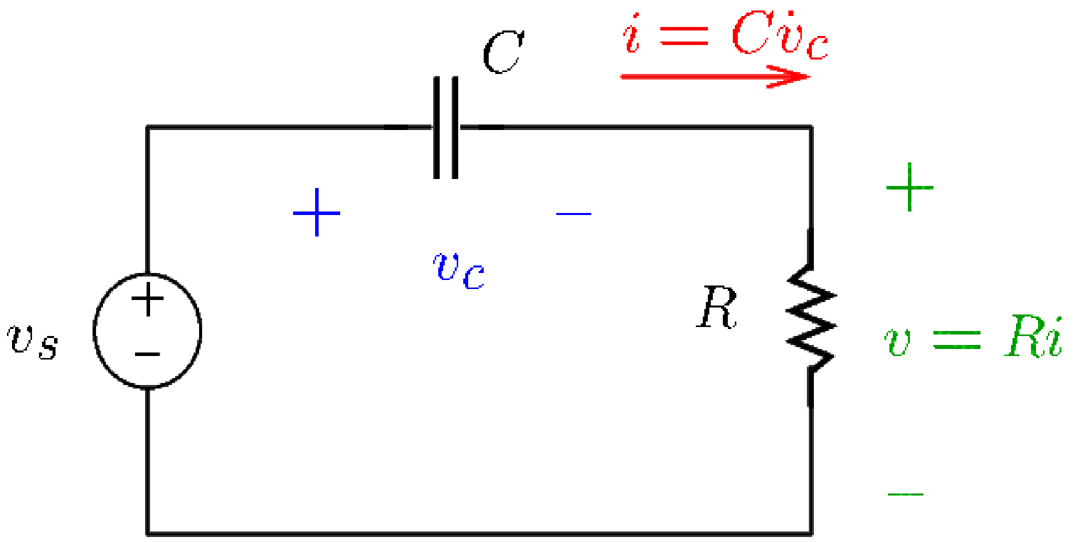
First Order Systems.
Thermal-Electrical Analogy.
Interference.

M.V. Iordache, *EEGR2051 Circuits and Measurements Lab*, Fall 2020, LeTourneau University
See https://mviordache.name/EEGR2051 for more information.

First Order Systems

- First order systems are described by first order differential equations.
- For example, the following circuit is a first order system.
  - By KVL, $v_s = v_c + v$.
  - Substituting $i = C\dot{v}_c$ and $v = Ri$, we obtain the first order equation:
    $$v_s = v_c + RC\dot{v}_c$$
  - Therefore, the circuit is a first order system.



Thermal Analysis

- Thermal analysis of electric circuits is important because electric components are damaged when the maximum admissible temperature is exceeded.
- Thermal analysis can be carried out using an electric analogy in which:
  - Temperature corresponds to electric potential.
  - Power corresponds to electric current.
  - Energy corresponds to electric charge.
  - Thermal resistance corresponds to electric resistance.
    - Thermal resistance is measured in Kelvin degrees per watt (*K/W*).
  - Thermal capacitance corresponds to capacitance.
    - Thermal capacitance is measured in joules per Kelvin degrees (*J/K*).

Thermal Analysis

- It is interesting to note the similarity of thermal and electric resistance:
  - The electric resistance of a conductor of length L, cross-sectional area A, and conductivity σ is $R = \frac{L}{\sigma A}$.
  - If σ denotes thermal conductivity, then thermal resistance is also $R = \frac{L}{\sigma A}$.
- The thermal capacitance of an object of mass m is $C = m \cdot c_p$, where c_p is the specific heat.

Example 1

Assume $T_2 = 30\,^{\circ}C$ *and* $T_1 = 10\,^{\circ}C$. *Find* $\Delta T = T_2 - T_1$ *in Kelvin degrees.*

- Let's verify that the answer is $20\ K$, not $20 + 273.15\ K = 293.15\ K$!
- $T_2 = 30\,^{\circ}C + 273.15 = 303.15\ K$.
- $T_1 = 10\,^{\circ}C + 273.15 = 283.15\ K$.
- $\Delta T = 303.15 - 283.15 = 20\ K$.
- In Celsius degrees, $\Delta T = 30\,^{\circ}C - 10\,^{\circ}C = 20\,^{\circ}C$.
- We have shown that $\Delta T = 20\ K = 20\,^{\circ}C$.
- Note that *temperature difference is the same in Kelvin and Celsius degrees*.

