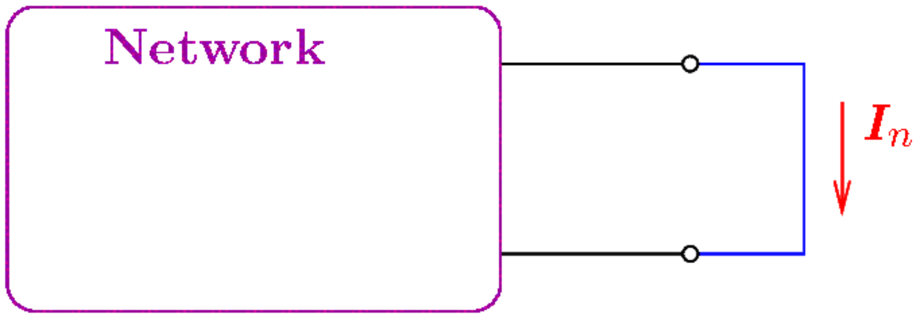
- The original network and the Thevenin/Norton networks are equivalent in the sense that they will produce the same voltage and current when identically connected.
- *This does not mean that they will dissipate the same amount of power!*



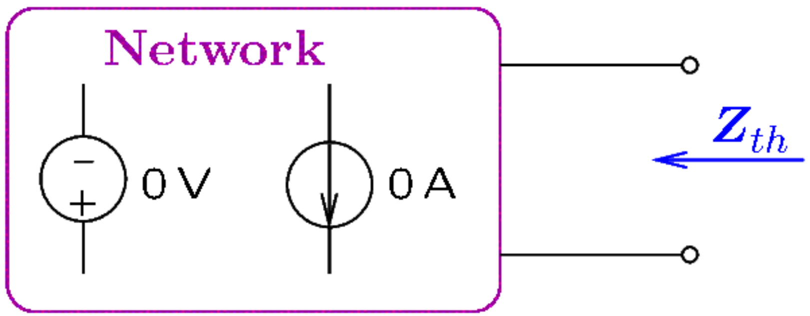
Thevenin/Norton Equivalents



V_{th} is the open-circuit voltage.



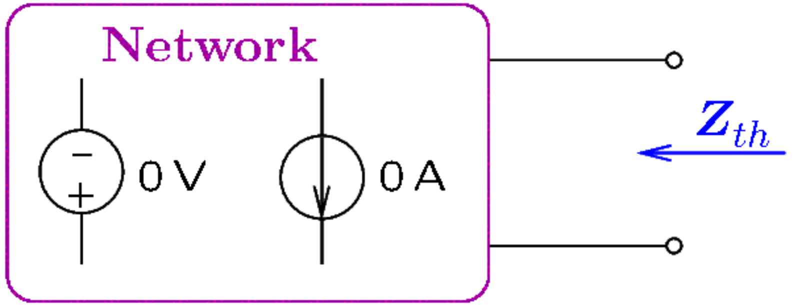
I_n is the short-circuit current.



Z_{th} is the impedance of the network when all independent sources are set to zero.

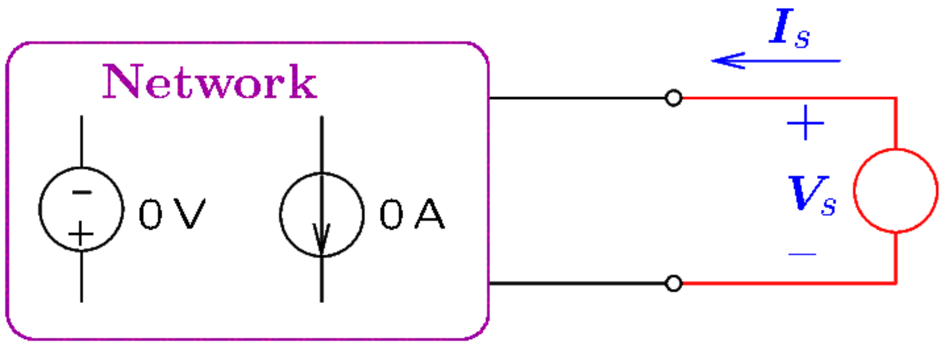
- *Dependent sources are not set to zero.*

Thevenin/Norton Equivalents—Finding Z_{th}



To find Z_{th}, set all independent sources to zero. Then:

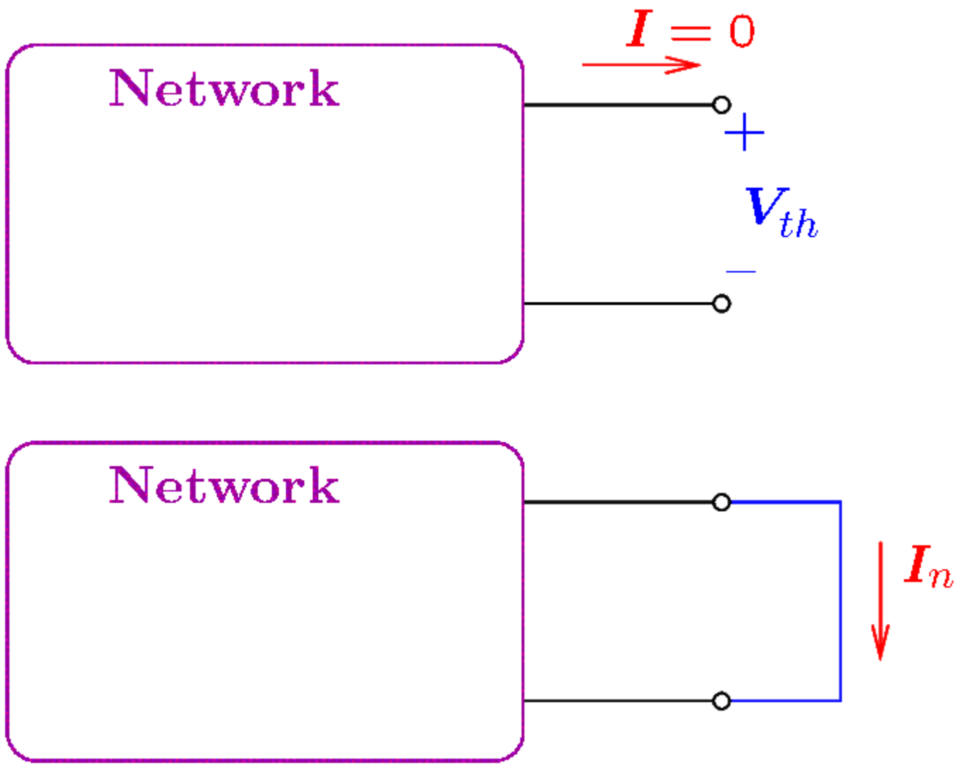
- *Use series and parallel combinations of impedances, if possible.*



- The general method is to add an external source and calculate Z_{th} *with the formula:*

$$Z_{th} = \frac{V_s}{I_s}$$

Thevenin/Norton Equivalents—Finding Z_{th}

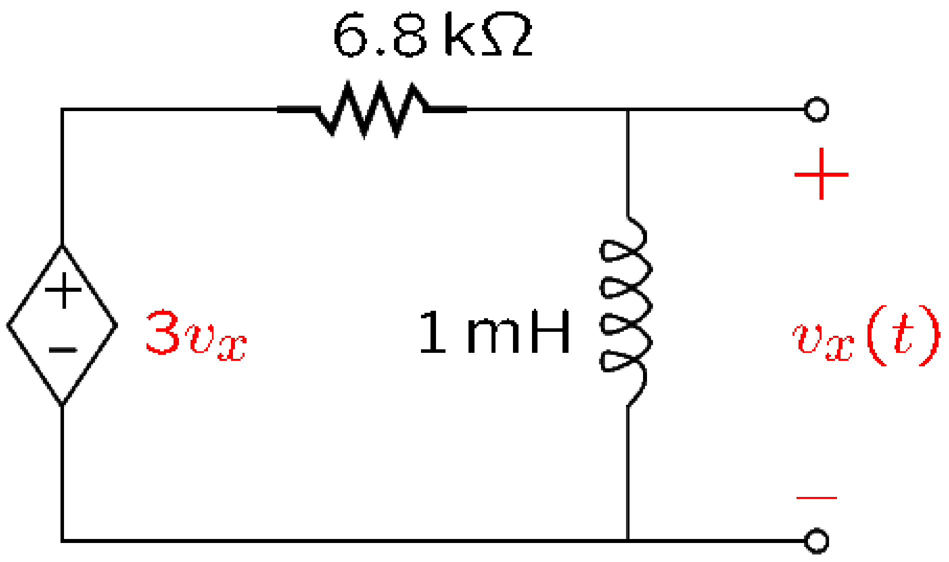


Another way to find Z_{th} is with the formula

$$Z_{th} = \frac{V_{th}}{I_n}$$

- This method can be used when the short-circuit current is not zero.
- Of course, all independent sources should be ON when finding V_{th} and I_n.

Thevenin Equivalent—Example



Find the Thevenin equivalent at the frequency $f = \frac{1}{2\pi}$ MHz.

