Data Acquisition Systems

Analog to Digital Converters, Digital to Analog Converters, Operational Amplifiers

M.V. Iordache, *EEGR2051 Circuits and Measurements Lab*, Fall 2020, LeTourneau University
See https://mviordache.name/EEGR2051 for more information.

Interfacing Analog and Digital Systems

- A *multifunction I/O device*, such as PCIe-6321 inside the lab computers, can
  - Digitize analog signals from the physical world and transmit them to computer software (such as LabVIEW).
  - Convert software-generated digital signals to physical analog signals.
- Some of the most important components needed to interface computers to the physical world are:
  - Analog to digital converters.
  - Digital to analog converters.
  - Electronic amplifiers.

PCIe-6321 from National Instruments
https://www.ni.com/en-us/support/model.pcie-6321.html

Number Representation

- The internal representation of numbers in digital systems is in base 2.
- Each digit of a base-two number is called *bit*.
- An n-bit base-2 number $b_{n-1}b_{n-2} \dots b_2b_1b_0$ corresponds in base 10 to a number

$$N = b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \cdots + b_2 2^2 + b_1 2^1 + b_0 2^0$$

For example, to convert the base-2 number $1011\ 0011$ *to base 10:*

- *Count the digits of the number from right to left, beginning with 0.*

7 6 5 4 3 2 1 0

1 0 1 1 0 0 1 1

- *Note that* $N = 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$.
- *The answer is* $N = 179$.

Number Representation

For example, let's write the number 138 in base 2:

- *Note that* $138 = 1 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$.
- *Therefore, the base-2 number is* $1000\ 1010$.
- *The decomposition into a sum of powers of 2 could be obtained by successive divisions by 2 and by reading the remainders in reverse order:*

$$
\begin{aligned}
138 \div 2 &= 69 \textit{ with remainder of } 0 && \Rightarrow b_0 = 0.\\
69 \div 2 &= 34 \textit{ with remainder of } 1 && \Rightarrow b_1 = 1.\\
34 \div 2 &= 17 \textit{ with remainder of } 0 && \Rightarrow b_2 = 0.\\
17 \div 2 &= 8 \textit{ with remainder of } 1 && \Rightarrow b_3 = 1.\\
8 \div 2 &= 4 \textit{ with remainder of } 0 && \Rightarrow b_4 = 0.\\
4 \div 2 &= 2 \textit{ with remainder of } 0 && \Rightarrow b_5 = 0.\\
2 \div 2 &= 1 \textit{ with remainder of } 0 && \Rightarrow b_6 = 0.\\
1 \div 2 &= 0 \textit{ with remainder of } 1 && \Rightarrow b_7 = 1.
\end{aligned}
$$

Number Representation

Consider a base-2 number.

- The *most significant bit (msb)* is the first bit.

1001 0010

- The last bit is the *least significant bit (lsb)*.

Digital to Analog Converters

- Digital instruments represent voltages with numbers.
- *Digital to Analog Converters (DACs)* convert numbers N to voltages.
- Given a number N, a DAC will output a voltage $\boxed{V_o = V_{LO} + step \cdot N}$ where
  - V_{LO} is the *low reference voltage*; typically, $V_{LO} = 0$.
  - $step$ is the *voltage resolution* of the converter, also known as *LSB*.
  - The voltage resolution is the amount by which the output voltage V changes when the least significant bit of N changes.
- If V_{HI} is the *high reference voltage* and the DAC works with m bit numbers, that is, m is the *bit resolution* of the DAC, then:

$$\boxed{step = \frac{V_{HI} - V_{LO}}{2^m}}$$

Examples

1) A DAC has a $20\ mV$ *voltage resolution. Assuming* $V_{LO} = 0$*, find the output voltage when the input is* $N = 0110\ 0011$.

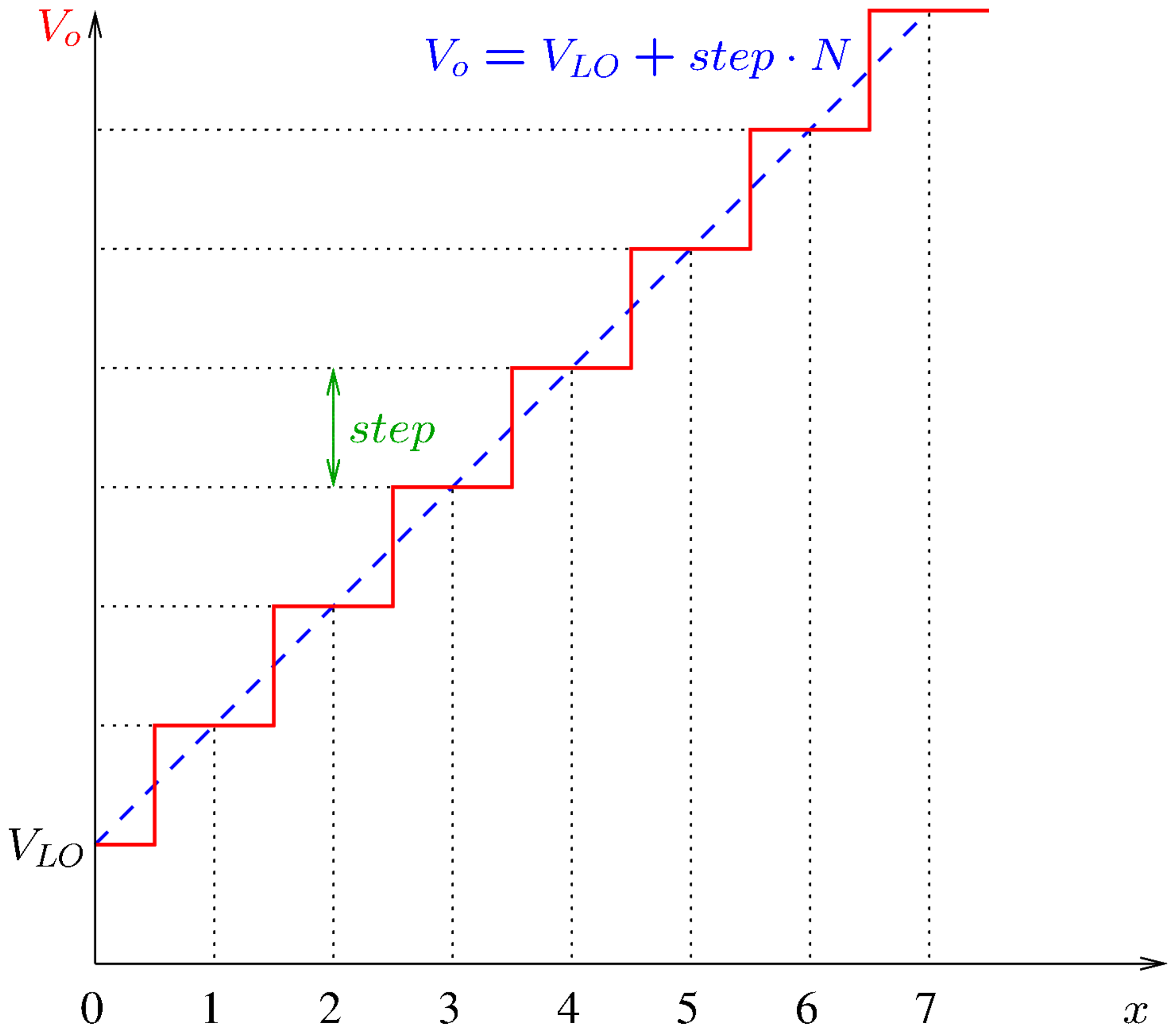
- *In base 10,* $N = 2^6 + 2^5 + 2^1 + 2^0 \Rightarrow N = 99$.
- Since $step = 20\ mV$ and $V_o = V_{LO} + step \cdot N$, *it follows that* $V_o = 1.98\ V$.