Example 2

A certain component dissipates an average power of $P = 3\ W$ *to the air. If the thermal resistance of the component to the air is* $R = 40\ K/W$ *and the temperature of the air is* $T_{air} = 20\ °C$*, what is the steady-state temperature of the component?*

- Let T be the temperature of the component.
- Note that T and T_{air} are analogous to electric potential.
- Therefore, $T - T_{air}$ is analogous to voltage, which is difference of potential.
- The power P is analogous to current.
- By Ohm's law, $T - T_{air} = R \cdot P \Rightarrow T - 20\ °C = 40\ K/W \cdot 3\ W$.
- The answer is $T = 140\ °C$.

Example 3

A certain component dissipates an average power of $P = 3\ W$ to the air. If the thermal resistance of the component to the air is $R = 40\ K/W$, the thermal capacitance is $C = 0.2\ J/K$, the air temperature is $T_{air} = 20\ °C$, and initially the component is at the air temperature, after how much time will the component be within $5\ °C$ of the steady-state temperature?

- The system is equivalent to a capacitor C in parallel with a resistor R.
- The time constant is $\tau = R \cdot C$.
- As found in the previous example, the steady-state temperature is $T(\infty) = 140\ °C$. The initial temperature is $T(t_0) = T_{air} = 20\ °C$.
- $T(t) = T(\infty) + \left(T(t_0) - T(\infty)\right)e^{-\frac{t-t_0}{\tau}}$.
- $135\ °C = T(\infty) + \left(T(t_0) - T(\infty)\right)e^{-\frac{t-t_0}{\tau}} \Rightarrow t - t_0 = 25.42\ s$.

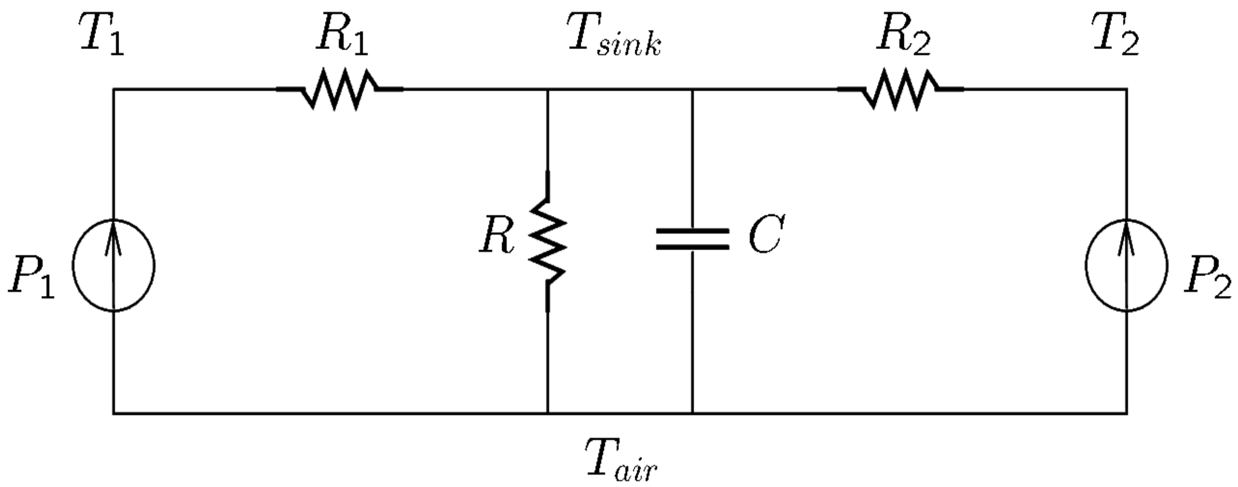
Example 4

- Two electronic components are placed on the same heat sink.
- One dissipates the power P_1 and the other P_2.
- The thermal resistances between the two components and the heat sink are R_1 and R_2, respectively.
- The thermal resistance of the heat sink with respect to the air is R and its thermal capacitance C.
- Let T_{air} be the air temperature.
- Let T_1 and T_2 be the temperatures of the two components.

Power transistors of a DC power supply connected to the same aluminum heat sink.