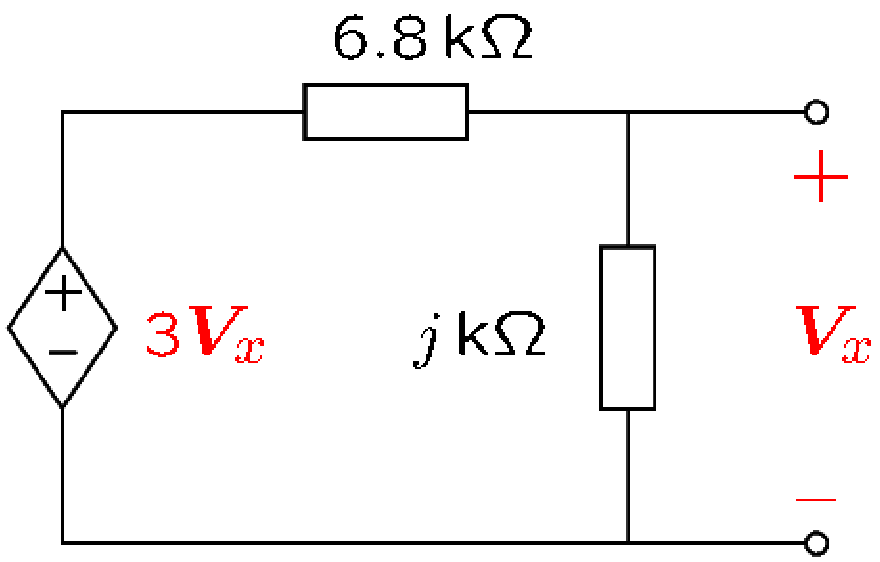
- The first step is to convert the circuit to the frequency domain.
- For this, we need to find the impedances.

$$\omega = 2\pi f = 10^6 \, rad/s$$

$$\boldsymbol{Z_{6.8}} = 6.8 \, k\Omega$$

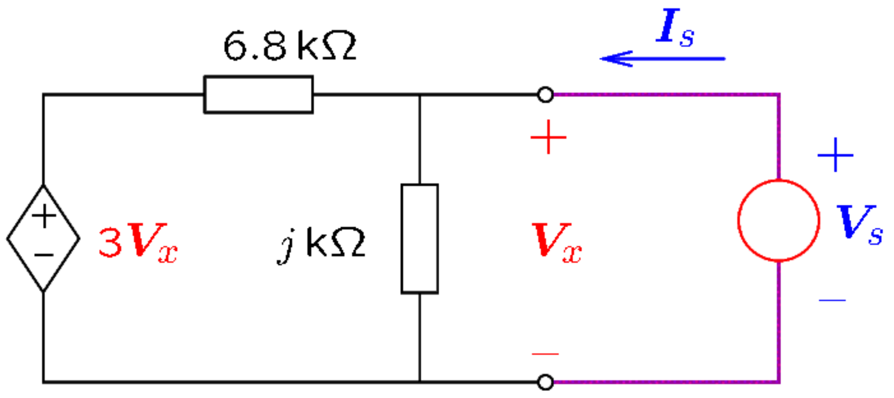
$$\boldsymbol{Z_{1m}} = j\omega L = j \, k\Omega$$

Thevenin Equivalent—Example



- The circuit has no independent source. Therefore, $V_{th} = 0$.
- For the same reason, $I_n = 0$. Therefore, the formula $Z_{th} = V_{th}/I_n$ cannot be used.
- Impedance combinations cannot be done either, because of the dependent source.
  - Recall that dependent sources cannot be set to zero! Only independent sources are set to zero.
- Therefore, we will use the general method with an external source.

Thevenin Equivalent—Example

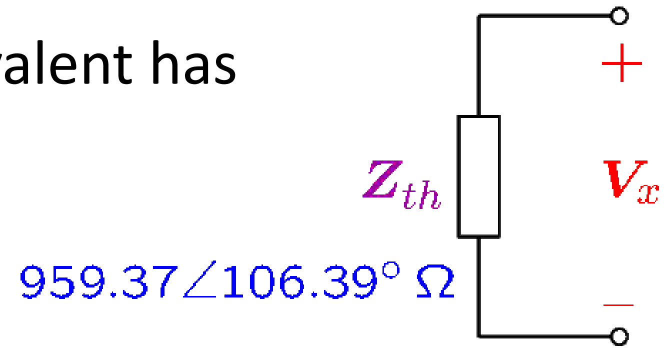


- The formula $Z_{th} = V_s/I_s$ will be used.
- Note that $V_s = V_x$.
- By KCL:

$$I_s = \frac{V_x}{j} + \frac{V_x - 3V_x}{6.8}$$

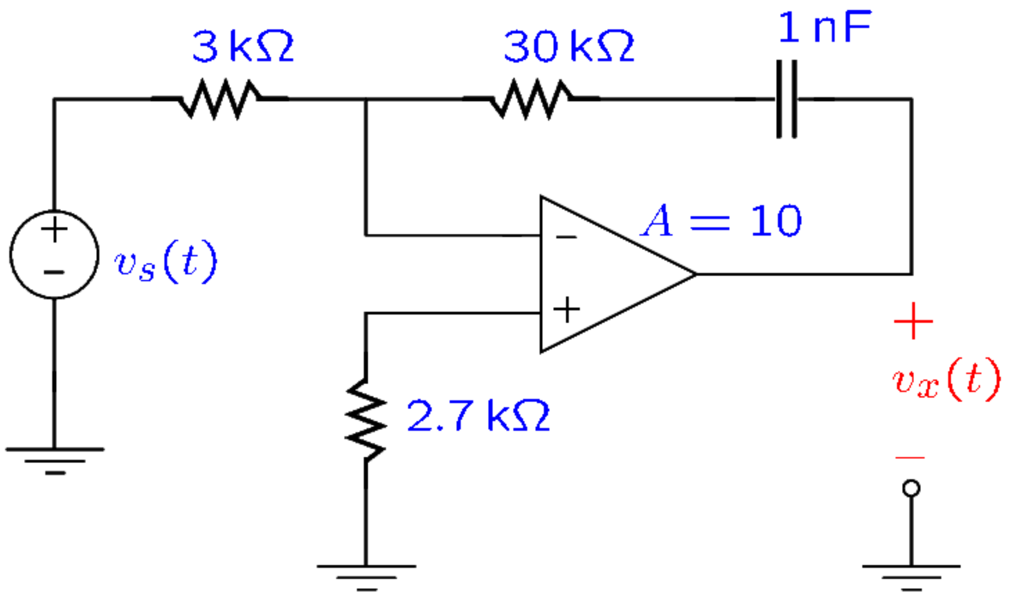
$$\Rightarrow \frac{V_x}{I_x} = \frac{1}{\frac{1}{j} - \frac{2}{6.8}} = -0.2707 + 0.92038j\ k\Omega$$

- Therefore, $Z_{th} = -270.7 + 920.38\ \Omega = 959.37\angle\ 106.39°\Omega$.
- Since $V_{th} = 0$, the Thevenin equivalent has the form shown in the figure.



Thevenin Equivalent—Example 2

Find the Thevenin equivalent assuming $v_s(t) = 50\, cos(\omega t)\, mV$, a frequency $f = 50\ kHz$, and an operational amplifier with an output resistance of $200\ \Omega$.



- *The first step is to find the impedances.*

$$\omega = 2\pi f = 314.1 \cdot 10^3\ rad/s$$

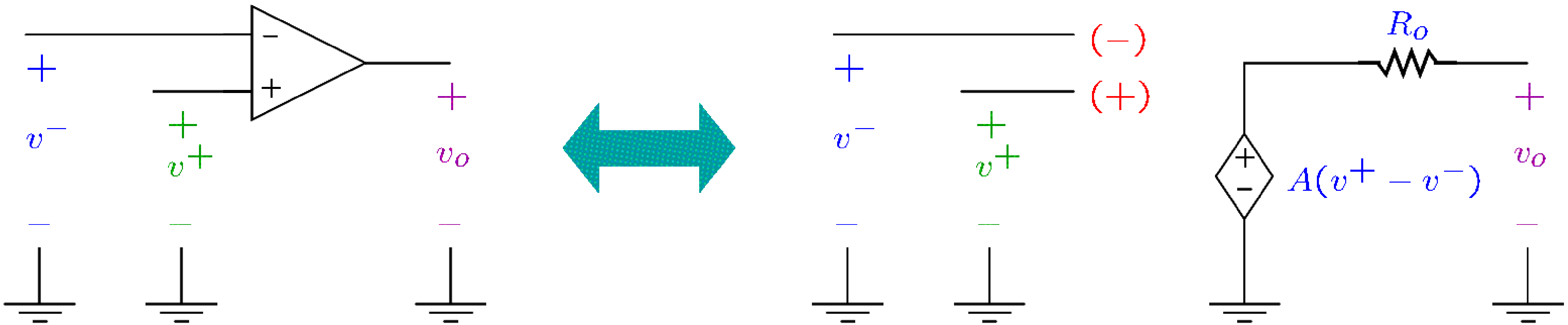
$$Z_{1n} = \frac{1}{j\omega C} = -3.18j\ k\Omega$$