2) A 10-bit DAC has $V_{LO} = 0$ *and* $V_{HI} = 15\ V$. *Find* N *for which* $V_o = 3.456\ V$.

- *Since* $step = \frac{V_{HI} - V_{LO}}{2^m}$ *and* $m = 10$, *it follows that* $step = \frac{15}{2^{10}}\ V$.
- *Note that* $3.456 = step \cdot N \Rightarrow N = 235.93$.
- *Rounding to the nearest integer,* $N = 236$.

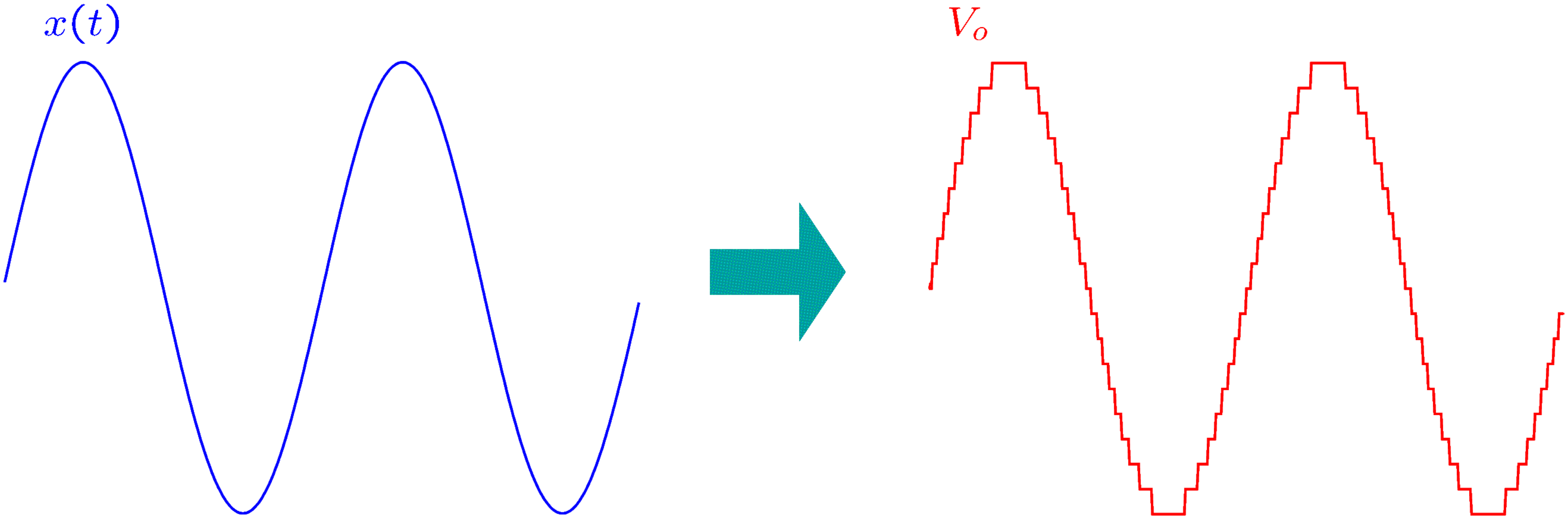
Digital to Analog Converters

- A signal $x(t)$, when applied to a DAC, will have its values rounded to the nearest integer N.
- Note that the output of the DAC will change in steps.



Digital to Analog Converters

- A signal $x(t)$, when applied to a DAC, will have its values rounded to the nearest integer N.
- Note that the output of the DAC will change in steps.



Analog to Digital Converters

- Digital to Analog Converters convert integer numbers to voltages.
- Analog to Digital Converters (ADCs) perform the converse operation, as they convert voltages to integer numbers.
- If an ADC outputs the number N, the input voltage V_{in} satisfies