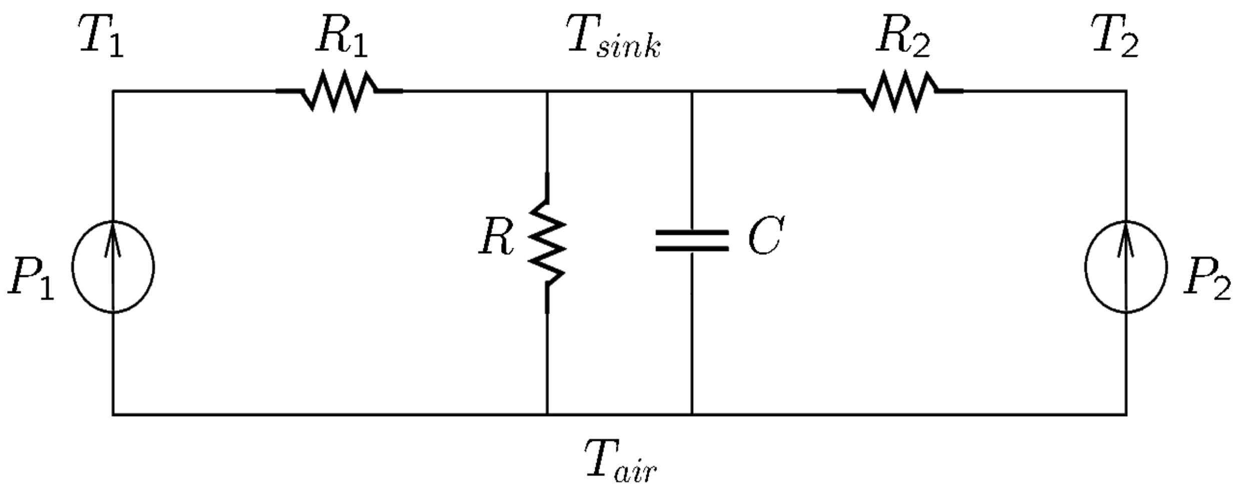
Example 4 (Continued)

- The heat transfer system is analogous to the following circuit.



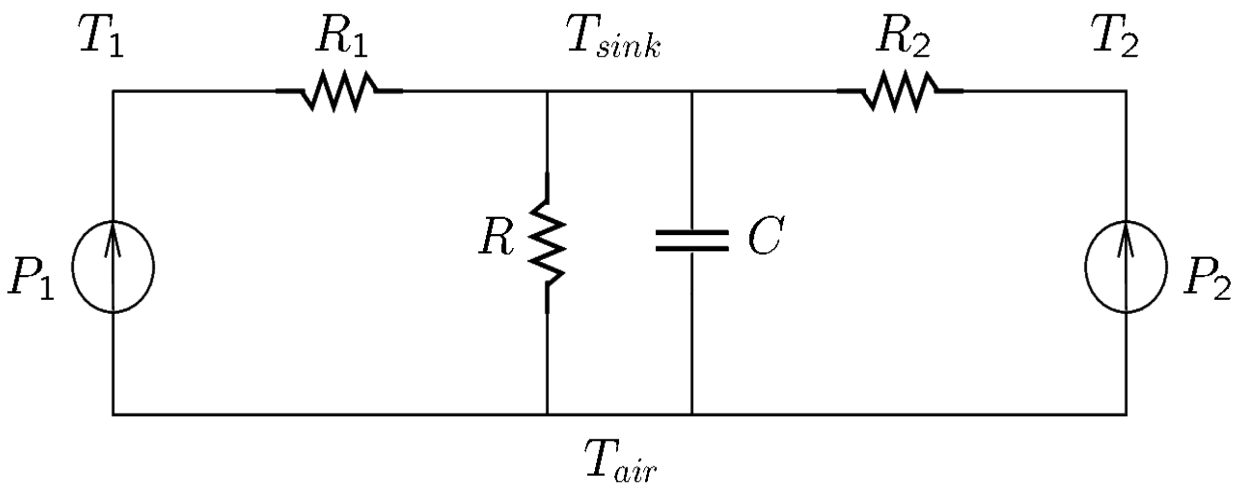
- T_{sink} is the temperature of the heat sink.
- Heat cannot be transferred instantly from the heat sink to the air.
- This is modeled by the thermal resistance R, which limits the rate at which energy can be transferred from the sink to the air.

