The Sinusoidal Function

An Outline

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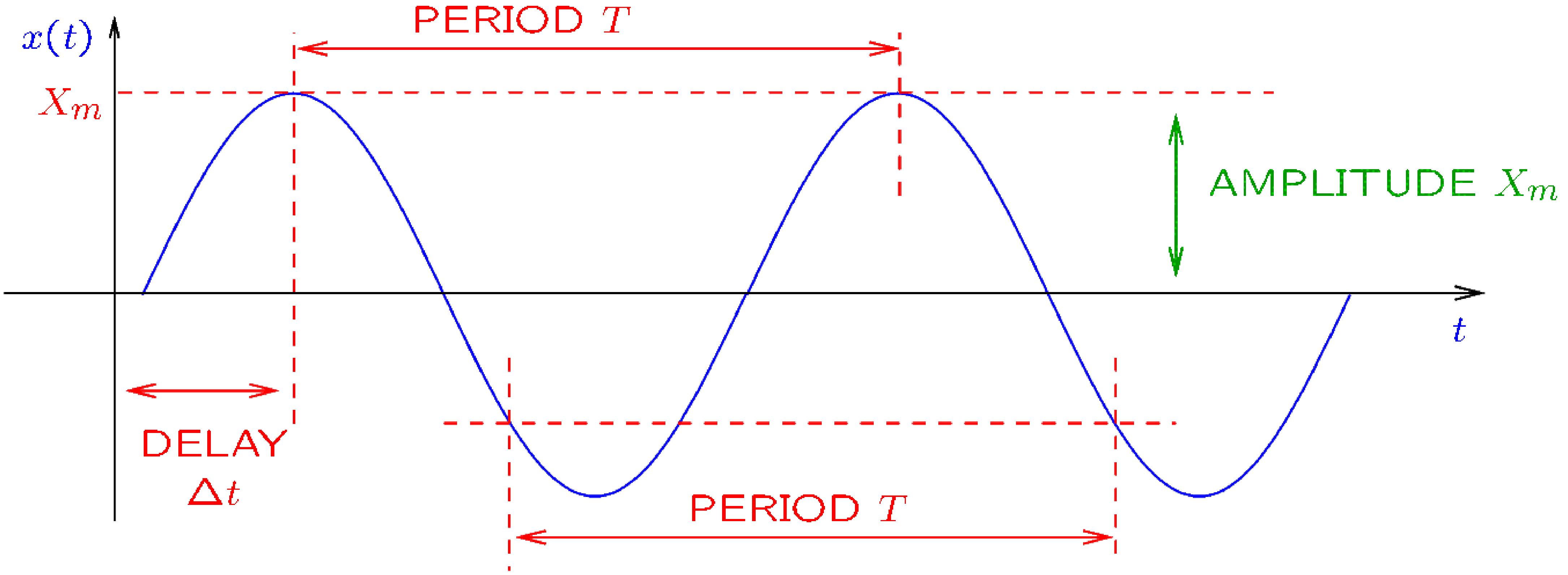
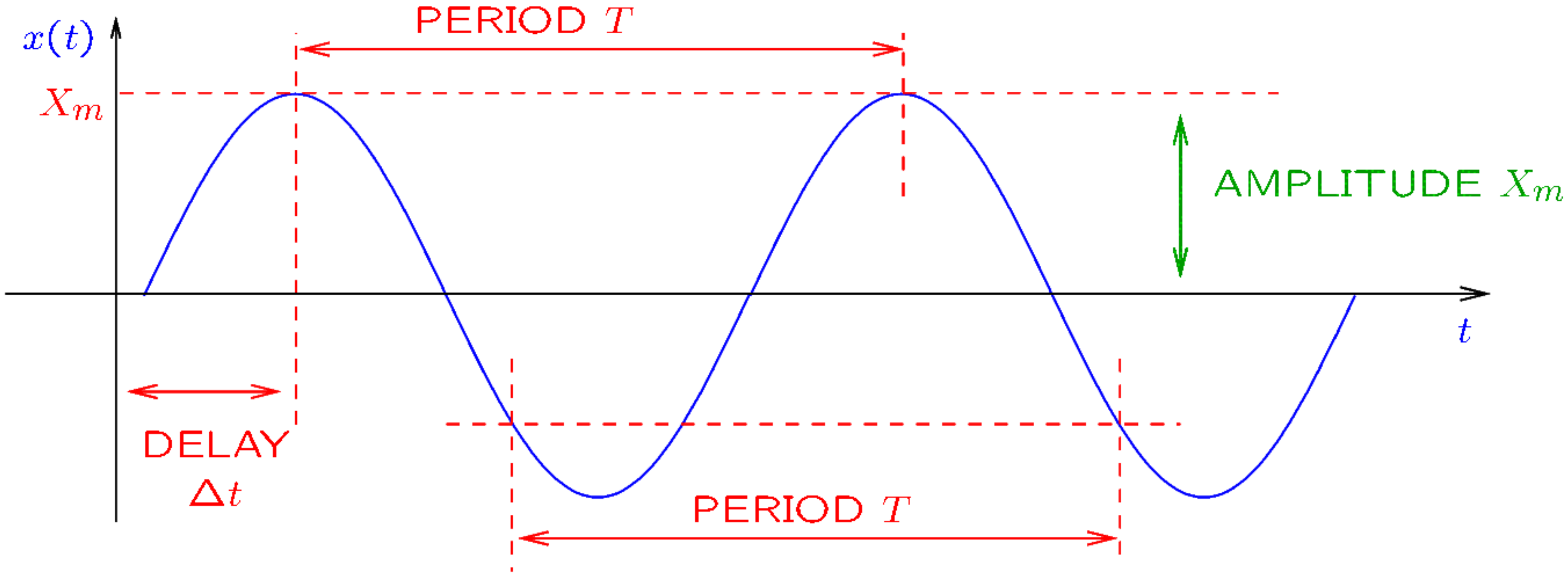
The Sinusoidal Function