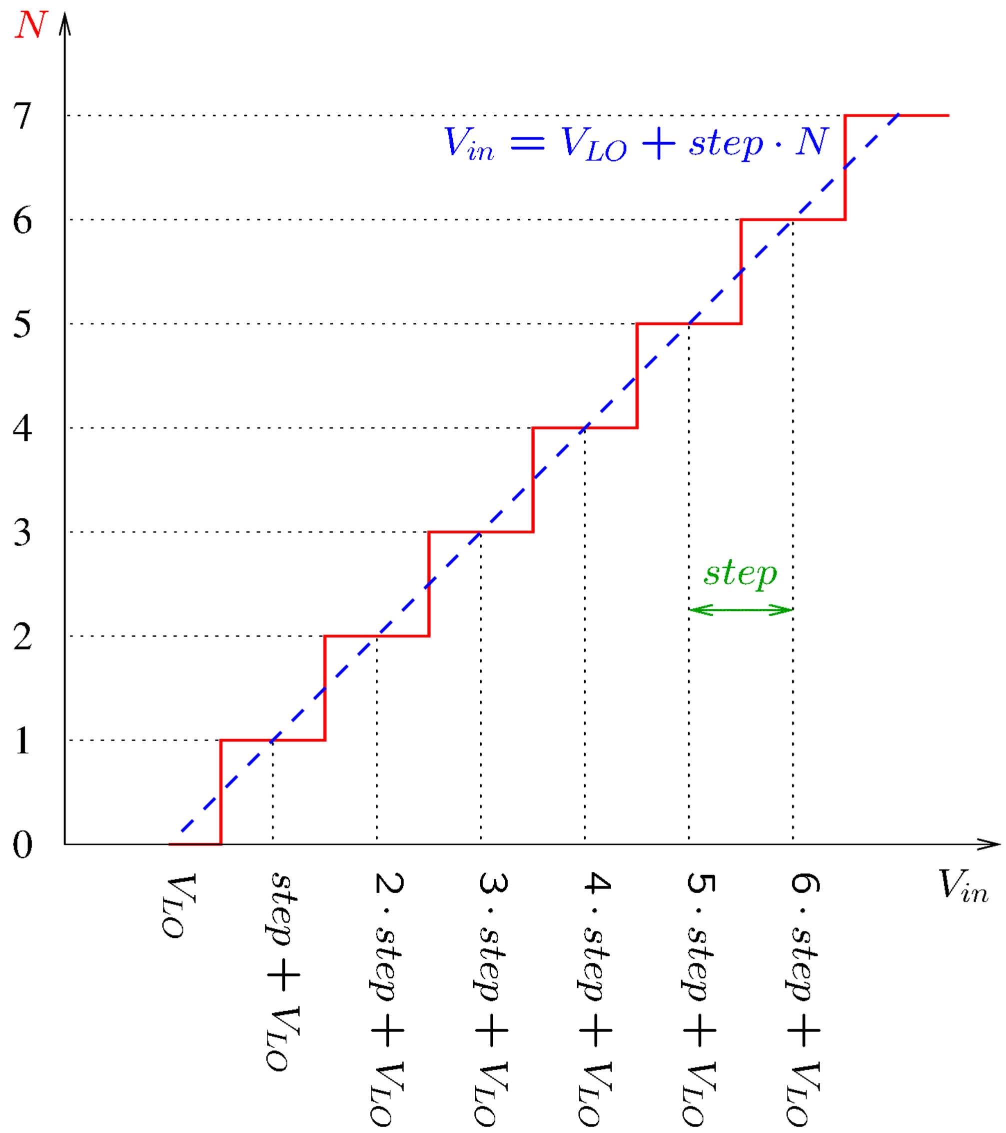
$$\boxed{V_{in} = V_{LO} + step \cdot N \pm \frac{step}{2}}$$

  - V_{LO} is the *low reference voltage*; typically, $V_{LO} = 0$.
  - $step$ is the *voltage resolution* of the converter, also known as *LSB*.
  - If V_{HI} is the *high reference voltage* and the DAC works with m bit numbers, that is, m is the *bit resolution* of the DAC, then:

$$\boxed{step = \frac{V_{HI} - V_{LO}}{2^m}}$$

Analog to Digital Converters

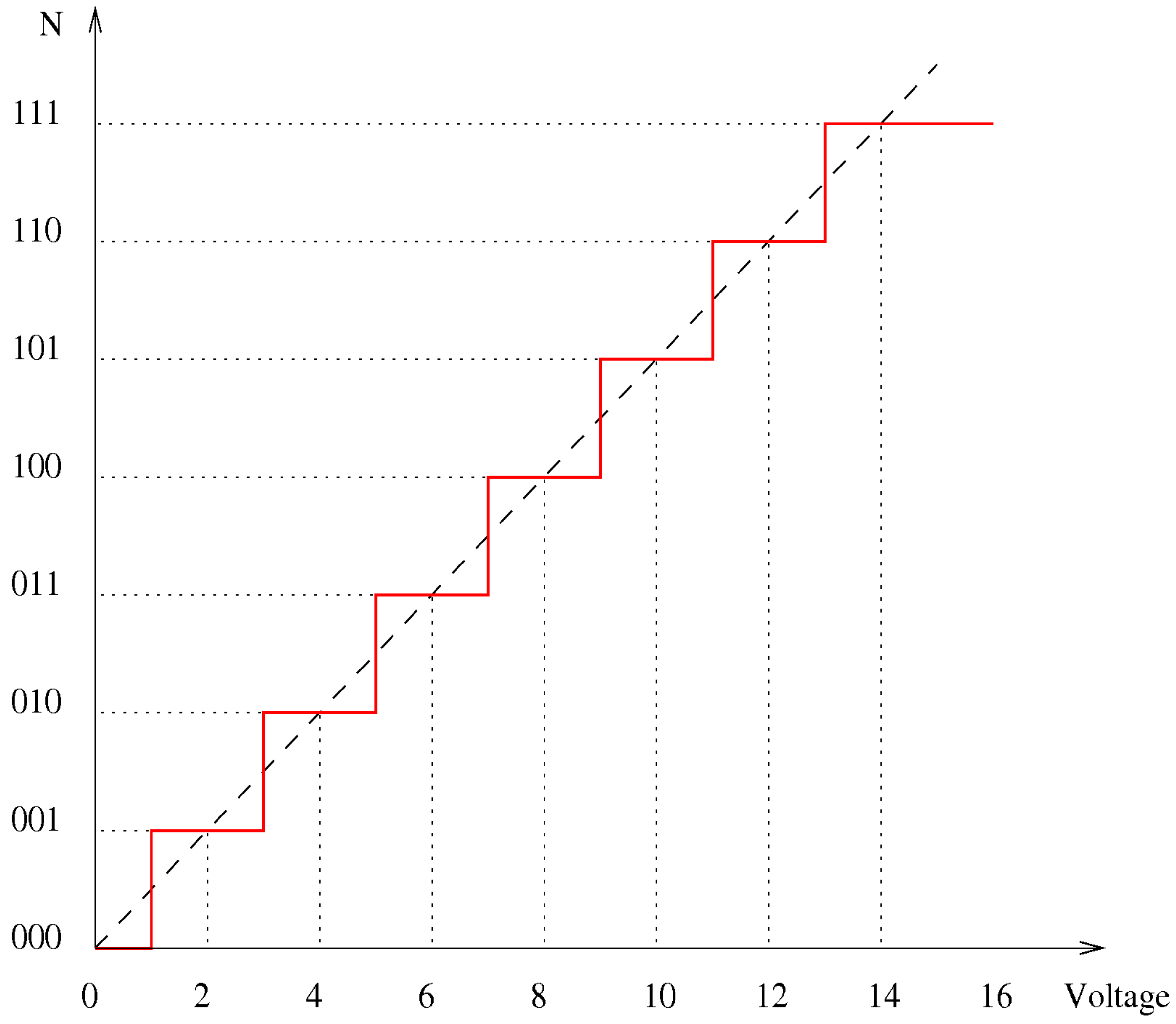
- The range of N is $0 \dots 2^m - 1$.
- As the graph shows, the equation $V_{in} = V_{LO} + step \cdot N$ predicts the input voltage V_{in} with an uncertainty of $\pm step/2$.
- $\pm step/2$ is the ideal uncertainty.
- A practical ADC has sources of error that make the uncertainty larger.