Example 4 (Continued)



- Suppose that the heat sink has reached a temperature $T_{sink} > T_{air}$.
- When power is turned off and $P_1 = P_2 = 0$, the heat sink will not cool instantly to T_{air}.
- Rather, it will cool gradually, as the heat sink releases its thermal energy (at a finite rate!) to the air.
- The capacitor C models the fact that the heat sink stores thermal energy (heat).
- *The circuit neglects the thermal capacitance of the two components.*

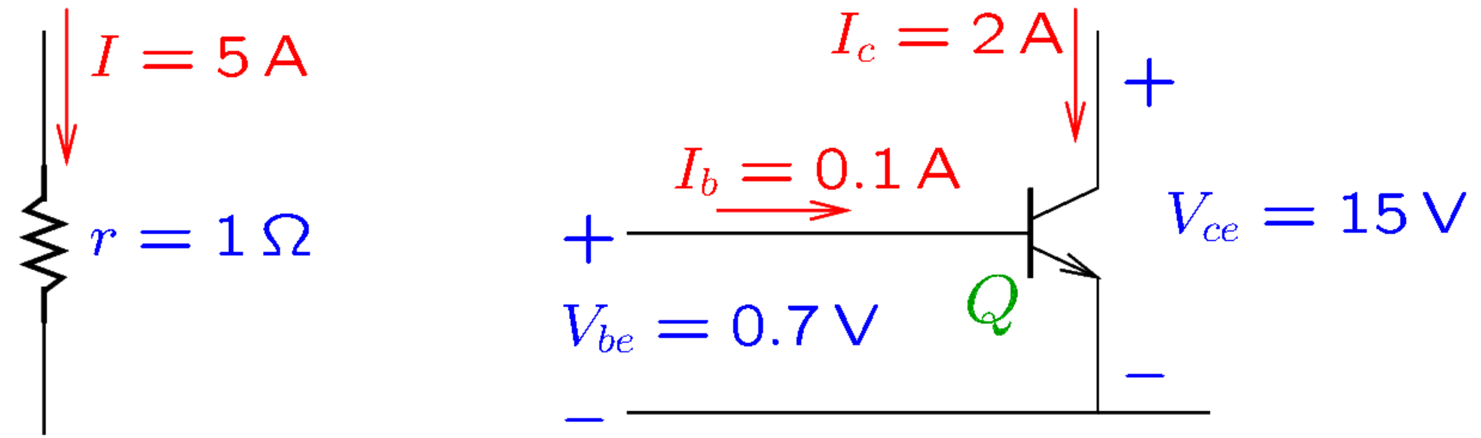
Example 4 (Continued)



- T_1 is the temperature of the first component.
- Since the contact between the component and the heat sink is imperfect, in general $T_1 \neq T_{sink}$.
- The thermal resistance R_1 models the imperfect contact between the component and the heat sink.
- P_1 is the power dissipated by the first component.

Example 5

Assume that in the previous example the components connected to the heat sink are a resistor $r = 1\,\Omega$ and a transistor Q. Assuming the shown voltages and currents, find P_1 and P_2 (the power dissipated by the two components).

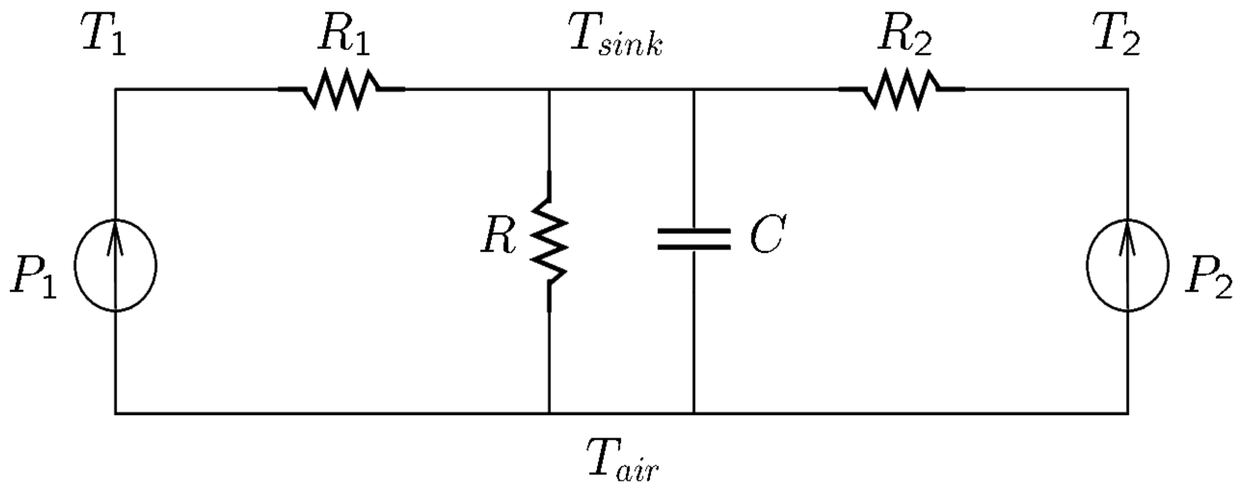


$P_1 = (rI)I = rI^2 = 25$ W.