$$x(t) = X_m \cos(\omega t + \alpha)$$



The Sinusoidal Function



- The figure shows a delayed cosine, $x(t) = X_m \cos(\omega(t - \Delta t))$.
- Let $\alpha = -\omega\Delta t$.
- Then $x(t) = X_m \cos(\omega t + \alpha)$

The Sinusoidal Function

Given $x(t) = X_m \cos(\omega t + \alpha)$ of period T:

- $\omega = \frac{2\pi}{T}$ is the angular frequency; unit: $\mathrm{rad/s}$.
- $f = \frac{1}{T}$ is the frequency; unit: Hz.
- α is the phase angle.
- X_m is the amplitude.

The Sinusoidal Function

Example: *Assume a voltage* $v(t) = 25\, cos(\omega t + 30°)$ *V of* 60 *Hz frequency.*

Note that $\omega = 2\pi f = 120\pi$ rad/s. Therefore, $v(t)$ can be written as

$$v(t) = 25\cos(120\pi t + 30°) \text{ V}$$

The value of $v(t)$ at time $t = 10$ ms is

$$v(0.01) = 25\cos(1.2\pi + 30°)$$

$$= 25\cos\left(1.2\pi + \frac{\pi}{6}\right)$$

$$= -10.17 \text{ V.}$$

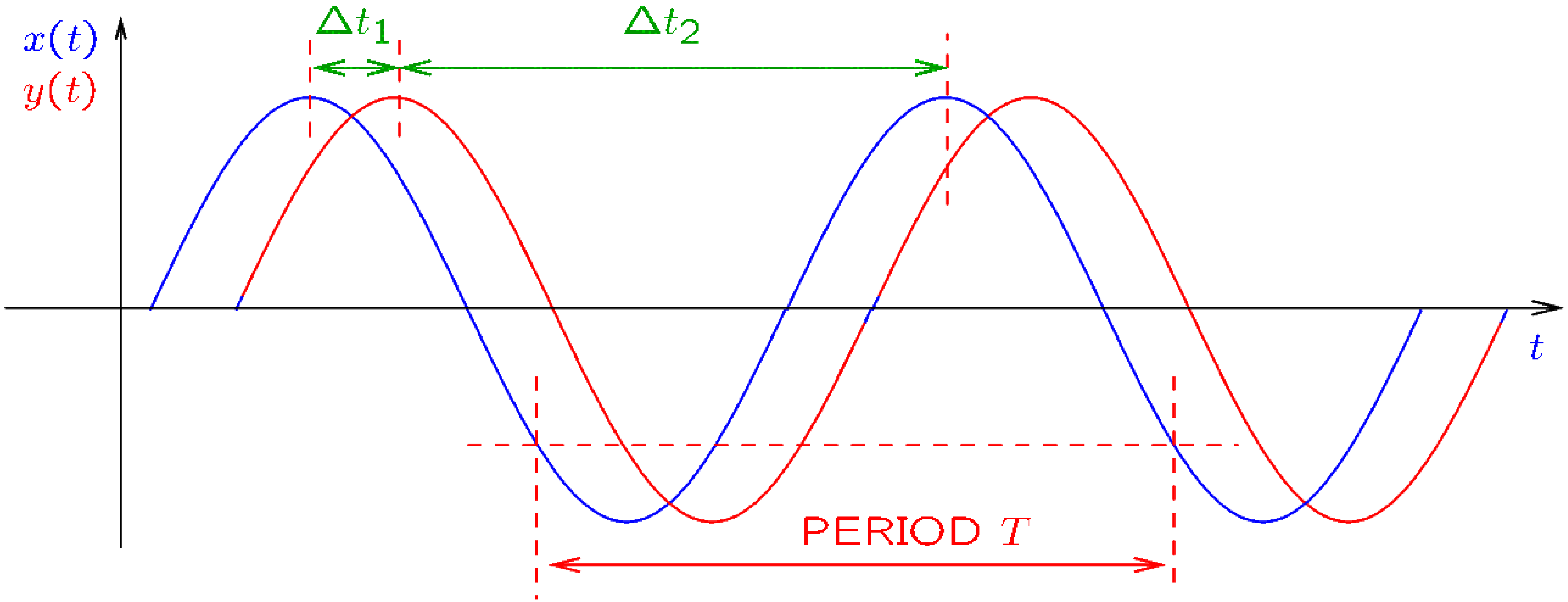
Lead/Lag relationships

- If two signals have the same frequency, their phase can be compared.
- Suppose

$$x(t) = X_m \cos(\omega t + \alpha_x)$$
$$y(t) = Y_m \cos(\omega t + \alpha_y)$$

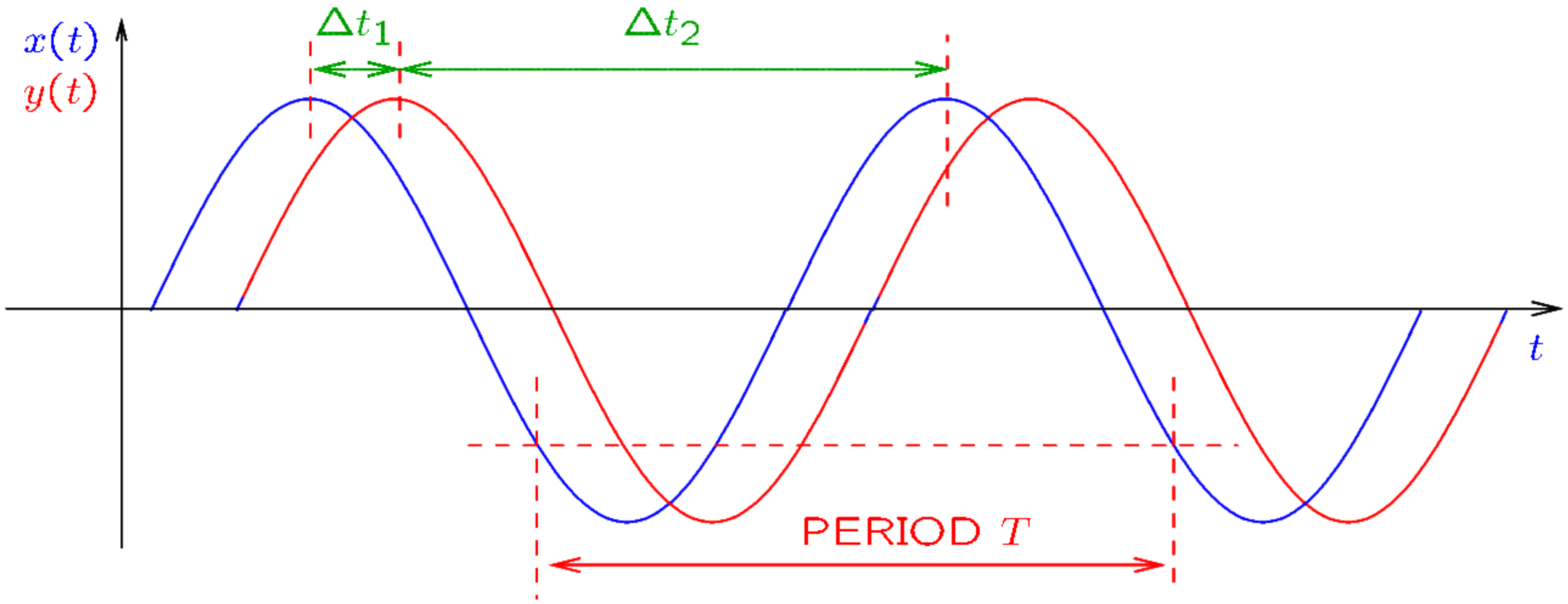
- We say that $x(t)$ leads $y(t)$ by an angle $\alpha = \alpha_x - \alpha_y$.
- We say that $x(t)$ lags $y(t)$ by an angle $\beta = \alpha_y - \alpha_x$.
- If $\alpha_x = \alpha_y$, then $x(t)$ and $y(t)$ are in phase.
- If $\alpha_x - \alpha_y = \pm 180°$, then $x(t)$ and $y(t)$ are $180°$ out of phase.