Example

The ADC shown in the figure outputs the following base-2 numbers: 000, 001, 010, 011, 100, 101, 110, and 111. Find V_{LO} and V_{HI}.

- *N has 3 digits in base 2, so $m = 3$.*
- *The line representing the equation $V_{in} = V_{LO} + step \cdot N$ goes through the origin, so $V_{LO} = 0$.*
- *The graph indicates that $step = 2\ V$.*
- *From $step = \frac{V_{HI} - V_{LO}}{2^m}$ we derive $V_{HI} = 16\ V$.*



Examples

1) An 8-bit ADC has $V_{LO} = 0$ *and* $V_{HI} = 10\ V$. *What is* V_{in} *when* $N = 20$?

- $step = \frac{V_{HI} - V_{LO}}{2^m} = 39.06\ mV.$
- $V_{in} = V_{LO} + step \cdot N \pm \frac{step}{2} \Rightarrow V_{in} = 781.25 \pm 19.53\ mV.$

2) Repeat the previous example assuming a 12-bit ADC.

- $step = \frac{V_{HI} - V_{LO}}{2^m} = 39.06\ mV.$
- $V_{in} = V_{LO} + step \cdot N \pm \frac{step}{2} \Rightarrow V_{in} = 48.83 \pm 1.22\ mV.$

Ideally, uncertainty decreases exponentially with the number of bits.

Differential Channels

- The voltage of a *single-ended channel* is measured with respect to GND.
- In a *differential channel*, voltage is measured with respect to an arbitrary reference.
- So far, we have considered ADCs with single-ended channels.
- ADCs are often used with differential channels, such as in the lab.
- With a differential channel, N may be positive or negative and is in the range $-2^{m-1} \ldots 2^{m-1} - 1$.
- V_{in} also may be positive or negative; it can be calculated with

$$V_{in} = step \cdot N \pm \frac{step}{2}, \text{ where } step = \frac{V_{HI} - V_{LO}}{2^{m-1}}$$

- Note 2^{m-1} instead of 2^m in the previous formula.

The Nyquist Frequency

- Note that any signal can be decomposed into a sum of sinusoidal components, each with a different frequency.
- Consider an ADC that samples an input signal at the rate of f_s samples per second.
- If f_{max} is the maximum frequency of the signal components, then the input signal can be reconstructed correctly from its samples if

$$\boxed{f_s \geq 2f_{max}}$$

- $f_N = 2f_{max}$ is known as the *Nyquist frequency*.

Example

A 12-bit successive approximation ADC operates at a clock frequency $f = 5\ MHz$*. What is the maximum frequency of the signal components for which the signal can be reconstructed correctly from its samples?*

- *Note that the conversion time of a successive approximation ADC is* $T_c \simeq \frac{m}{f}$*, where m is the number of bits.*
- *The maximum rate at which the ADC can sample the input is* $f_{s,max} = \frac{1}{T_c}$*.*
- *Sampled signals can be reconstructed correctly from their samples if* $f_s \geq 2f_{max}$*.*
- *We derive that* $f_{max} \leq \frac{f_s}{2} \leq \frac{f_{s,max}}{2} \simeq \frac{f}{2m}$*.*
- *Numerically,* $f_{max} \leq 208.33\ kHz$*.*

Electronic Amplifiers

- Amplifiers are used to increase the voltage or power of an input signal.
- If V_{out} is the amplitude of the output signal and V_{in} the amplitude of the input signal, then the voltage gain of the amplifier is

$$A = \frac{V_{out}}{V_{in}}$$

- The decibel gain of the amplifier is

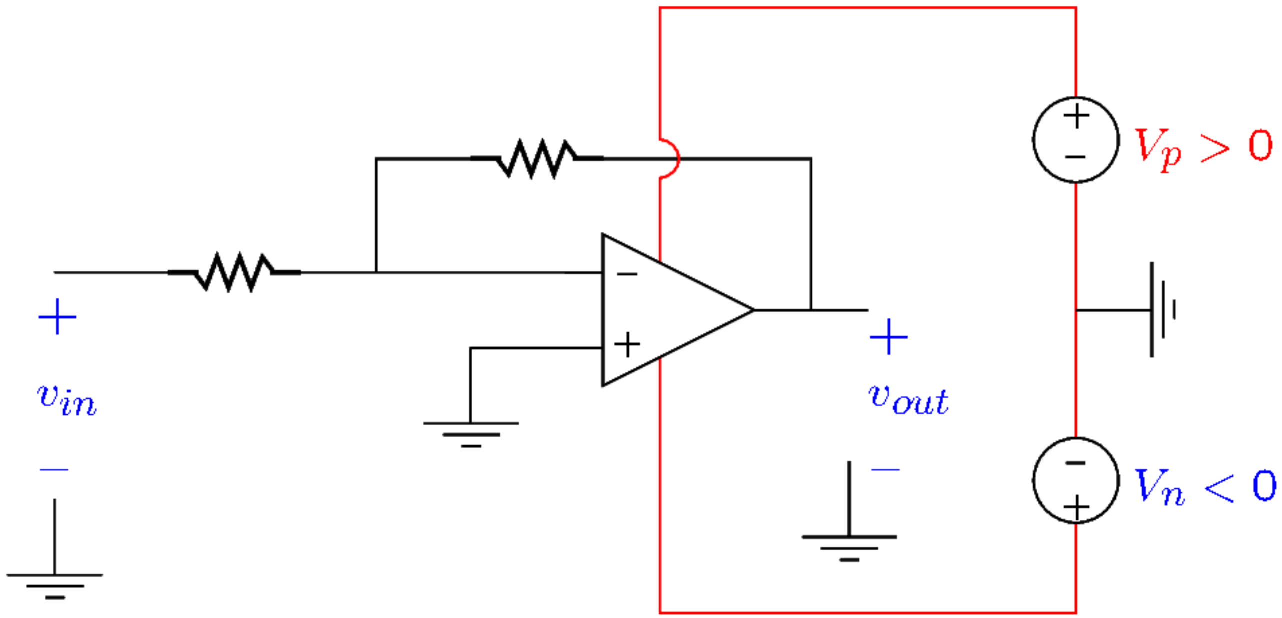
$$A_{dB} = 20 \cdot \log_{10} \frac{V_{out}}{V_{in}}$$

Example: A $3\ mV$ peak-to-peak signal is applied to a $40\ dB$ amplifier. What is the peak-to-peak amplitude of the amplifier output?