$P_2 = V_{be}I_b + V_{ce}I_c = 0.07 + 30 \simeq 30$ W.

Example 6

Find the steady-state temperatures T_1 and T_2 when $P_1 = 25$ W, $P_2 = 30$ W, $R_1 = 2$ K/W, $R_2 = 1.5$ K/W, $R = 1$ K/W, and $T_{air} = 20\,°C$.

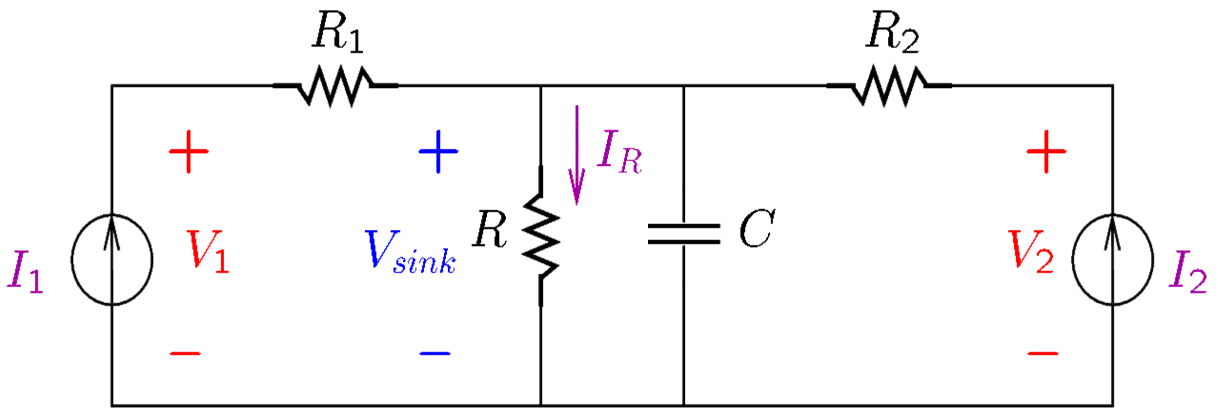


In the electric analogy of the heat transfer system:

- The powers P_1 and P_2 are represented by currents.
- Temperature differences are represented by voltages.
- Let us use the common V and I notation for the "voltages" and "currents" of the electric analogy.

Example 6 (Continued)

- Let's use the common V and I notation for the "voltages" and "currents" of the electric analogy:



$V_1 = T_1 - T_{air}$ $\quad I_1 = P_1$

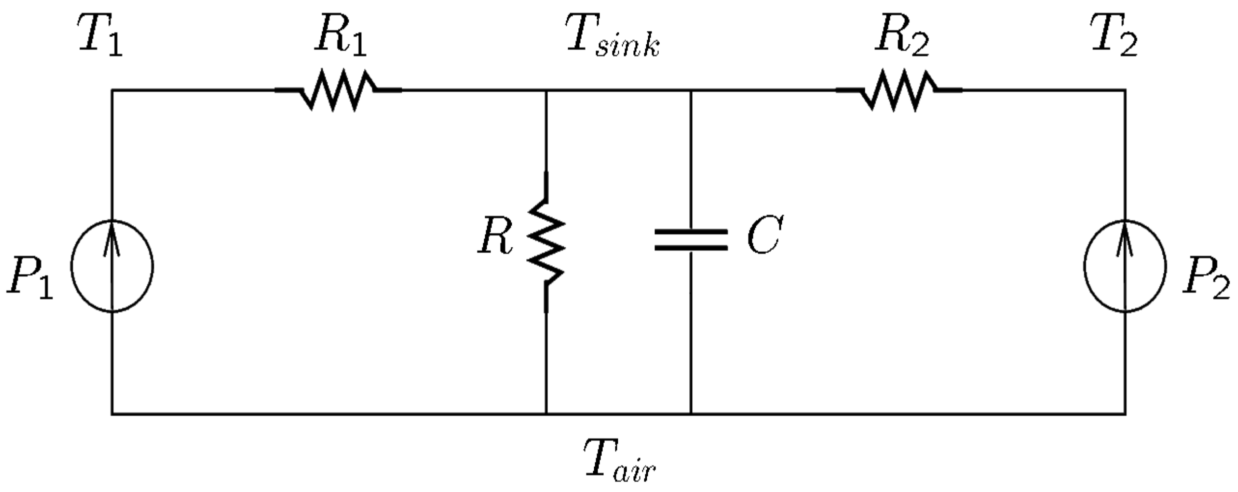
$V_2 = T_2 - T_{air}$ $\quad I_2 = P_2$

$V_{sink} = T_{sink} - T_{air}$

- At steady state, the capacitor current is zero.
- Therefore, $I_R = I_1 + I_2$, that is, $I_R = P_1 + P_2$.
- $V_{sink} = (I_1 + I_2)R$, that is, $T_{sink} - T_{air} = (P_1 + P_2)R \Rightarrow T_{sink} = 75\,°C$.
- $V_1 = I_1 R_1 + V_{sink}$, that is, $T_1 - T_{air} = P_1 R_1 + T_{sink} - T_{air} \Rightarrow T_1 = 125\,°C$.
- $V_2 = I_2 R_2 + V_{sink}$, that is, $T_2 - T_{air} = P_2 R_2 + T_{sink} - T_{air} \Rightarrow T_2 = 120\,°C$.

Example 7

If $R_1 = 2$ K/W, $R_2 = 1.5$ K/W, $R = 1$ K/W, and $C = 10\ J/K$, what is the time constant?



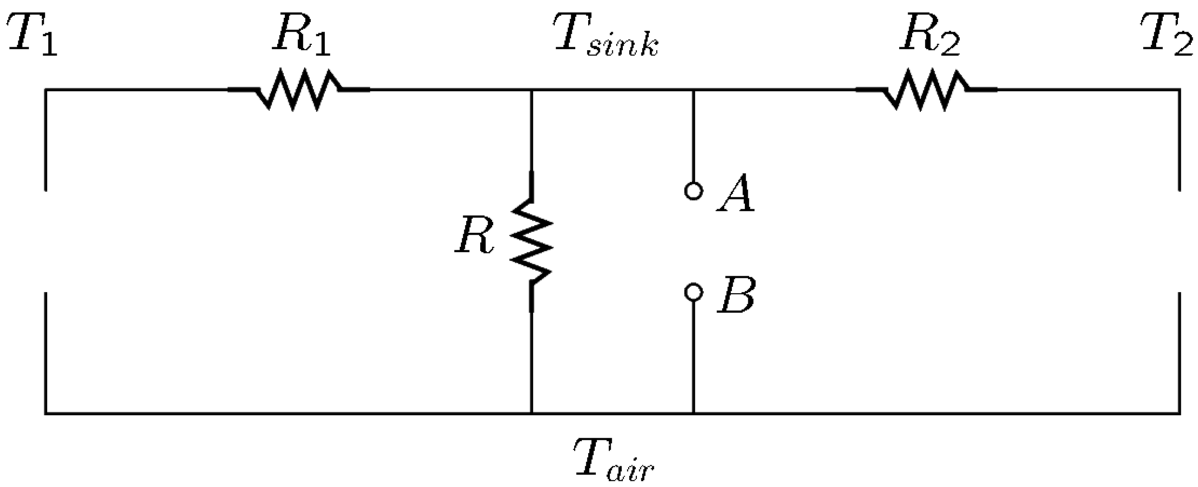
- When power is turned off, or when it is turned on, the temperatures will follow an equation of the form

$$T(t) = T(\infty) + \left(T(t_0) - T(\infty)\right)e^{-\frac{t-t_0}{\tau}}$$

  where $\tau = R_{th}C$ is the time constant.
- R_{th} is the Thevenin resistance seen by the capacitor C.

Example 7 (Continued)

- R_{th} is the resistance between the nodes of the capacitor when
  - Every independent source is set to zero.
  - The capacitor is removed from the circuit.
- Note that a current source of value zero is an open circuit.



- R_{th} is the resistance between points A and B.
- Since disconnected resistors do not matter, $R_{th} = R$ and so $\tau = 10$ s.