Lead/Lag relationships



- $x(t)$ leads $y(t)$ by Δt_1, so $x(t) = y(t + \Delta t_1)$.
- $y(t)$ leads $x(t)$ by Δt_2, so $y(t) = x(t + \Delta t_2)$.
- $y(t)$ lags $x(t)$ by Δt_1, so $y(t) = x(t - \Delta t_1)$.
- $x(t)$ lags $y(t)$ by Δt_2, so $x(t) = y(t - \Delta t_2)$.

Lead/Lag relationships



It is common to express lead/lag relationship in terms of a phase angle:

- $x(t)$ leads $y(t)$ by an angle $\alpha_1 = \omega\Delta t_1$.
- $y(t)$ leads $x(t)$ by an angle $\alpha_2 = \omega\Delta t_2$.
- $y(t)$ lags $x(t)$ by an angle $\alpha_1 = \omega\Delta t_1$.
- $x(t)$ lags $y(t)$ by an angle $\alpha_2 = \omega\Delta t_2$.

Lead/Lag relationships

Example: $\sin(\omega t)$ lags $\cos(\omega t)$ by $90°$. Therefore, $\sin(\omega t) = \cos(\omega t - 90°)$.

Example: $\cos(\omega t)$ leads $\sin(\omega t)$ by $90°$. Therefore, $\cos(\omega t) = \sin(\omega t + 90°)$.

Example: Indicate the phase relationship of $v(t) = 30\cos(\omega t + 215°)$ V and $i(t) = 5\cos(\omega t - 450°)$ A.

Solution: $v(t)$ leads $i(t)$ by $215° - (-450°) = 665°$, that is, by $665° - 2 \cdot 360° = -55°$. Alternatively, we could say that $i(t)$ leads $v(t)$ by $+55°$.

Example: Indicate the angle by which $v(t) = 30\cos(\omega t - 80°)$ V lags $v'(t) = 20\sin(\omega t + 30°)$ V.

Solution: $v(t)$ lags $v'(t)$ by $30° - (-80°) = 110°$. We could also say that $v'(t)$ lags $v(t)$ by $-110°$.

Lead/Lag relationships

Example: Indicate the angle by which $v(t) = -30\cos(\omega t - 80°)$ V lags $v'(t) = 20\sin(\omega t + 30°)$ V.

Solution: We need first to write both voltages in the form $X_m\cos(\omega t + \alpha)$.

$$v(t) = -30\cos(\omega t - 80°)$$
$$= 30\cos(\omega t - 80° + 180°)$$
$$= 30\cos(\omega t + 100°) \text{ V.}$$

$$v'(t) = 20\sin(\omega t + 30°)$$
$$= 20\cos(\omega t + 30° - 90°)$$
$$= 20\cos(\omega t - 60°) \text{ V.}$$

$v(t)$ lags $v'(t)$ by $-60° - 100° = -160°$, that is, it leads $v'(t)$ by $160°$.