$$40\ dB = 20 \cdot \log_{10} \frac{V_{out}}{3\ mV} \Rightarrow V_{out} = 300\ mV$$ *peak-to-peak.*

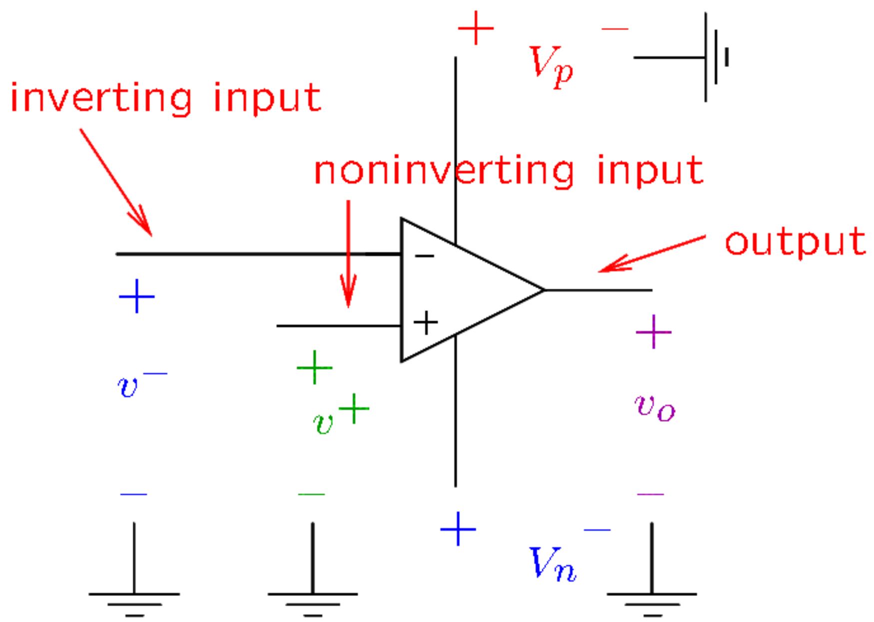
Operational Amplifiers

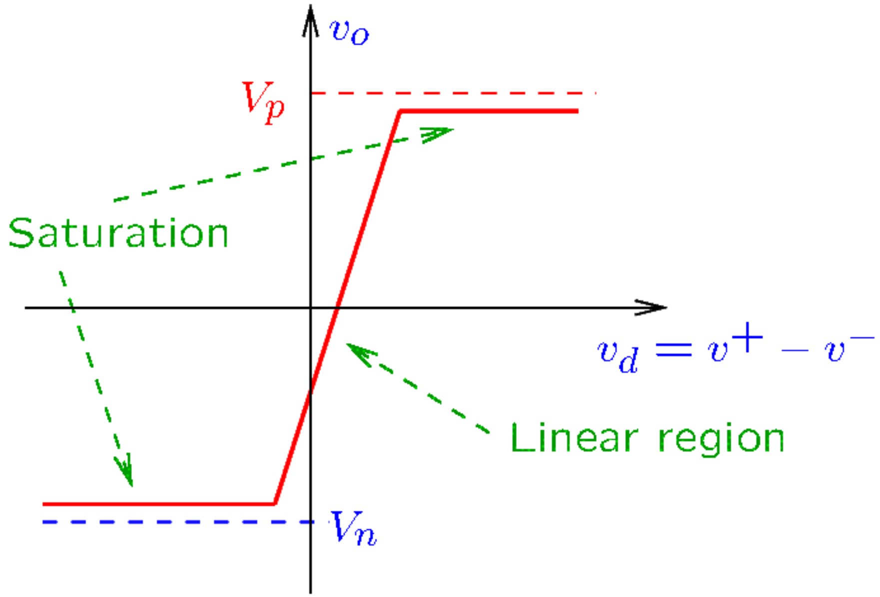
- The figure shows an amplifier circuit that uses an operational amplifier.
- Operational amplifiers are commonly used to amplify signals.
- They have pins for the input and output signals (shown in black) and supply pins (shown in red) used to power the operational amplifier.



Operational Amplifiers

- Let v^+ be the *noninverting input* voltage and v^- the *inverting input* voltage.
- Let V_p be the voltage on the positive supply pin and V_n the voltage on the negative supply pin.
- In *saturation*, the output voltage is $v_o \simeq V_p$ or $v_o \simeq V_n$.
- In the *linear region*, $v_o \simeq A(v^+ - v^-)$, where A is the operational amplifier gain.





Operational Amplifiers

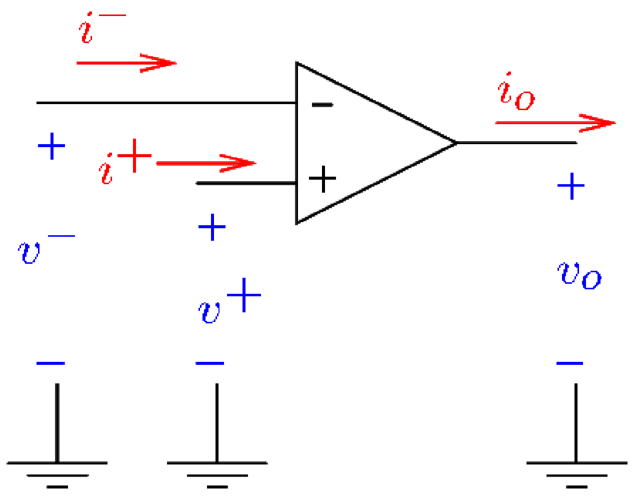
- Because the gain A is typically very large:

  $$v^+ \simeq v^- \text{ in the linear region}.$$

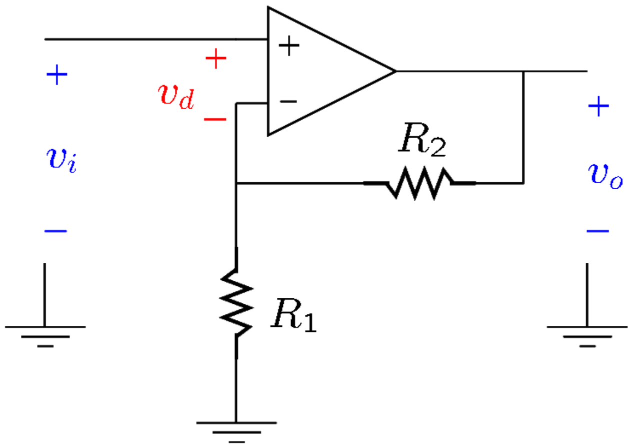
- The input currents are normally negligible:

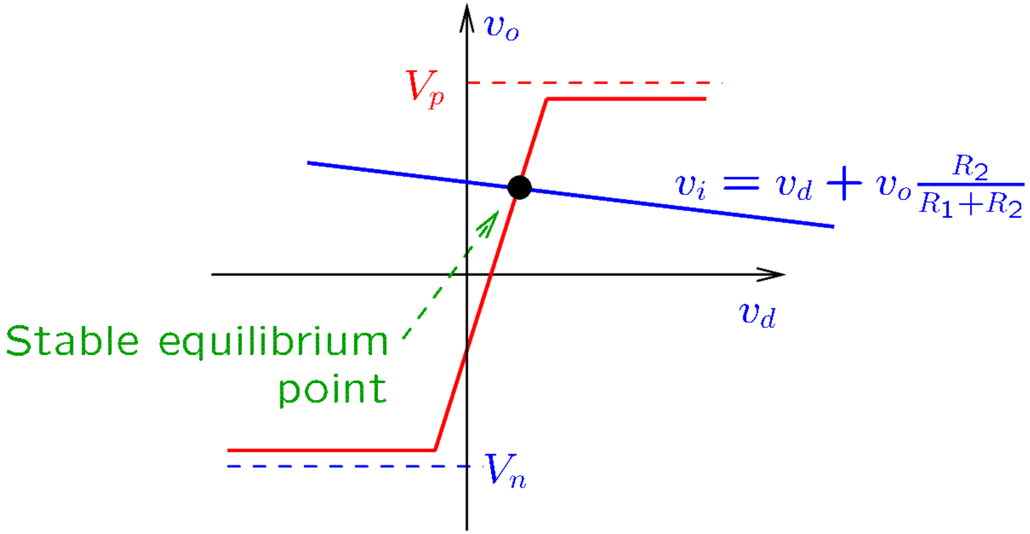
  $$i^+ \simeq i^- \simeq 0.$$

- The equations $v^+ = v^-$ and $i^+ = i^- = 0$ can be used to solve operational amplifier circuits.



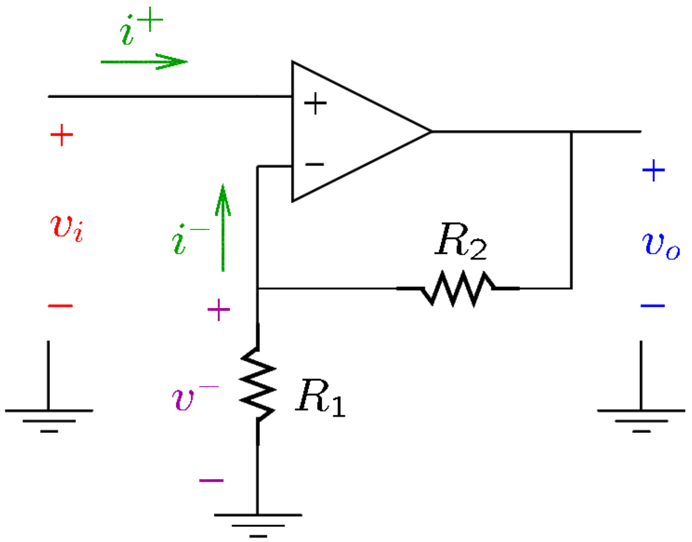
The Noninverting Amplifier

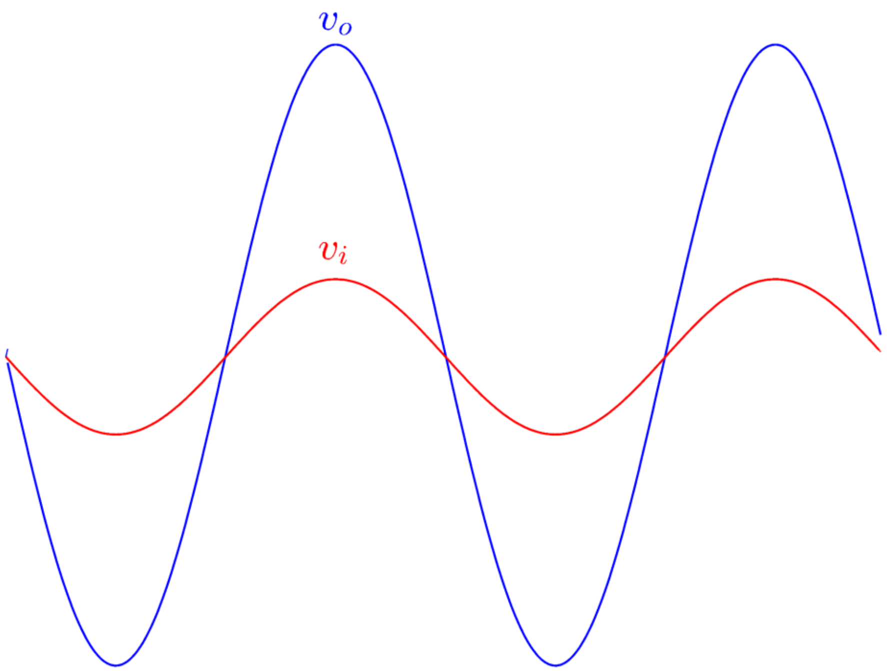




- Operational amplifiers are normally used with *feedback*.
- A feedback network connects the output of the amplifier to the input so as to ensure that the operational amplifier works in the linear region.
- In the figure, the resistor R_2 connects the output of the amplifier to the inverting input, creating in this way feedback.
- The circuit shown in the figure is known as the *noninverting amplifier*.