Interference

Interference

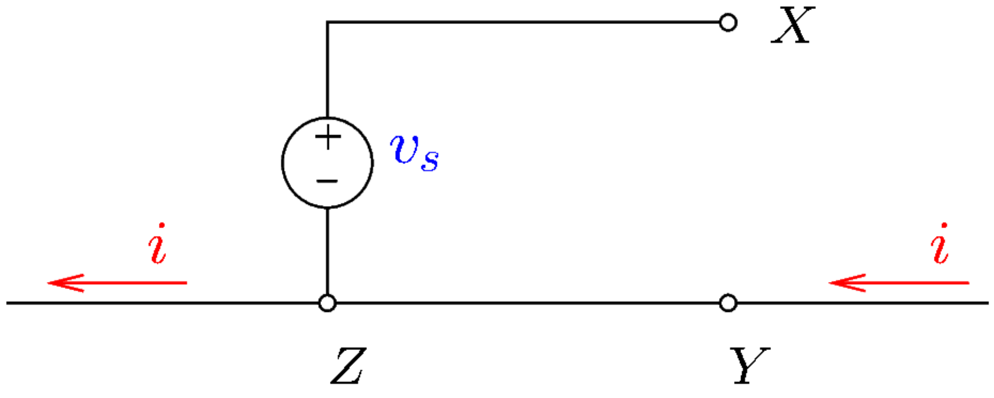
- In a measurement system, in addition to the signals that are measured, there are also unwanted electrical signals.
- Some unwanted signals are due to interference from nearby circuit networks.
- Interference signals are classified based on the source of interference and the way they are transmitted:
  - Capacitive interference
  - Inductive interference
  - Electromagnetic interference
  - Conductively coupled interference
  - Ground-loop interference
- At this time we will consider the *conductively coupled interference*.

Conductively Coupled Interference

- We normally assume that wires have zero resistance and thus zero voltage.
- In practice, however, the resistance of the conductors may not always be negligible.
- For example, a pulse of $2\ A$ flowing through a $100\ m\Omega$ wire will result in a voltage of 200 mV on the wire. *A 200 mV voltage is rarely negligible!*

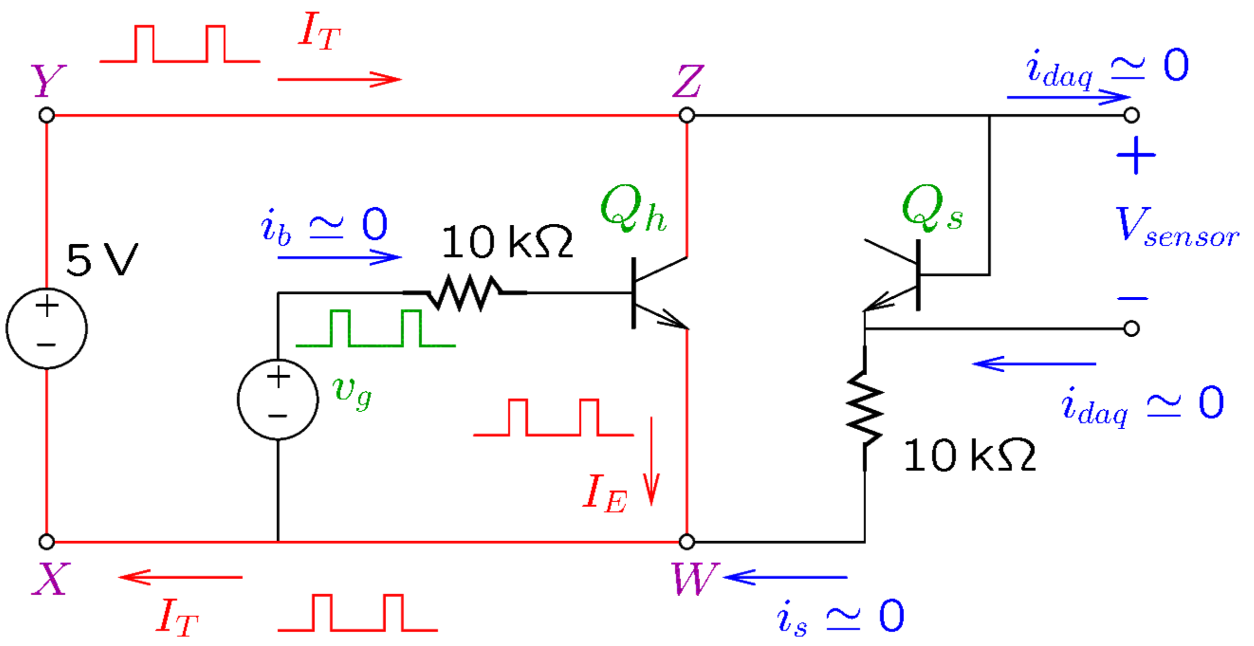
Example: A sensor outputs a voltage $v_s = 4$ mV. The sensor voltage is measured between the points X and Y. If the resistance of the conductor between Y and Z is $r = 10\ m\Omega$ and $i = 100\ mA$, what is the measured voltage?