- *The total impedance of the $30\ k\Omega$ resistor and the capacitor is:*

$$\boldsymbol{Z_t} = 30 - 3.18j\ k\Omega$$

Thevenin Equivalent—Example 2

- *Here, the equivalent model of the operational amplifier includes also the output resistance R_o, which has been often neglected in homework problems.*



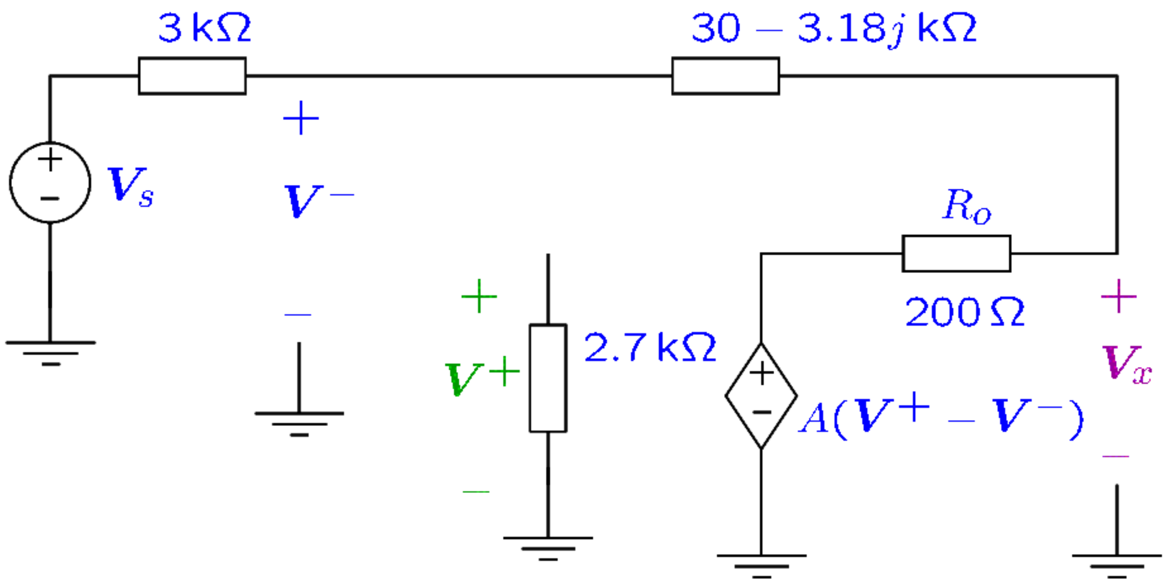
Thevenin Equivalent—Example 2

- *To find the Thevenin equivalent, we will first derive* $\boldsymbol{V_{th}}$.
- *Note that*

$$\boldsymbol{V_s} = 50\angle 0°\ mV$$

$$\boldsymbol{V}^{+} = 0$$

$$A(\boldsymbol{V}^{+} - \boldsymbol{V}^{-}) = -A\boldsymbol{V}^{-}$$



- *The nodal equations are:*

$$\frac{\boldsymbol{V_s} - \boldsymbol{V}^{-}}{3\ k\Omega} + \frac{(-A\boldsymbol{V}^{-}) - \boldsymbol{V}^{-}}{200\ \Omega + 30 - 3.18j\ k\Omega} = 0 \Rightarrow \boldsymbol{V}^{-} = 23.99\angle -3.13°\ mV$$