The Noninverting Amplifier



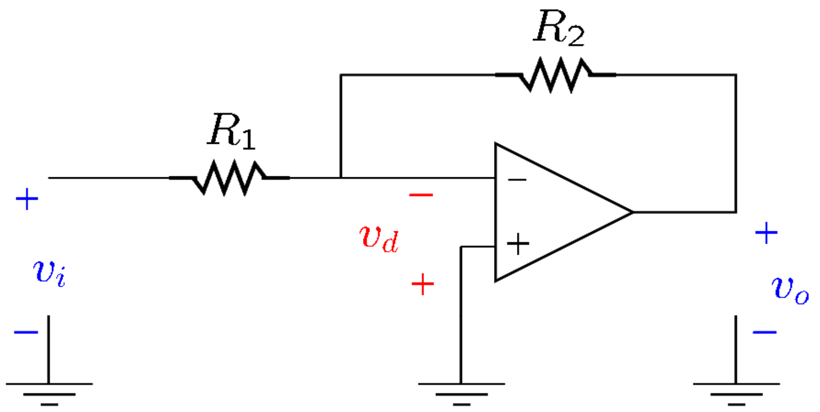


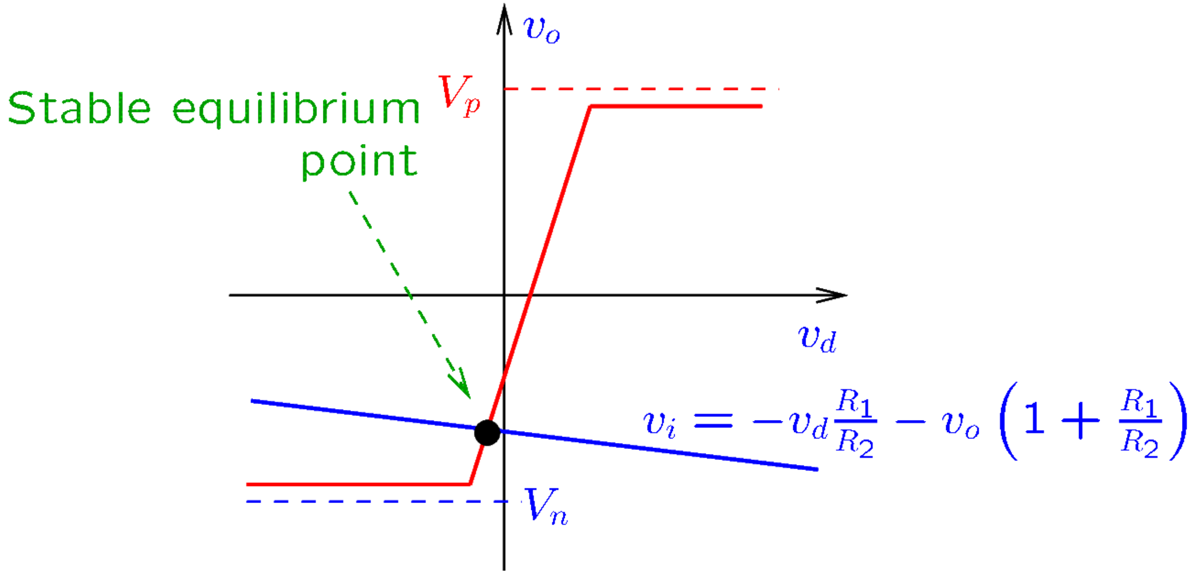
- The gain of the noninverting amplifier is

$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}$$

- The formula can be easily proven:
  - Note that $v^+ = v_i$.
  - $i^- = 0 \Rightarrow v^- = v_o \frac{R_1}{R_1+R_2}$
  - $v^+ = v^- \Rightarrow v_i = v_o \frac{R_1}{R_1+R_2}$, from which the gain formula follows.

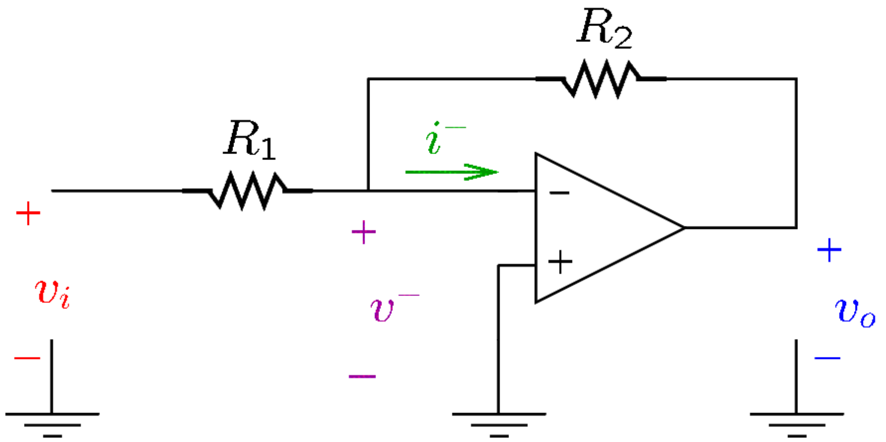
The Inverting Amplifier

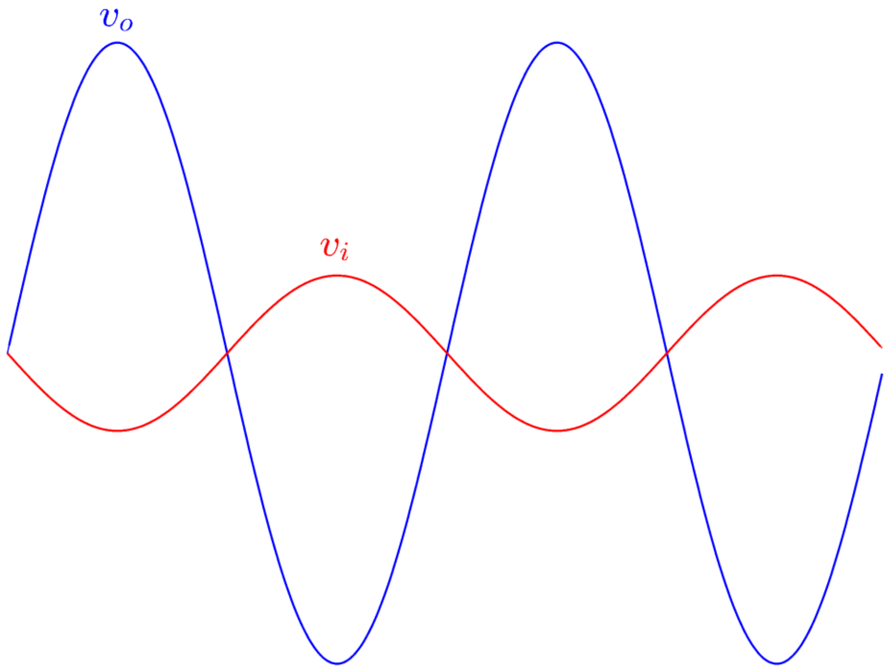




- In the figure, the resistor R_2 connects the output of the amplifier to the inverting input, creating in this way feedback.
- This feedback ensures that the operational amplifier works in the linear region.
- The circuit shown in the figure is known as the *inverting amplifier*.