- The measured voltage is
$$v_m = v_s + r \cdot i = 5\ mV.$$
- The measured voltage has 25% error!



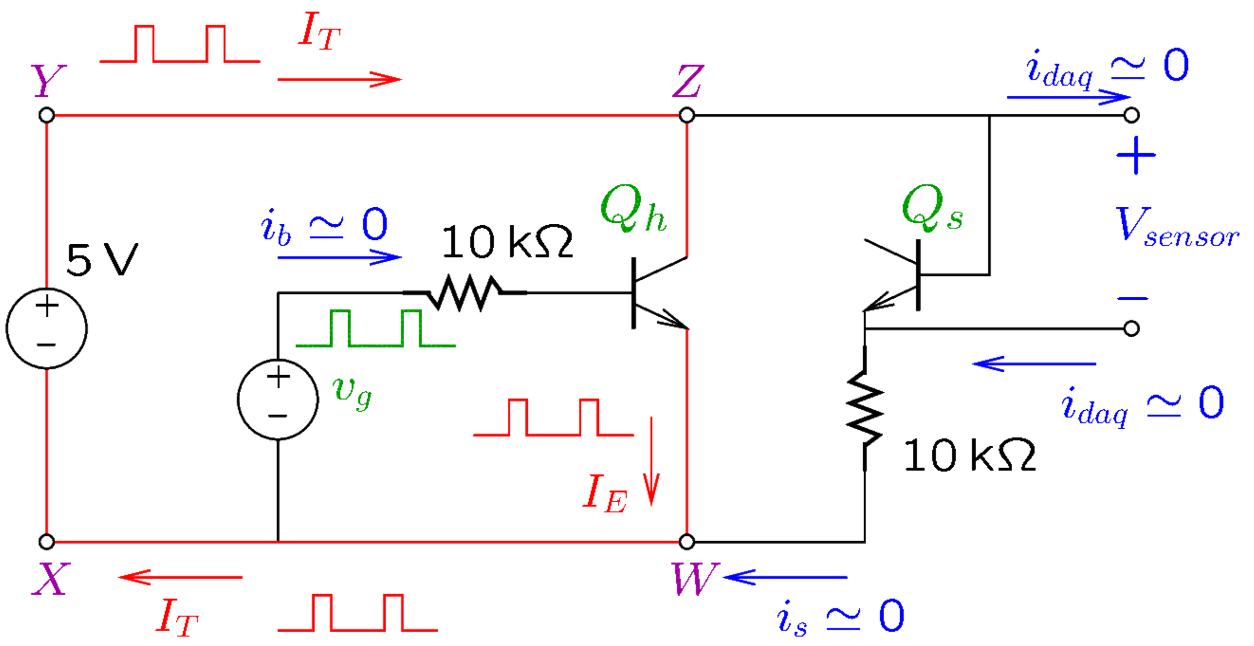
Example

- Consider a system that regulates the temperature of an object.
- The transistor Q_h is used as the heat element (to generate heat).
- The transistor Q_s is used as a temperature sensor.



- The voltage V_{sensor} of Q_s is measured with a data acquisition system (DAQ).
- The DAQ also sends a pulsating voltage v_g that turns on the heat element for the duration of a pulse.
- By adjusting the pulse width, it is possible to control the average power of the heat element.

Example (Continued)



- The current I_E of the heat element is large for the duration of a pulse, but zero otherwise.
- The total current $I_T \simeq I_E$.
- Temperature changes are measured by observing small changes in V_{sensor}.
- V_{sensor} depends on the voltage between Z and W.
- Ideally, the voltage between Z and W is constant, namely $5\ V$.
- In practice, due to the resistance $r \neq 0$ of the conductors on the path WXYZ, the voltage will be $5 - rI_T$. *The voltage will pulsate, since I_T is pulsating!*
- Therefore, temperature measurements will have significant error!

Example (Continued)

- A possible solution is to ensure that the sensor network does not use any of the conductors of the heat element (shown in red).
- To avoid interference, conductors that carry large amounts of current should not be part of a low-voltage network.