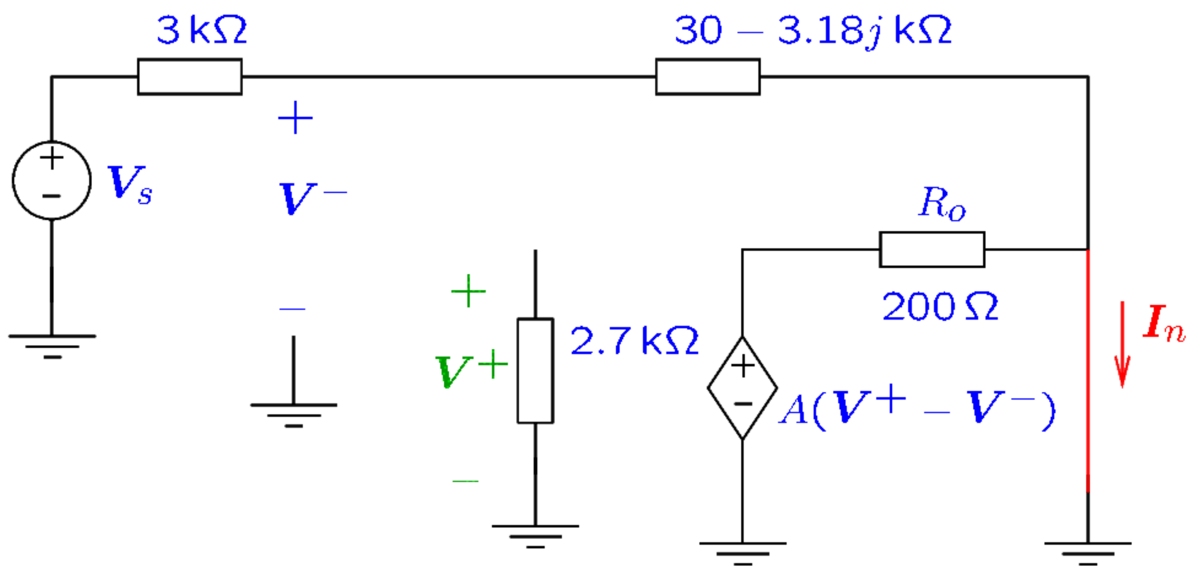
$$\frac{\boldsymbol{V_x} - (-A\boldsymbol{V}^{-})}{200\ \Omega} + \frac{\boldsymbol{V_x} - \boldsymbol{V_s}}{30 - 3.18j + 3\ k\Omega} = 0 \Rightarrow \boldsymbol{V_{th}} = \boldsymbol{V_x} = 238.21\angle 176.83°\ mV$$

Thevenin Equivalent—Example 2

- *To find $\boldsymbol{Z_{th}}$, note that series and parallel combinations of impedances will not work, due to the dependent source.*
- *The short-circuit current method is illustrated here.*
- *By voltage division,*

$$\boldsymbol{V}^{-} = \boldsymbol{V_s} \cdot \frac{30 - 3.18j}{3 + 30 - 3.18j}$$
$$= 45.50\angle -0.55°\ mV$$

- *The short-circuit current is:*



$$\boldsymbol{I_n} = \frac{\boldsymbol{V_s}}{3 + 30 - 3.18j\ k\Omega} + \frac{-A\boldsymbol{V}^{-}}{200\ \Omega} = 2.27\angle 179.45°\ mA$$

$$\boldsymbol{Z_{th}} = \frac{\boldsymbol{V_{th}}}{\boldsymbol{I_n}} = 104.78\angle -2.62°\Omega$$

Thevenin Equivalent—Example 2

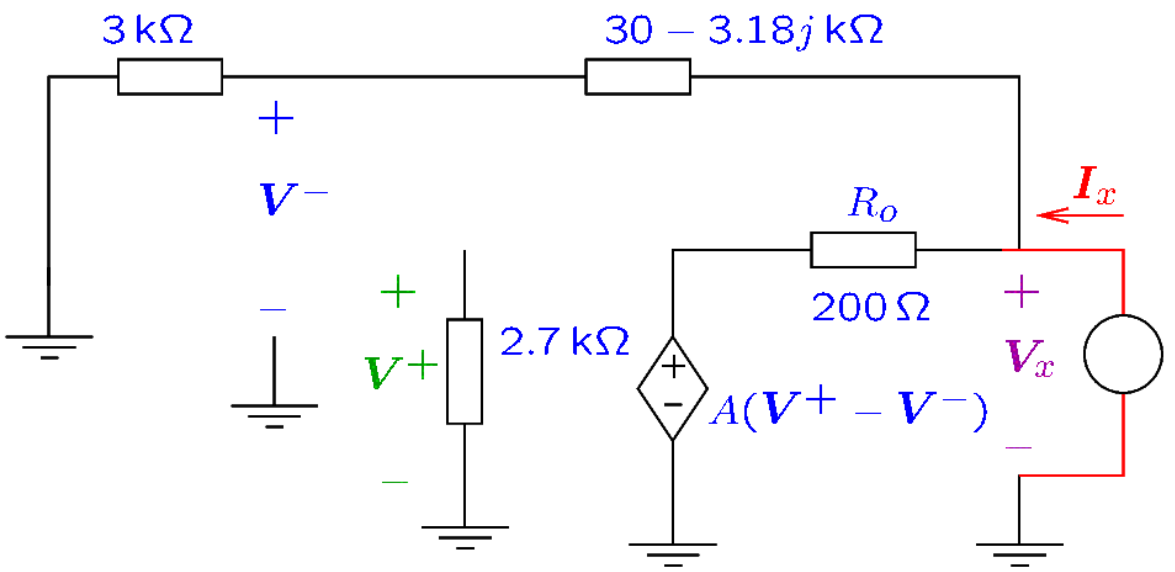
- *The general method of finding* $\boldsymbol{Z_{th}}$ *is with an external source.*
- *This method is illustrated here.*
- *By voltage division,*

$$V^- = V_x \cdot \frac{3}{3+30-3.18j}$$

- *By KVL,*

$$I_x = \frac{V_x}{33-3.18j\ k\Omega} + \frac{V_x - (-AV^-)}{200\ \Omega}$$

- *After substituting* $\boldsymbol{V^-}$*:*



$$I_x = V_x\left(\frac{1}{33-3.18j\ k\Omega} + \frac{1}{200\ \Omega} + \frac{A}{33-3.18j\ k\Omega}\cdot\frac{3\ k\Omega}{200\ \Omega}\right)$$