The Inverting Amplifier



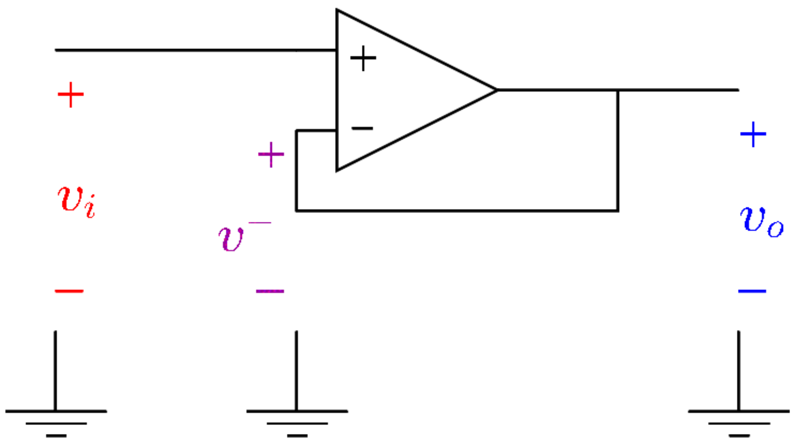


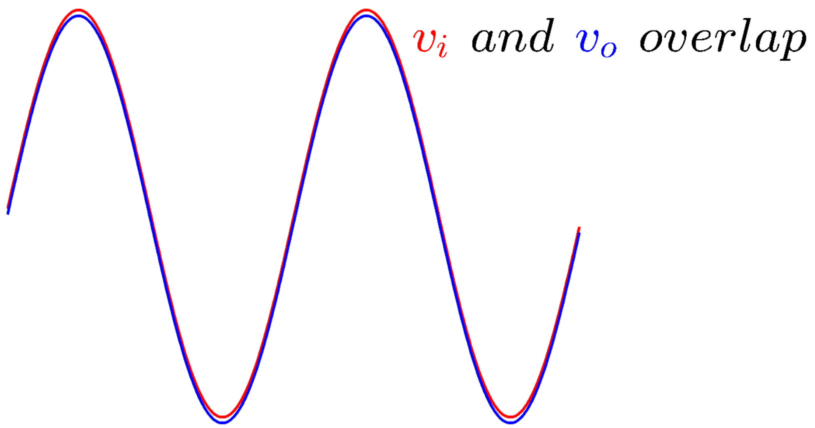
- The gain of the inverting amplifier is

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

- The formula can be easily proven:
  - The nodal equation is $\frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = i^-$.
  - The gain formula follows immediately by substituting $i^- = 0$ and $v^- = v^+ = 0$.

The Voltage Follower



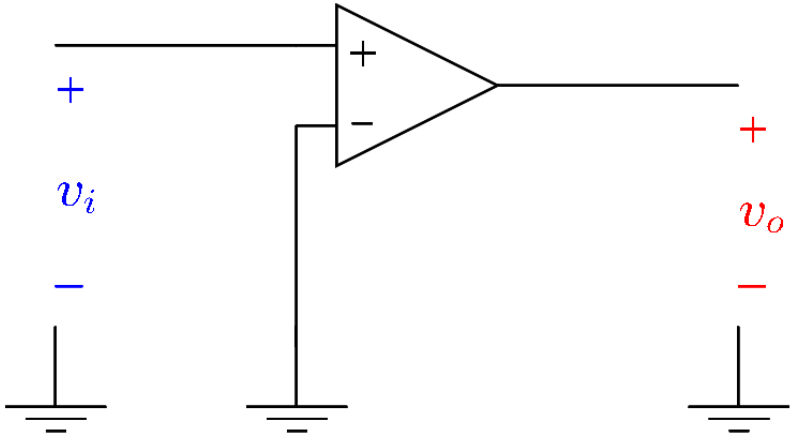


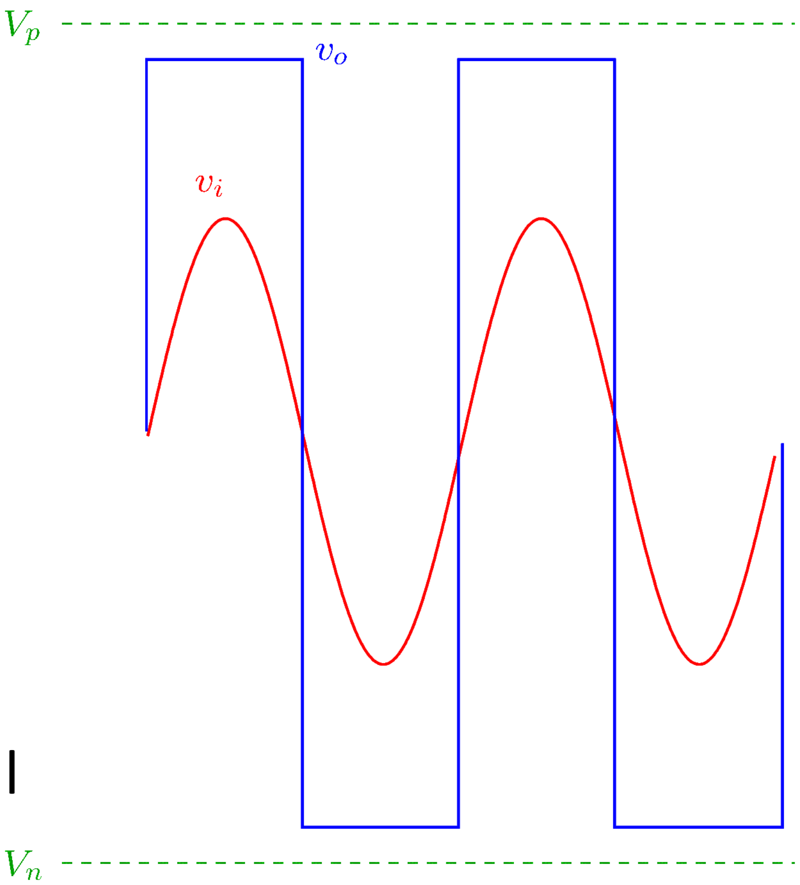
- Feedback ensures that the operational amplifier is in the linear region.
- The circuit shown in the figure is known as a *voltage follower*.
- The voltage follower has the gain

$$\frac{v_o}{v_i} = 1$$

- It has a negligible input current, while delivering the required output current.
- The voltage gain formula can be easily proven:
  - $v_i = v^+$, $v^+ = v^-$, and $v^- = v_o$ imply $v_o = v_i$.

Voltage Comparator





- Since there is no feedback, the operational amplifier will be saturated.
- Let V_p be the positive supply voltage of the operational amplifier and V_n the negative supply voltage.
- If $v_i > 0$, the operational amplifier will output maximum voltage; $v_o \simeq V_p$.
- If $v_i < 0$, the operational amplifier will output the most negative voltage; $v_o \simeq V_n$.
- The circuit compares the voltage of the noninverting input to the voltage of the inverting input and outputs V_p if the former is larger and V_n if smaller.