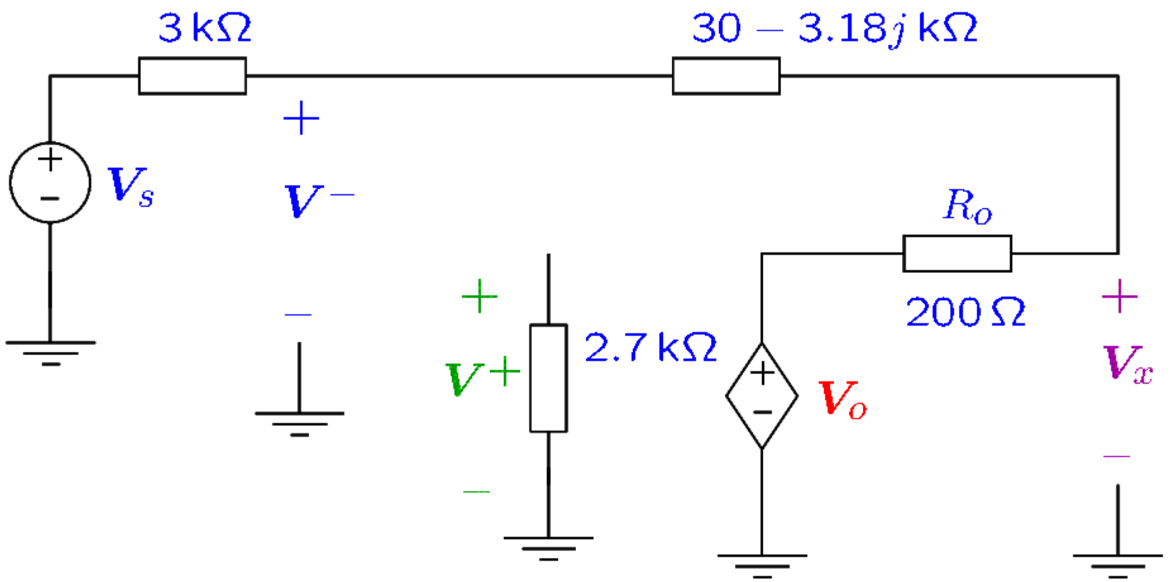
$$\Rightarrow Z_{th} = \frac{V_x}{I_x} = 104.78\angle -2.62°\Omega$$

Thevenin Equivalent—Example 3

Repeat the previous example when $A \to \infty$.

- *Note that practical operational amplifiers of high gain can be approximated reasonably well using $A \to \infty$.*
- *The approximation $A \to \infty$ is used because it simplifies calculations.*
- *It implies that*

$$V^+ - V^- = \frac{V_o}{A} = 0$$



- *This does not mean that the (+) and (-) terminals of the operational amplifier are shorted!*
- *Rather, the circuit will adjust V_o so that $V^+ = V^-$.*

Thevenin Equivalent—Example 3

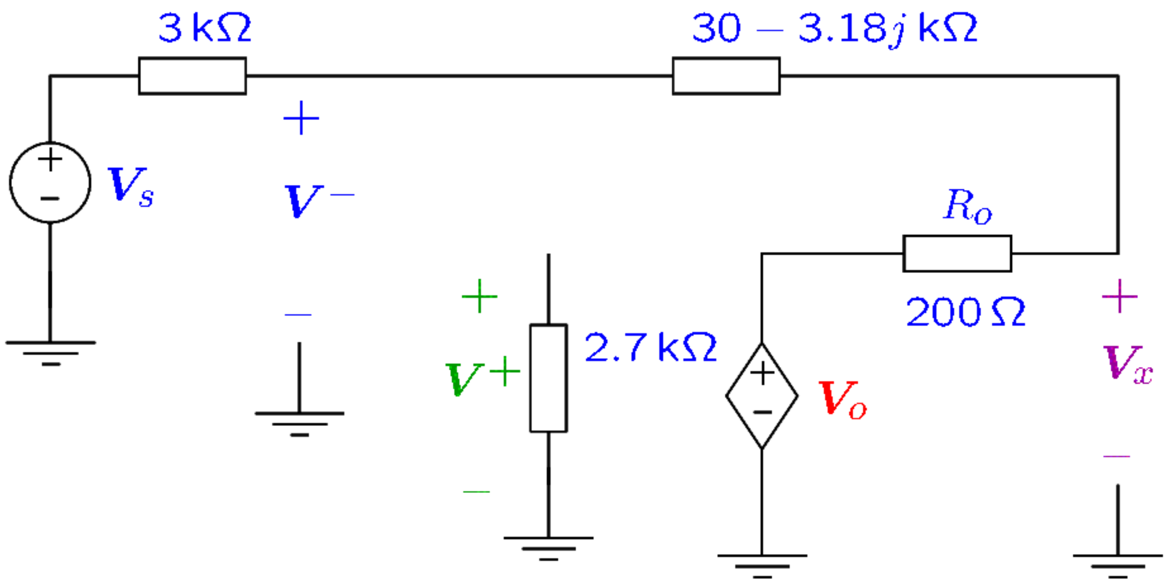
- *To find $\boldsymbol{V_{th}}$, note that:*

$$V^- = V^+ = 0$$

- *From nodal analysis:*

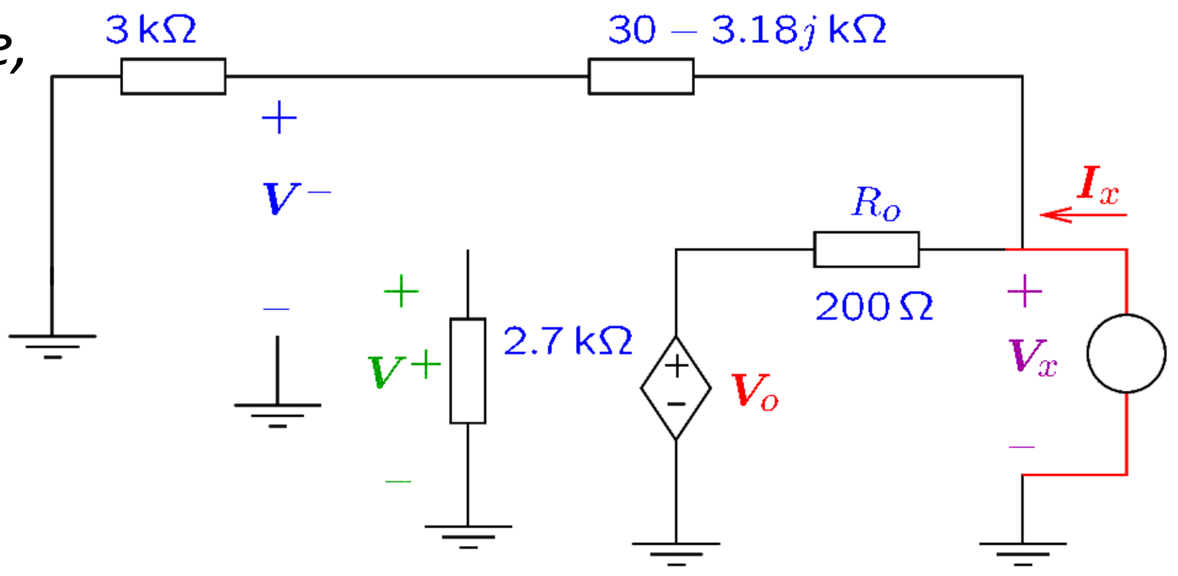
$$\frac{V_x - 0}{30 - 3.18j} + \frac{V_s - 0}{3} = 0$$

$$\Rightarrow V_{th} = V_x = 402.24\angle\, 173.95°\ mV$$



Thevenin Equivalent—Example 3

- *To find $\boldsymbol{Z_{th}}$, the short-circuit method will not work! This is because, as we will prove, $\boldsymbol{Z_{th}} = 0$.*
- *The general method of finding $\boldsymbol{Z_{th}}$ is with an external source.*
- *This method is illustrated here.*
- *Voltage division implies that $\boldsymbol{V^-}$ is a fraction of $\boldsymbol{V_x}$. However, $\boldsymbol{V^-} = \boldsymbol{V^+} = 0$. It follows that $\boldsymbol{V_x} = 0$.*



- *The source does not have to output $I_x = 0$; for example, it could be a current source of $1\ A$. Therefore,*

$$\boldsymbol{Z_{th}} = \frac{\boldsymbol{V_x}}{\boldsymbol{I_x}} = 0\ \Omega$$